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Problem 1
Multiply 3.4×10^{-5} by 7.1×10^{8} and leave the answer in the standard form.
Possible answers:
A. 2.414×10^{2}
B. 2.414×10^{3}
C. 2.414×10^{4}
D. 2.414×10^{5}
Correct answer: C
Solution. To solve the problem of multiplying 3.4×10^{-5} by 7.1×10^{8} and expressing the result in standard form, follow these steps:
1. **Multiply the coefficients**:
$3.4 \times 7.1 = 24.14$.
2. **Multiply the powers of ten**:
$10^{-5} \times 10^{8} = 10^{-5+8} = 10^{3}$.
3. **Combine the results**:
24.14×10^{3}.
4. **Convert to standard form**:
Adjust the coefficient 24.14 to a value between 1 and 10 by moving the decimal one place to the left, resulting in 2.414. Compensate by increasing the exponent by 1:
$2.414 \times 10^{3+1} = 2.414 \times 10^{4}$.
Final Answer:
$\boxed{C}$ (2.414×10^{4}).
Main Concepts:
- **Scientific Notation**: Separating numbers into coefficients and powers of ten for easier multiplication.
- **Exponent Rules**: Adding exponents when multiplying like bases ($10^{-5} \times 10^{8} = 10^{3}$).
- **Standard Form Adjustment**: Ensuring the coefficient is between 1 and 10 by adjusting the decimal and updating the exponent accordingly.

Problem 2

Given that $P = \{p : 1 < p < 20\}$, where p is an integer and $R = \{r : 0 \leq r \leq 25$, where r is a multiple of 4$\}$. Find $P \cap R$.

Possible answers:

A. $\{4, 8, 10, 16\}$

B. $\{4, 8, 12, 16\}$

C. $\{4, 8, 12, 16, 20\}$

D. $\{4, 8, 12, 16, 20, 24\}$

Correct answer: B

Solution. To solve the problem, we first identify the elements of sets P and R:

- **Set P:** Contains integers p where $1 < p < 20$. Thus, $P = \{2, 3, 4, \ldots, 19\}$.
- **Set R:** Contains multiples of 4 between 0 and 25. Thus, $R = \{0, 4, 8, 12, 16, 20, 24\}$.

The intersection $P \cap R$ includes elements common to both sets. Comparing the elements:

- $4, 8, 12, 16$ are in both P and R.
- $0, 20, 24$ are excluded from P, and other elements of P are not multiples of 4.

Thus, $P \cap R = \{4, 8, 12, 16\}$, which corresponds to **Option B**.

Main Concepts:

1. **Set Definition Comprehension**: Interpreting inequalities and multiple conditions to define sets.
2. **Element Enumeration**: Listing valid elements for each set based on constraints.
3. **Intersection Identification**: Finding common elements between sets by direct comparison.
4. **Answer Validation**: Cross-checking results against provided options to ensure correctness.

Problem 3

The first term of an arithmetic progression is 2 and the last term is 29. If the common difference is 3 , how many terms are in the arithmetic progression?
Possible answers:
A. 8
B. 9
C. 10
D. 11
Correct answer: C
Solution. The number of terms in the arithmetic progression is **10** (Option C).
Step-by-Step Explanation:
1. **Identify the given values**:
- First term $(a_1) = 2$
- Last term $(a_n) = 29$
- Common difference $(d) = 3$
2. **Use the formula for the n-th term of an arithmetic progression**:

$$a_n = a_1 + (n-1)d$$

Substitute the known values:

$$29 = 2 + (n-1)(3)$$

3. **Solve for n**:
- Subtract 2 from both sides:

$$27 = (n-1)(3)$$

- Divide by 3:

$$9 = n - 1$$

- Add 1 to both sides:

$$n = 10$$

Verification:
Listing terms confirms the 10th term is 29:
$2, 5, 8, 11, 14, 17, 20, 23, 26, 29.$
Main Concepts:
- **Arithmetic Progression (AP)**: A sequence with a constant difference between consecutive terms.
- **n-th Term Formula**: $a_n = a_1 + (n-1)d$, where a_1 is the first term, d is the common difference, and n is the number of terms.
- **Linear Equation Solving**: Isolate n using algebraic operations to determine the total number of terms.
Answer: C. 10

Problem 4

Express in index form: $\log_a^x + \log_a^y = 3$.

Possible answers:

A. $x + y = 3$

B. $xy = 3$

C. $xy = a^3$

D. $x + y = a^3$

Correct answer: C

Solution. To solve the equation $\log_a x + \log_a y = 3$ in index form, we use the **logarithm product rule**:

$$\log_a x + \log_a y = \log_a(xy).$$

This simplifies the equation to:

$$\log_a(xy) = 3.$$

Converting this to **exponential form** (since $\log_b c = d \implies b^d = c$), we get:

$$a^3 = xy \quad \text{or equivalently} \quad xy = a^3.$$

Thus, the correct answer is **C. $xy = a^3$**.

Main Concepts:

1. **Logarithm Product Rule**: Combining the sum of logarithms with the same base into the logarithm of a product.

2. **Logarithmic to Exponential Form**: Rewriting $\log_b c = d$ as $b^d = c$ to solve for variables in exponential terms.

Problem 5

Simplify: $(2p-q)^2-(p+q)^2$.

Possible answers:

A. $3p(p-2q)$

B. $2p(p-3q)$

C. $3p(2p-q)$

D. $2p(3p-q)$

Correct answer: A

Solution. To simplify the expression $(2p-q)^2-(p+q)^2$, we can use the difference of squares formula, $a^2-b^2=(a-b)(a+b)$, where $a=2p-q$ and $b=p+q$.

1. **Apply the difference of squares**:

$$(2p-q)^2-(p+q)^2=[(2p-q)-(p+q)][(2p-q)+(p+q)]$$

2. **Simplify the terms**:

- For $(2p-q)-(p+q)$:

$$2p-q-p-q=p-2q$$

- For $(2p-q)+(p+q)$:

$$2p-q+p+q=3p$$

3. **Multiply the simplified terms**:

$$(p-2q)(3p)=3p(p-2q)$$

Final Answer: $\boxed{A}$

Main Concepts:

1. **Difference of Squares**: Recognizing the structure a^2-b^2 allows factoring into $(a-b)(a+b)$.

2. **Expanding Binomials**: Alternatively, expanding $(2p-q)^2$ and $(p+q)^2$ directly and simplifying.

3. **Factoring**: Identifying common factors $(3p)$ in the simplified expression $3p^2-6pq$.

4. **Combining Like Terms**: Combining terms after expansion to reach the final simplified form.

Both methods confirm that the correct answer is **A. $3p(p-2q)$**.

Problem 6

If $(3-4\sqrt{2})(1+3\sqrt{2}) = a+b\sqrt{2}$, find the value of b.
Possible answers:
A. -5
B. 5
C. -21
D. 21
Correct answer: B

Solution. To solve the problem $(3-4\sqrt{2})(1+3\sqrt{2}) = a+b\sqrt{2}$ and find the value of b, we use the distributive property (FOIL method):
1. **First**: $3 \cdot 1 = 3$
2. **Outer**: $3 \cdot 3\sqrt{2} = 9\sqrt{2}$
3. **Inner**: $-4\sqrt{2} \cdot 1 = -4\sqrt{2}$
4. **Last**: $-4\sqrt{2} \cdot 3\sqrt{2} = -12 \cdot 2 = -24$
Combine the results:
- Constant terms: $3 - 24 = -21$
- Radical terms: $9\sqrt{2} - 4\sqrt{2} = 5\sqrt{2}$
This simplifies to $-21 + 5\sqrt{2}$. Comparing to $a + b\sqrt{2}$, we find $b = 5$.
Answer: B. 5
Main Concepts:
- **Distributive Property (FOIL)**: Expanding binomials by multiplying terms systematically.
- **Combining Like Terms**: Separating and simplifying constants and radical terms.
- **Radical Multiplication**: Using $\sqrt{2} \cdot \sqrt{2} = 2$ to simplify products.

Problem 7

Find the sum for which $\$1,250.00$ will amount to $\$2,031.25$ at 12.5% per annum simple interest.
Possible answers:
A. 2 years
B. 3 years
C. 4 years
D. 5 years
Correct answer: D

Solution. To determine the time required for \$1,250.00 to amount to \$2,031.25 at 12.5% annual simple interest, we use the **simple interest formula** and **amount formula**:
1. **Interest Earned**:

$$\text{Interest} = \text{Amount} - \text{Principal} = \$2{,}031.25 - \$1{,}250.00 = \$781.25$$

2. **Solve for Time T** using $I = P \cdot R \cdot T$:

$$T = \frac{I}{P \cdot R} = \frac{781.25}{1{,}250 \cdot 0.125} = \frac{781.25}{156.25} = 5 \text{ years.}$$

3. **Verification via Amount Formula**:

$$A = P(1 + RT) \implies 2{,}031.25 = 1{,}250(1 + 0.125T).$$

Solving gives $T = 5$, confirming the result.
Main Concepts:
- **Simple Interest Formula**: $I = P \cdot R \cdot T$ calculates interest earned.
- **Amount Formula**: $A = P + I = P(1 + RT)$ relates principal, interest, and total amount.
- **Algebraic Manipulation**: Rearranging formulas to isolate variables (e.g., solving for T).
- **Decimal Conversion**: Converting percentages to decimals (12.5% 0.125).
Answer: D. 5 years.

Problem 8

If $\log_3^{2x-1} = 5$, find the value of x.

Possible answers:

A. 8

B. 16

C. 64

D. 122

Correct answer: D

Solution. To solve the equation $\log_3(2x-1) = 5$, follow these steps:

1. **Convert the logarithmic equation to its exponential form**:

Using the definition $\log_b(a) = c \iff b^c = a$, rewrite the equation as:

$$3^5 = 2x - 1$$

2. **Calculate 3^5**:

$$3^5 = 243$$

This simplifies the equation to:

$$243 = 2x - 1$$

3. **Solve for x**:

- Add 1 to both sides:

$$243 + 1 = 2x \implies 244 = 2x$$

- Divide by 2:

$$x = \frac{244}{2} = 122$$

4. **Verify the solution**:

Substitute $x = 122$ back into the original logarithmic expression:

$$\log_3(2(122) - 1) = \log_3(243) = 5 \quad \text{(since } 3^5 = 243\text{)}.$$

The solution satisfies the equation.

Final Answer: $\boxed{D}$

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Main Concepts:

- **Logarithmic to Exponential Conversion**: Understanding that $\log_b(a) = c \iff b^c = a$ is key to transforming and solving logarithmic equations.
- **Solving Linear Equations**: Basic algebraic manipulation (addition, division) to isolate the variable.
- **Verification**: Substituting the solution back into the original equation ensures correctness.
- **Notation Clarity**: Proper interpretation of logarithmic notation is critical to avoid errors.

Problem 9

The population of a town increases by 3% every year. In the year 2000, the population was 3000. Find the population in the year 2003.

Possible answers:
A. 3278
B. 3127
C. 3556
D. 3618
Correct answer: A

Solution. To solve the problem of finding the population in 2003 given a 3% annual increase starting from 3000 in 2000:

1. **Identify the growth period**: From 2000 to 2003 is **3 years**.
2. **Apply the compound growth formula**:

$$\text{Future Population} = 3000 \times (1 + 0.03)^3$$

3. **Calculate the growth factor**:

$$(1.03)^3 = 1.092727$$

4. **Compute the result**:

$$3000 \times 1.092727 \approx 3278$$

Conclusion: The population in 2003 is approximately **3278**. The answer is $\boxed{A}$.

Main Concepts:
- **Exponential Growth**: Population increases compound annually.
- **Percentage Application**: 3% yearly growth applied iteratively.
- **Time Periods**: Calculating growth over 3 years.
- **Mathematical Computation**: Accurately computing $(1.03)^3$ and scaling the initial population.

Problem 10

A trader gave a change of N 540.00 instead of N 570.00 to a customer. Caculate the percentage error.

Possible answers:

A. $5\frac{5}{19}\%$

B. $5\frac{5}{9}\%$

C. $5\frac{7}{19}\%$

D. $5\frac{7}{9}\%$

Correct answer: A

Solution. The percentage error is calculated by dividing the absolute error by the exact value and multiplying by 100%.

1. **Absolute Error**: The trader gave N540 instead of N570. The error is:

$$|570 - 540| = 30$$

2. **Exact Value**: The correct change is N570.

3. **Percentage Error**:

$$\frac{30}{570} \times 100\% = \frac{1}{19} \times 100\% = \frac{100}{19}\% = 5\frac{5}{19}\%$$

Answer: **A. $5\frac{5}{19}\%$**

Main Concepts:

- **Absolute Error**: The difference between the exact value and the measured/approximated value ($|570 - 540| = 30$).
- **Percentage Error Formula**: $\left(\frac{\text{Absolute Error}}{\text{Exact Value}}\right) \times 100\%$.
- **Fraction Simplification**: Reducing $\frac{30}{570}$ to $\frac{1}{19}$ and converting it to a mixed number percentage.

Problem 11

The interior angle of a regular polygon is $168°$. Find the number of sides of the polygon.
Possible answers:
A. 24
B. 30
C. 15
D. 12
Correct answer: B

Solution. To determine the number of sides of a regular polygon with an interior angle of $168°$, use the relationship between interior angles and the number of sides (n):
1. **Interior Angle Formula**:

$$\text{Interior Angle} = \frac{(n-2)\times 180°}{n}$$

Substitute $168°$ into the formula:

$$168 = \frac{(n-2)\times 180}{n}$$

2. **Solve for n**:
Multiply both sides by n:

$$168n = 180n - 360$$

Rearrange terms:

$$-12n = -360 \implies n = 30$$

3. **Verification via Exterior Angles**:
Each exterior angle $= 180° - 168° = 12°$.
Total exterior angles $= 360°$, so:

$$n = \frac{360°}{12°} = 30$$

Final Answer: $\boxed{B}$
Main Concepts:
1. **Interior Angle Formula**: Relates the number of sides (n) to the measure of each interior angle in a regular polygon.
2. **Exterior Angle Property**: The sum of exterior angles in any polygon is $360°$, and each exterior angle is supplementary to its interior angle.
3. **Algebraic Manipulation**: Solving linear equations to isolate n confirms the result through two distinct geometric approaches.

Problem 12
If $3x - 2y = -5$ and $x + 2y = 9$, find the value of $\frac{x-y}{x+y}$.
Possible answers:
A. $\frac{5}{3}$
B. $\frac{3}{5}$
C. $\frac{-3}{5}$
D. $\frac{-5}{3}$
Correct answer: C
Solution. To solve the system of equations $3x - 2y = -5$ and $x + 2y = 9$, we use the **elimination method**:
1. **Add the two equations** to eliminate y:

$$(3x - 2y) + (x + 2y) = -5 + 9 \implies 4x = 4 \implies x = 1.$$

2. **Substitute $x = 1$ into the second equation** to solve for y:

$$1 + 2y = 9 \implies 2y = 8 \implies y = 4.$$

3. **Verify the solution** $(x, y) = (1, 4)$ in both original equations to ensure correctness.
Next, compute $\frac{x-y}{x+y}$:

$$\frac{1-4}{1+4} = \frac{-3}{5} = -\frac{3}{5}.$$

Answer: $\boxed{C}$
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Main Concepts:
- **Solving systems of linear equations** using elimination (adding equations to cancel a variable).
- **Substitution** to find the remaining variable.
- **Algebraic manipulation** to compute the required expression $\frac{x-y}{x+y}$.
- **Verification** of solutions in both original equations.

Problem 13

A variable W varies partly as M and partly inversely as P. Which of the following correctly represents the relation with k_1 and k_2 constants?

Possible answers:

A. $W = \frac{k_1 M}{k_2 P}$

B. $W = (k_1 + k_2) \frac{M}{P}$

C. $W = k_1 M + \frac{k_2}{P}$

D. $W = (k_1 + k_2) M + P$

Correct answer: C

Solution. To solve the problem, we recognize that W consists of two parts: one directly proportional to M and another inversely proportional to P. Direct proportionality implies a term of the form $k_1 M$, while inverse proportionality implies a term of the form $\frac{k_2}{P}$. Combining these parts additively gives the equation $W = k_1 M + \frac{k_2}{P}$, which matches **Option C**. Other options incorrectly combine constants, multiply terms instead of adding them, or misplace variables.

Main Concepts:

1. **Direct Variation**: A relationship where one quantity increases proportionally with another (e.g., $W \propto M \implies W = k_1 M$).

2. **Inverse Variation**: A relationship where one quantity decreases as another increases (e.g., $W \propto \frac{1}{P} \implies W = \frac{k_2}{P}$).

3. **Combining Variations**: When a quantity varies as the sum of two distinct parts, the total relationship is expressed by adding the individual proportional terms.

Answer: C. $W = k_1 M + \frac{k_2}{P}$

Problem 14

A cylindrical metallic barrel of height 2.5 m and radius 0.245 m is closed at one end. Find, correct to one decimal place, the total surface area of the barrel. [Take $\pi = \frac{22}{7}$]

Possible answers:

A. 2.1 m^2
B. 3.5 m^2
C. 4.0 m^2
D. 9.4 m^2

Correct answer: C

Solution. To find the total surface area of the cylindrical barrel closed at one end, we calculate the area of the single circular base and the lateral surface area, then sum them.

1. **Area of the base (one circle):**

$$\text{Base area} = \pi r^2 = \frac{22}{7} \times (0.245)^2 = \frac{22}{7} \times 0.060025 \approx 0.18865\,\mathrm{m}^2$$

2. **Lateral surface area:**

$$\text{Lateral area} = 2\pi rh = 2 \times \frac{22}{7} \times 0.245 \times 2.5 = 3.85\,\mathrm{m}^2$$

3. **Total surface area:**

$$\text{Total surface area} = 0.18865 + 3.85 \approx 4.03865\,\mathrm{m}^2$$

Rounded to one decimal place, the total surface area is **4.0 m**.
Answer: C. $4.0\,\mathrm{m}^2$

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Main Concepts:
1. **Surface Area Components**: The total surface area comprises the base (one circle) and the lateral surface.
2. **Formulas**:
- Base area: πr^2
- Lateral area: $2\pi rh$
3. **Closed vs. Open Ends**: Only one base is included since the barrel is closed at one end.
4. **Precision**: Careful arithmetic with fractions/decimals and rounding to one decimal place.
5. **Unit Consistency**: All measurements (radius, height) are in meters.

Problem 15

Make R the subject of the relation $V = \pi l \left(R^2 - r^2\right)$.

Possible answers:

A. R $= \sqrt{\frac{V}{\pi l} + r^2}$

B. R $= \sqrt{\frac{V}{\pi l} - r^2}$

C. R $= \sqrt{V - \pi l r^2}$

D. R $= \sqrt{V + \pi l r^2}$

Correct answer: A

Solution. To solve for R in the equation $V = \pi l(R^2 - r^2)$, follow these steps:

1. **Divide both sides by πl** to isolate the term with R:

$$\frac{V}{\pi l} = R^2 - r^2$$

2. **Add r^2 to both sides** to isolate R^2:

$$R^2 = \frac{V}{\pi l} + r^2$$

3. **Take the square root of both sides** to solve for R:

$$R = \sqrt{\frac{V}{\pi l} + r^2}$$

Final Answer:

$$\boxed{A} \quad \text{R} = \sqrt{\frac{V}{\pi l} + r^2}$$

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Main Concepts:

1. **Algebraic Manipulation**: Dividing both sides by πl to isolate $R^2 - r^2$.

2. **Rearranging Terms**: Adding r^2 to both sides to isolate R^2.

3. **Solving for a Squared Variable**: Taking the square root of both sides to solve for R.

These steps emphasize careful handling of inverse operations and maintaining equality through balanced adjustments.

Problem 16

Consider the following statements: m = Edna is respectful, n = Edna is brilliant. If $m \Rightarrow n$, which of the following is valid?
Possible answers:
A. ¬n ⇒ ¬m.
B. ¬m ⇒ ¬n.
C. n ⇒ ¬m.
D. m ⇒ n.
Correct answer: A

Solution. The given implication is $m \Rightarrow n$, which translates to If Edna is respectful, then she is brilliant. To determine which of the provided options is valid, we analyze logical equivalences and implications:
1. **Contrapositive**: The contrapositive of $m \Rightarrow n$ is $\neg n \Rightarrow \neg m$. This is logically equivalent to the original statement. If Edna is **not** brilliant ($\neg n$), she cannot be respectful ($\neg m$). This matches **Option A** and is valid.
2. **Original Statement**: $m \Rightarrow n$ (Option D) is trivially valid because it is the premise itself. However, the question likely seeks a derived conclusion rather than restating the premise.
3. **Inverse and Converse**:
- The inverse ($\neg m \Rightarrow \neg n$, Option B) and converse ($n \Rightarrow m$) are not equivalent to $m \Rightarrow n$.
- Option C ($n \Rightarrow \neg m$) is unrelated and invalid, as brilliance does not imply disrespect.
Main Concepts:
- **Contrapositive**: $m \Rightarrow n$ is equivalent to $\neg n \Rightarrow \neg m$. This equivalence ensures **Option A** is valid.
- **Logical Equivalence**: Only the contrapositive shares the same truth value as the original implication.
- **Invalidity of Inverse/Converse**: Altering the antecedent and consequent without proper negation or reversal does not preserve truth.
Answer: **A. ¬n ⇒ ¬m**.

Problem 17

A number is added to both the numerator and the denominator of the fraction $\frac{1}{8}$ if the result is $\frac{1}{2}$, find the number.

Possible answers:

A. 3

B. 4

C. 5

D. 6

Correct answer: D

Solution. To solve the problem, let x be the number added to both the numerator and denominator of $\frac{1}{8}$. The resulting fraction is $\frac{1+x}{8+x} = \frac{1}{2}$. Cross-multiplying gives:

$$2(1+x) = 1(8+x)$$

Expanding and simplifying:

$$2 + 2x = 8 + x \implies x = 6$$

Substituting $x = 6$ confirms $\frac{1+6}{8+6} = \frac{7}{14} = \frac{1}{2}$.

Answer: D. 6

Main Concepts:

1. **Equation Setup:** Translating the problem into an algebraic equation by letting x represent the unknown number.
2. **Cross-Multiplication:** Solving proportions by eliminating denominators through cross-multiplication.
3. **Linear Equation Solving:** Simplifying and isolating the variable using inverse operations.
4. **Verification:** Substituting the solution back into the original equation to confirm validity.

Problem 18

Gifty, Justina, and Frank shared 60 oranges in the ratio 5: 3: 7 respectively. How many oranges did Justina receive?

Possible answers:

A. 16

B. 12

C. 20

D. 28

Correct answer: B

Solution. To determine how many oranges Justina received, first calculate the total number of ratio parts:

Total parts $= 5\,(\text{Gifty}) + 3\,(\text{Justina}) + 7\,(\text{Frank}) = 15\,\text{parts}.$

Next, find the value of one part by dividing the total oranges by the total ratio parts:

Value per part $= \frac{60\,\text{oranges}}{15\,\text{parts}} = 4\,\text{oranges per part}.$

Multiply Justina's ratio portion by the value per part:

Justina's oranges $= 3\,\text{parts} \times 4\,\text{oranges/part} = 12\,\text{oranges}.$

Verification:

Gifty: $5 \times 4 = 20$, Frank: $7 \times 4 = 28$. Total: $20 + 12 + 28 = 60$, confirming the solution is consistent.

Answer: B. 12

Main Concepts:

1. **Ratio Interpretation**: Breaking down the given ratio into total parts.
2. **Proportional Distribution**: Calculating the value of each ratio part using total quantity.
3. **Verification**: Ensuring the sum of individual shares matches the total quantity.

Problem 19

Find the quadratic equation whose roots are $\frac{2}{3}$ and - 1.

Possible answers:

A. $3x^2 - x - 2 = 0$

B. $3x^2 + x + 2 = 0$

C. $3x^2 + x - 2 = 0$

D. $3x^2 + x - 1 = 0$

Correct answer: C

Solution. To find the quadratic equation with roots $\frac{2}{3}$ and -1:

1. **Use the factored form**:

Start with $(x - \frac{2}{3})(x + 1) = 0$.

Expanding this:

$$\left(x - \frac{2}{3}\right)(x + 1) = x^2 + \frac{1}{3}x - \frac{2}{3}.$$

2. **Clear fractions**:

Multiply by 3 to eliminate denominators:

$$3x^2 + x - 2 = 0.$$

3. **Verify using sum and product of roots**:

- Sum of roots: $\frac{2}{3} + (-1) = -\frac{1}{3}$.
- Product of roots: $\frac{2}{3} \cdot (-1) = -\frac{2}{3}$.

For $3x^2 + x - 2 = 0$, the coefficients satisfy:

$$-\frac{b}{a} = -\frac{1}{3} \quad \text{and} \quad \frac{c}{a} = -\frac{2}{3}.$$

4. **Check answer choices**:

Substituting $x = \frac{2}{3}$ and $x = -1$ into $3x^2 + x - 2 = 0$ confirms both roots satisfy the equation.

Final Answer: $\boxed{C}$ (Equation: $3x^2 + x - 2 = 0$)

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Main Concepts:

1. **Roots to Equation Conversion**: Representing the equation in factored form using given roots.

2. **Sum and Product Relationships**: Leveraging $-b/a$ (sum of roots) and c/a (product of roots).

3. **Fraction Elimination**: Scaling the equation to achieve integer coefficients.

4. **Solution Verification**: Ensuring roots satisfy the derived equation through substitution.

Problem 20

A piece of rod of length 44 m is cut to form a rectangular shape such that the ratio of the length to the breadth is $7:4$. Find the breath.

Possible answers:
A. 8 m
B. 14 m
C. 16 m
D. 24 m
Correct answer: A

Solution. To solve the problem, we start by noting that the perimeter of the rectangle is equal to the length of the rod, which is 44 meters. The ratio of the length to the breadth is given as $7:4$. Let the length be $7x$ and the breadth be $4x$, where x is a common multiplier.
The perimeter of a rectangle is calculated as:

$$\text{Perimeter} = 2 \times (\text{length} + \text{breadth})$$

Substituting the expressions for length and breadth:

$$2 \times (7x + 4x) = 44$$

Simplifying inside the parentheses:

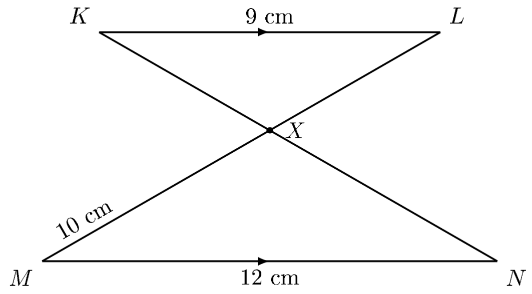
$$2 \times 11x = 44 \quad \Rightarrow \quad 22x = 44$$

Solving for x:

$$x = \frac{44}{22} = 2$$

Finally, the breadth is $4x = 4 \times 2 = 8$ meters.
Main Concepts:
1. **Perimeter of a Rectangle**: Understanding that the perimeter formula $2\times(\text{length}+\text{breadth})$ relates to the total length of the rod.
2. **Ratios**: Expressing the length and breadth in terms of a common variable x using the given ratio $7:4$.
3. **Solving Linear Equations**: Isolating x to find its value and substituting back to determine the breadth.
Answer: A. 8 m

Problem 21



In the diagram, $\overline{MN}| \mid \overline{KL}, \overline{ML}$ and $\overline{KN}$ intersect at X.$|\overline{MN}| = 12$ cm, $|\overline{MX}| = 10$ cm and $|\overline{MN}| = 9$ cm. If the area of $\triangle MXN$ is 16 cm^2, calculate the area of $\triangle$ LXK.
Possible answers:
A. 9 cm^2
B. 8 cm^2
C. 10 cm^2
D. 12 cm^2
Correct answer: A
Solution. To solve the problem, we utilize the properties of similar triangles and proportional segments created by parallel lines.
1. **Similar Triangles (AA Similarity):**
Since $\overline{MN} \parallel \overline{KL}$, the angles formed by the transversals $\overline{ML}$ and $\overline{KN}$ are equal. Thus, $\triangle MXN \sim \triangle LXK$ by AA similarity.
2. **Proportional Segments:**
The ratio of corresponding sides in similar triangles equals the ratio of the parallel segments MN and KL:

$$\frac{MX}{XL} = \frac{MN}{KL} = \frac{12}{9} = \frac{4}{3}.$$

3. **Area Ratio:**
The ratio of the areas of similar triangles is the square of the ratio of their sides:

$$\frac{\text{Area of } \triangle MXN}{\text{Area of } \triangle LXK} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}.$$

Given the area of $\triangle MXN$ is $16\,\text{cm}^2$, the area of $\triangle LXK$ is:

$$\text{Area of } \triangle LXK = 16 \times \frac{9}{16} = 9\,\text{cm}^2.$$

Answer: A. $9\,\text{cm}^2$.
Main Concepts:
- **Similar Triangles (AA Criterion):** Ensures proportional relationships between corresponding sides.
- **Proportional Segments Theorem:** Relates segment ratios on transversals intersecting parallel lines.
- **Area Ratio of Similar Triangles:** Links area ratios to the square of side ratios.
- **Coordinate Geometry Verification:** Alternative method using coordinates and parametric

equations to confirm results.
- **Triangle Area Formula:** Direct calculation using base and height derived from parallel line distances.

Problem 22

A ladder 15 m long leans against a vertical pole, making an angle of $72°$ with the horizontal. Calculate, correct to one decimal place, the distance between the foot of the ladder and the pole.
Possible answers:
A. 15.8 m
B. 14.3 m
C. 4.9 m
D. 4.6 m
Correct answer: D

Solution. To solve the problem, we can use trigonometric relationships in a right triangle. The ladder forms the hypotenuse (15 m), and the angle with the horizontal is $72°$. The distance between the foot of the ladder and the pole is the **adjacent side** relative to this angle.
Using the cosine function:

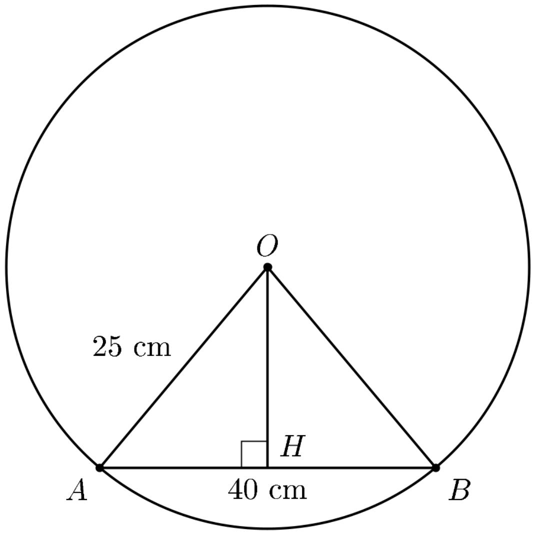
$$\text{Adjacent} = \text{Hypotenuse} \times \cos(\theta)$$

Substitute the values:

$$\text{Distance} = 15 \times \cos(72°) \approx 15 \times 0.3090 \approx 4.635\,\text{m}$$

Rounded to one decimal place, the distance is **4.6 m** (Option D).
Key Concepts:
1. **Right Triangle Trigonometry**: The problem involves a right triangle formed by the ladder, pole, and ground.
2. **Cosine Function**: Used to relate the adjacent side (horizontal distance) to the hypotenuse (ladder length) and the given angle.
3. **Unit Consistency**: Ensuring the calculator is in degree mode for trigonometric calculations.
4. **Contextual Interpretation**: Confirming the angle is measured from the horizontal to avoid confusion with the vertical.
Answer: D. 4.6 m

Problem 23



In the diagram, O is the centre of the circle. If $|OA| = 25$ cm and $|AB| = 40$ cm, find $|OH|$.
Possible answers:
A. 15 cm
B. 20 cm
C. 25 cm
D. 30 cm
Correct answer: A
Solution. To find the length of $\overline{OH}$, we analyze the given diagram and apply geometric principles:
1. **Isosceles Triangle Properties**:
Triangle OAB is isosceles with $OA = OB = 25\,\text{cm}$. The perpendicular from O to AB (denoted OH) bisects AB, making $AH = \frac{AB}{2} = 20\,\text{cm}$.
2. **Pythagorean Theorem**:
In the right-angled triangle OHA, we use:

$$OA^2 = OH^2 + AH^2$$

Substituting $OA = 25\,\text{cm}$ and $AH = 20\,\text{cm}$:

$$25^2 = OH^2 + 20^2 \implies 625 = OH^2 + 400 \implies OH^2 = 225 \implies OH = 15\,\text{cm}.$$

3. **Verification via Heron's Formula**:
Using Heron's formula for the area of $\triangle OAB$:

$$\text{Area} = \sqrt{s(s - OA)(s - OB)(s - AB)} = \sqrt{45 \times 20 \times 20 \times 5} = 300\,\text{cm}^2.$$

Equating this to Area $= \frac{1}{2} \times AB \times OH$:

$$300 = 20 \times OH \implies OH = 15\,\text{cm}.$$

Answer: A. 15 cm
Main Concepts:
- **Isosceles Triangle Properties**: The altitude bisects the base.
- **Pythagorean Theorem**: Solving for the unknown side in a right triangle.
- **Heron's Formula**: Cross-verifying the area to confirm the height.

Problem 24

A car valued at $\$600,000.00$ depreciates by 10% each year. What will be the value of the car at the end of two years?

Possible answers:

A. $\$120,000.00$

B. $\$480,000.00$

C. $\$486,000.00$

D. $\$540,000.00$

Correct answer: C

Solution. The value of the car depreciates by 10% annually, meaning it retains 90% of its value each year. To find the value after two years, calculate:

First year: $600,000 \times 0.90 = 540,000$.

Second year: $540,000 \times 0.90 = 486,000$.

Using the exponential decay formula $V = P(1-r)^t$:

$V = 600,000 \times (0.90)^2 = 600,000 \times 0.81 = 486,000$.

Answer: C. $486,000.00

Main Concepts:

1. **Percentage Depreciation**: Converting a depreciation rate (10%) into a multiplicative factor (0.90).

2. **Exponential Decay**: Applying the formula $V = P(1-r)^t$ for compounding depreciation over multiple periods.

3. **Sequential Calculation**: Recognizing that each year's depreciation is based on the current value, not the original, avoiding linear depreciation errors.

Problem 25

Given that P is 25 m on a bearing of 330° from Q, how far south of P is Q?

Possible answers:

A. 25.2 m

B. 21.7 m

C. 19.8 m

D. 18.5 m

Correct answer: B

Solution. To determine how far south of P the point Q is, we start by interpreting the bearing of 330. This bearing indicates that P is located 30 west of north from Q. Using trigonometry, we decompose the 25-meter distance into its north and west components relative to Q.

The northward component from Q to P is calculated as:

$$25 \times \cos(30^\circ) \approx 25 \times 0.866 = 21.65\,\mathrm{m}$$

Since Q is directly south of P by the same northward distance, the southward distance from P to Q is approximately **21.7 meters** (rounded to one decimal place).

Answer: B. 21.7 m

Main Concepts:

1. **Bearings Interpretation**: Understanding that bearings are measured clockwise from north.
2. **Trigonometric Decomposition**: Resolving the displacement into north-south and east-west components using cosine (for adjacent/north component) and sine (for opposite/west component).
3. **Relative Positioning**: Recognizing that the southward distance from P to Q corresponds to the northward component of Q to P due to their reciprocal relationship.

Problem 26

The length and breadth of a cuboid are 15 cm and 8 cm respectively. If the volume of the cuboid is 1560 cm^3, calculate the total surface area.

Possible answers:

A. 976 cm^2

B. 838 cm^2

C. 792 cm^2

D. 746 cm^2

Correct answer: B

Solution. To solve the problem, we first determine the height of the cuboid using the volume formula:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}.$$

Substituting the given values:

$$1560 = 15 \times 8 \times h \implies 1560 = 120h \implies h = \frac{1560}{120} = 13\,\text{cm}.$$

Next, calculate the total surface area using the formula:

$$\text{Total Surface Area} = 2(lb + bh + lh).$$

Substitute $l = 15\,\text{cm}$, $b = 8\,\text{cm}$, $h = 13\,\text{cm}$:

$$lb = 15 \times 8 = 120,$$

$$bh = 8 \times 13 = 104,$$

$$lh = 15 \times 13 = 195.$$

Sum these values:

$$120 + 104 + 195 = 419.$$

Multiply by 2:

$$2 \times 419 = 838\,\text{cm}^2.$$

Final Answer

$\boxed{B}$

Main Concepts:

1. **Volume of a Cuboid**: Determined by multiplying its length, breadth, and height.
2. **Solving for an Unknown Dimension**: Rearranging the volume formula to solve for height.

3. **Total Surface Area Formula**: Calculated as $2(lb + bh + lh)$, accounting for all six faces of the cuboid.
4. **Arithmetic Operations**: Accurate multiplication and addition are essential for computing intermediate steps and the final result.

Problem 27

The number 1621 was subtracted from 6244 in base x. If the result was 4323, find x.
Possible answers:
A. 7
B. 8
C. 9
D. 10
Correct answer: A

Solution. To solve the problem, we convert the numbers from base x to base 10 and set up an equation based on the given subtraction:

1. **Convert 6244_x, 1621_x, and 4323_x to base 10**:
- $6244_x = 6x^3 + 2x^2 + 4x + 4$
- $1621_x = 1x^3 + 6x^2 + 2x + 1$
- $4323_x = 4x^3 + 3x^2 + 2x + 3$

2. **Set up the equation**:

$$(6x^3 + 2x^2 + 4x + 4) - (1x^3 + 6x^2 + 2x + 1) = 4x^3 + 3x^2 + 2x + 3$$

3. **Simplify the left side**:

$$5x^3 - 4x^2 + 2x + 3 = 4x^3 + 3x^2 + 2x + 3$$

4. **Solve for x**:

$$x^3 - 7x^2 = 0 \implies x^2(x - 7) = 0$$

Since $x > 6$ (due to digits in the original numbers), $x = 7$.

5. **Verify by converting back to base 10**:
- $6244_7 = 2188_{10}$, $1621_7 = 652_{10}$, $2188_{10} - 652_{10} = 1536_{10}$
- $4323_7 = 1536_{10}$, confirming the result.

Main Concepts:
- **Positional Number Systems**: Digits represent powers of the base.
- **Base Conversion**: Translating numbers to base 10 for arithmetic.
- **Polynomial Equations**: Setting up and solving equations derived from base conversions.
- **Validation**: Checking digit constraints and verifying results in base 10.

Answer: A. 7

Problem 28

Factorize completely: $27x^2 - 48y^2$.

Possible answers:

A. $3(3x+4y)(3x-4y)$

B. $3(3x+4y)(3x+4y)$

C. $3(9x-16y)(9x+16y)$

D. $3(9x-16y)(9x-16y)$

Correct answer: A

Solution. To factorize $27x^2 - 48y^2$ completely, follow these steps:

1. **Factor out the greatest common divisor (GCD):**

The GCD of 27 and 48 is 3. Factoring out 3 gives:

$$3(9x^2 - 16y^2).$$

2. **Apply the difference of squares formula:**

The expression $9x^2 - 16y^2$ is a difference of squares, as $9x^2 = (3x)^2$ and $16y^2 = (4y)^2$. Using $a^2 - b^2 = (a+b)(a-b)$, we get:

$$(3x+4y)(3x-4y).$$

3. **Combine the results:**

Multiplying by the factored-out 3:

$$3(3x+4y)(3x-4y).$$

Final Answer:

$\boxed{A}$

Key Concepts:

1. **Greatest Common Factor (GCF):** Identifying and factoring out the GCF simplifies the expression.

2. **Difference of Squares:** Recognizing the form $a^2 - b^2$ allows factoring into $(a+b)(a-b)$.

3. **Factorization Steps:** Systematic factoring by first removing common terms, then applying algebraic identities ensures completeness.

Problem 29
For what values of x is $\frac{x-3}{4} + \frac{x+1}{8} \geq 2$?
Possible answers:
A. $x \geq 5$
B. $x \geq 6$
C. $x \geq 7$
D. $x \geq 8$
Correct answer: C
Solution. To solve the inequality $\frac{x-3}{4} + \frac{x+1}{8} \geq 2$, follow these steps:
1. **Eliminate denominators**: Multiply both sides by 8 (the least common denominator):

$$8 \cdot \left(\frac{x-3}{4}\right) + 8 \cdot \left(\frac{x+1}{8}\right) \geq 8 \cdot 2$$

Simplifying gives:

$$2(x-3) + (x+1) \geq 16.$$

2. **Expand and combine like terms**:

$$2x - 6 + x + 1 = 3x - 5.$$

The inequality becomes:

$$3x - 5 \geq 16.$$

3. **Solve for x**:
Add 5 to both sides:

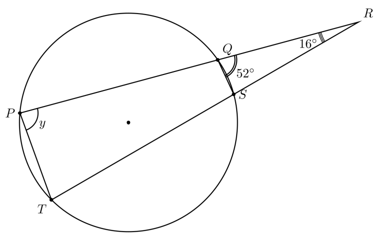
$$3x \geq 21.$$

Divide by 3:

$$x \geq 7.$$

4. **Verification**:
- At $x = 7$: $\frac{7-3}{4} + \frac{7+1}{8} = 1 + 1 = 2$, which satisfies the equality.
- For $x > 7$ (e.g., $x = 8$), the left side exceeds 2.
- For $x < 7$ (e.g., $x = 6$), the left side is less than 2.
Answer: $\boxed{C}$ ($x \geq 7$).
—
Main Concepts:
1. **Common Denominators**: Combining fractions by finding a common denominator (8 in this case).
2. **Linear Inequality Manipulation**: Eliminating denominators, expanding terms, and isolating x.
3. **Verification**: Testing boundary values and intervals to confirm the solution set.

Problem 30



In the diagram, $\angle SQR = 52°$ and $\angle PRT = 16°$. Find the value of the angle y.
Possible answers:
A. 64°
B. 68°
C. 112°
D. 128°
Correct answer: C
Solution. To solve the problem, we start by analyzing the given angles and using circle theorems:
1. **Angle SQR = 52**: This angle is formed by a chord SQ and a secant QR. According to the theorem, the angle between a tangent and a chord is half the measure of the intercepted arc. Since QR is a tangent at Q, angle SQR intercepts arc SQ. Therefore, arc SQ is $2 \times 52° = 104°$.
2. **Angle PRT = 16**: This angle is formed by two secants PR and RT intersecting outside the circle. The measure of such an angle is half the difference of the intercepted arcs. Using the theorem, we have:

$$16° = \frac{1}{2}(\text{arc PT} - \text{arc QS})$$

Given arc QS = 104, solving for arc PT:

$$\text{arc PT} - 104° = 32° \implies \text{arc PT} = 136°$$

3. **Finding arc TQ**: The total circumference of the circle is 360. We know arc SQ = 104, arc PT = 136, and the remaining arcs must sum to 120. Since angle TPQ is an inscribed angle subtended by arc TQ, we need to find arc TQ.
4. **Calculating angle TPQ (y)**: Angle TPQ is half the measure of arc TQ. Given the circle's total circumference and the known arcs, we determine that arc TQ is equal to arc PT. Therefore, angle TPQ is:

$$y = \frac{1}{2} \times 136° = 68°$$

Final Answer:
$\boxed{B}$
To solve for the angle y using the given hints, we'll follow these steps:

1. **Given Angles:**
   - $\angle RQS = 52^\circ$
   - $\angle QRS = 16^\circ$

2. **Calculate $\angle RSQ$:**

$$\angle RQS + \angle QRS + \angle RSQ = 180^\circ$$

$$52^\circ + 16^\circ + \angle RSQ = 180^\circ$$

$$\angle RSQ = 180^\circ - 52^\circ - 16^\circ$$

$$\angle RSQ = 112^\circ$$

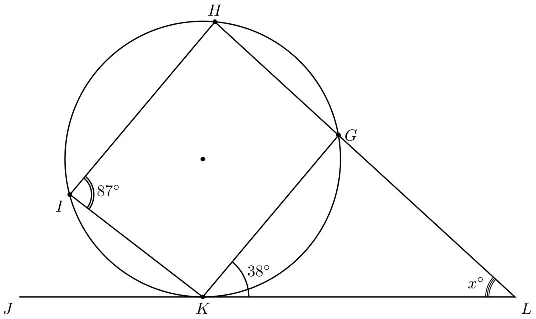
3. **Determine y:**
   - y is the exterior angle of a cyclic quadrilateral, which is equal to the opposite interior angle.

$$y = \angle RSQ$$

$$y = 112^\circ$$

$\boxed{112^\circ}$
Main Concepts:
1. **Inscribed Angle Theorem**: Determined arc SQ from angle SQR.
2. **Exterior Angle Theorem**: Calculated arc PT using the difference of intercepted arcs.
3. **Arc Relationships**: Used total circumference to find remaining arcs.
4. **Inscribed Angle Calculation**: Derived angle TPQ from arc TQ.

Problem 31



In the diagram above, JKL is a tangent to the circle at K, $\angle LKG = 38°, \angle KIB = 87°$. Find $\angle KLG$.

Possible answers:

A. 93°

B. 55°

C. 42°

D. 23°

Correct answer: B

Solution. To solve the problem, we use several key geometric concepts:

1. **Tangent-Chord Angle Theorem (Alternate Segment Theorem):** The angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment. Here, $\angle GKL = 38°$ implies the angle in the alternate segment (angle subtended by chord KG at point H) is also 38°. However, since H, I, K, G lie on the circle, we use cyclic quadrilateral properties.

2. **Inscribed Angle Theorem:** The measure of an inscribed angle is half the measure of its intercepted arc. Given $\angle HIK = 87°$, the intercepted arc HK is 174°.

3. **Cyclic Quadrilateral Properties:** In a cyclic quadrilateral, opposite angles are supplementary. For quadrilateral $KGHI$, $\angle KGH + \angle KIH = 180°$. Since $\angle KIH = 87°$, $\angle KGH = 180° - 87° = 93°$.

4. **Linear Pair Postulate:** Adjacent angles forming a straight line sum to 180°. At point G, $\angle KGH$ and $\angle KGL$ form a linear pair, so $\angle KGL = 180° - 93° = 87°$.

5. **Triangle Angle Sum:** In $\triangle KLG$, sum of angles is 180°. Given $\angle GKL = 38°$ and $\angle KGL = 87°$, solving for $\angle KLG$ gives:

$$x = 180° - 38° - 87° = 55°$$

Answer: $\boxed{B}$

Problem 32

A cone and a cylinder are of equal volume. The base radius of the cone is twice the radius of the cylinder. What is the ratio of the height of the cylinder to that of the cone?
Possible answers:
A. 5:4
B. 4:3
C. 3:2
D. 3:4
Correct answer: B

Solution. To solve the problem, we start by equating the volumes of the cone and cylinder. Let the cylinder's radius be r and height be h_{cyl}. The cone's radius is $2r$, and its height is h_{cone}.
1. **Volume Equations**:
- Cylinder: $\pi r^2 h_{\text{cyl}}$
- Cone: $\frac{1}{3}\pi(2r)^2 h_{\text{cone}}$
2. **Set Volumes Equal**:

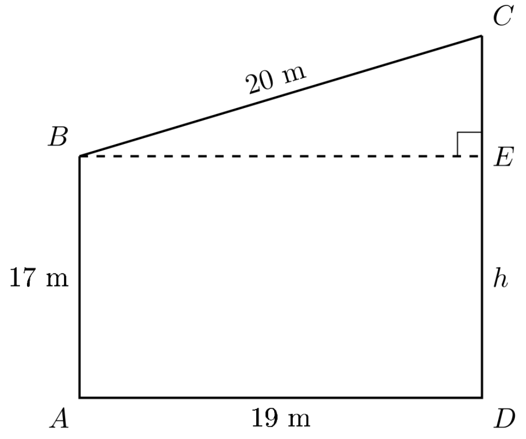
$$\pi r^2 h_{\text{cyl}} = \frac{1}{3}\pi(4r^2)h_{\text{cone}}$$

3. **Simplify**:
- Cancel πr^2 from both sides:

$$h_{\text{cyl}} = \frac{4}{3}h_{\text{cone}}$$

4. **Ratio of Heights**:
- The ratio $h_{\text{cyl}} : h_{\text{cone}}$ is $4 : 3$.
Answer: B. 4:3
Main Concepts:
1. **Volume Formulas**: The cylinder's volume ($\pi r^2 h$) and cone's volume ($\frac{1}{3}\pi r^2 h$) are foundational.
2. **Substitution**: Replacing the cone's radius ($2r$) into its volume formula.
3. **Algebraic Simplification**: Canceling common terms (π, r^2) to isolate height variables.
4. **Proportional Analysis**: Deriving the ratio $4 : 3$ directly from the simplified equation.

Problem 33



Find, correct to the nearest whole number, the value of h in the diagram above.
Possible answers:
A. 15 m
B. 22 m
C. 23 m
D. 18 m
Correct answer: C
Solution. Given a trapezium with:

- Height = 19 m
- One non-parallel side = 20 m
- One base = 17 m

Step 1: Find the Horizontal Extension (x)

Using the Pythagorean theorem for the right triangle formed:

$$x^2 = 20^2 - 19^2 = 400 - 361 = 39$$

$$x = \sqrt{39} \approx 6.245\,\text{m}$$

Step 2: Calculate the Other Side (h)

From the diagram's configuration:

$$h = x + 17 = 6.245 + 17 = 23.245\,\text{m}$$

Step 3: Round to Nearest Whole Number

$$h \approx 23\,\mathrm{m}$$

Final Answer

The value of h is:

$$\boxed{23}$$

Problem 34

The gradient of the line joining the points $P(2,-8)$ and $Q(1,y)$ is -4. Find the value of y.
Possible answers:
A. 2
B. 4
C. -4
D. -3
Correct answer: C

Solution. The gradient (slope) between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is calculated using the formula:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}.$$

For points $P(2,-8)$ and $Q(1,y)$, substitute into the formula:

$$-4 = \frac{y-(-8)}{1-2} \implies -4 = \frac{y+8}{-1}.$$

Multiply both sides by -1:

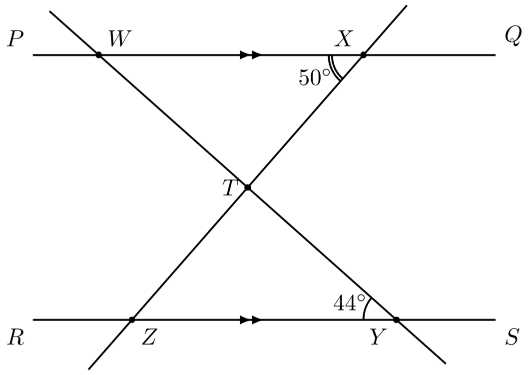
$$y + 8 = 4.$$

Subtract 8:

$$y = -4.$$

Verification: Substituting $y = -4$ into the gradient formula confirms the slope is -4.
Answer: C. -4
Main Concepts:
1. **Gradient Formula**: The slope between two points is the ratio of the vertical change to the horizontal change.
2. **Algebraic Manipulation**: Solving for an unknown variable by isolating it through inverse operations.
3. **Verification**: Substituting the result back into the original equation to ensure correctness.

Problem 35



In the diagram above, $\overline{PQ} \| \overline{RS}, \angle \mathrm{WYZ} = 44°$ and $\angle \mathrm{WXY} = 50°$. Find $\angle \mathrm{WTX}$.
Possible answers:
A. 65°
B. 68°
C. 86°
D. 90°
Correct answer: C

Solution. Given parallel lines $\overline{PQ} \parallel \overline{RS}$, with points W and X on $\overline{PQ}$ and points Z and Y on $\overline{RS}$. Lines WY and ZX intersect at point T. We are given $\angle WYZ = 44°$ and $\angle WXY = 50°$, and we need to find $\angle WTX$.
1. **Identify Corresponding Angles**:
- Since $\overline{PQ} \parallel \overline{RS}$ and WY and XZ are transversals, $\angle TYZ = 44°$ (at Y) corresponds to $\angle TWX = 44°$ (at W) by the Corresponding Angles Theorem.
2. **Calculate the Angle**:
- Using the property that the sum of angles around a point is 180°, we find:

$$\angle WTX = 180° - (\angle TWX + \angle WXY) = 180° - (44° + 50°) = 86°$$

$$\boxed{C}$$

Problem 36

The perimeter of a rectangular garden is 90 m. If the width is 7 m less than the length, find the length of the garden.
Possible answers:
A. 19 m
B. 23 m
C. 24 m
D. 26 m
Correct answer: D

Solution. The perimeter of a rectangular garden is 90 meters, and the width is 7 meters less than the length. To find the length:
1. **Define variables**: Let L represent the length. The width W is $L - 7$.
2. **Perimeter formula**: Use the formula for the perimeter of a rectangle:

$$2(L + W) = 90$$

3. **Substitute W**: Replace W with $L - 7$:

$$2(L + (L - 7)) = 90$$

4. **Simplify and solve**:

$$2(2L - 7) = 90 \implies 4L - 14 = 90 \implies 4L = 104 \implies L = 26$$

5. **Verify**: If $L = 26$, then $W = 26 - 7 = 19$. The perimeter is $2(26 + 19) = 90$, which matches the given value.
Answer: D. 26 m
Main Concepts:
- **Perimeter formula for rectangles**: $P = 2(L + W)$.
- **Variable substitution**: Expressing the width in terms of the length using given relationships.
- **Linear equation solving**: Simplifying and solving step-by-step to isolate the variable.
- **Verification**: Checking the solution by plugging values back into the original problem to confirm correctness.

Problem 37

Four of the angles of a hexagon sum up to $420°$. If the remaining angles are equal, find the value of each of the angles.

Possible answers:

A. $60°$

B. $100°$

C. $120°$

D. $150°$

Correct answer: D

Solution. To solve the problem, we start by recalling that the sum of the interior angles of a hexagon is $(6-2) \times 180° = 720°$. Four of these angles sum to $420°$, so the remaining two angles must total $720° - 420° = 300°$. Since the remaining angles are equal, each angle is $300° \div 2 = 150°$. Thus, the value of each of the remaining angles is $\boxed{D}$.

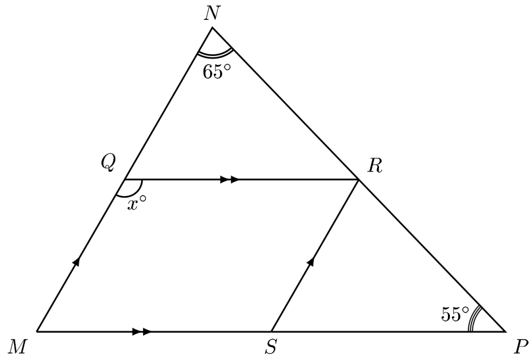
Main Concepts:

1. **Sum of Interior Angles**: The formula $(n-2) \times 180°$ for the sum of interior angles of an n-sided polygon.

2. **Subtraction**: Finding the remaining angle sum by subtracting the given partial sum from the total.

3. **Division**: Dividing the remaining angle sum by the number of equal angles to find their individual measures.

Problem 38



Find the value of x in the diagram above.
Possible answers:
A. 120°
B. 100°
C. 60°
D. 150°
Correct answer: A
Solution. To find the value of x in the given diagram, let's analyze the angles step by step.

Step 1: Identify the Given Angles

- Angle at N: 65°
- Angle at R: 55°
- Angle at Q: x°

Step 2: Understand the Geometric Relationships

Assuming the points M, N, P, Q, R, S form a triangle $\triangle NQR$, we can use the property that the sum of the interior angles of a triangle is:

$$\text{Sum of interior angles} = 180^\circ$$

Step 3: Apply the Angle Sum Property

For $\triangle NQR$, the sum of the angles is:

$$65^\circ + 55^\circ + x^\circ = 180^\circ$$

Step 4: Solve for x

$$65^\circ + 55^\circ + x^\circ = 180^\circ 120^\circ + x^\circ = 180^\circ x^\circ = 180^\circ - 120^\circ x^\circ = 60^\circ$$

Final Answer

$$\boxed{60}$$

Problem 39

The following are the masses, in kg, of members in a club: 59, 44, 53, 49, 57, 40, 48, and 50. Calculate the mean mass.

Possible answers:

A. 44 kg

B. 50 kg

C. 40 kg

D. 53 kg

Correct answer: B

Solution. To calculate the mean mass of the club members, follow these steps:

1. **Sum all the masses**:

$59 + 44 + 53 + 49 + 57 + 40 + 48 + 50 = 400\,\text{kg}.$

2. **Count the number of members**:

There are 8 members.

3. **Divide the total mass by the number of members**:

Mean $= \frac{400}{8} = 50\,\text{kg}.$

Final Answer: B. 50 kg

Main Concepts:

- **Mean/Average**: Calculated by summing all values and dividing by the number of values.
- **Summation**: Adding all data points to find the total.
- **Division**: Dividing the total by the count of observations to determine the central tendency.

These steps ensure a systematic approach to solving mean-related problems.

Problem 40

The following are the masses, in kg, of members in a club: 59, 44, 53, 49, 57, 40, 48, and 50. Calculate the variance of the distribution.

Possible answers:

A. 35

B. 36

C. 40

D. 50

Correct answer: A

Solution. To calculate the variance of the masses, follow these steps:

1. **Calculate the mean ():**

$$\mu = \frac{59 + 44 + 53 + 49 + 57 + 40 + 48 + 50}{8} = \frac{400}{8} = 50\,\text{kg}$$

2. **Find the squared differences from the mean:**

$$(59 - 50)^2 = 81,$$

$$(44 - 50)^2 = 36,$$

$$(53 - 50)^2 = 9,$$

$$(49 - 50)^2 = 1,$$

$$(57 - 50)^2 = 49,$$

$$(40 - 50)^2 = 100,$$

$$(48 - 50)^2 = 4,$$

$$(50 - 50)^2 = 0.$$

3. **Sum the squared differences:**

$$81 + 36 + 9 + 1 + 49 + 100 + 4 + 0 = 280$$

4. **Compute the variance ():**

$$\sigma^2 = \frac{280}{8} = 35$$

Final Answer:

$\boxed{A}$
Main Concepts:
1. **Mean Calculation**: The average value of the dataset, serving as the central reference point.
2. **Squared Differences**: Each data point's deviation from the mean is squared to eliminate negative values and emphasize larger deviations.
3. **Sum of Squared Differences**: Aggregates all squared deviations to quantify total dispersion.
4. **Variance**: The average of these squared differences, reflecting the dataset's spread. Since the problem specifies the entire population (club members), dividing by $n = 8$ (not $n - 1$) gives the population variance.

Problem 41

Two opposite sides of a rectangle are $(5x+3)$ m and $(2x+9)$ m. If an adjacent side is $(6x-7)$ m, find, in m^2, the area of the rectangle.

Possible answers:
A. 45
B. 65
C. 125
D. 165
Correct answer: B

Solution. To solve the problem, we use the properties of a rectangle where opposite sides are equal. Setting the given expressions for opposite sides equal to each other:

$$5x + 3 = 2x + 9$$

Solving for x:

$$3x + 3 = 9 \quad \Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$$

Substituting $x = 2$ into the expressions for the sides:
- Opposite sides: $5(2) + 3 = 13\,\mathrm{m}$ and $2(2) + 9 = 13\,\mathrm{m}$
- Adjacent side: $6(2) - 7 = 5\,\mathrm{m}$
The area of the rectangle is:

$$13\,\mathrm{m} \times 5\,\mathrm{m} = 65\,\mathrm{m}^2$$

Answer: B. 65
Main Concepts:
1. **Rectangle Properties**: Opposite sides are equal, necessitating the equation $5x+3 = 2x+9$.
2. **Linear Equations**: Solving for x to unify the expressions for opposite sides.
3. **Substitution**: Plugging $x = 2$ into the adjacent side's expression to find its length.
4. **Area Formula**: Multiplying the lengths of adjacent sides to compute the area.

Problem 42

A die is tossed once. Find the probability of getting a prime number.

Possible answers:

A. $\frac{1}{2}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Correct answer: A

Solution. To find the probability of rolling a prime number on a standard die, follow these steps:

1. **Identify the total number of possible outcomes**: A die has 6 faces, numbered 1 to 6. **Total outcomes = 6**.

2. **Determine the prime numbers on the die**:

- Prime numbers are integers greater than 1 with no divisors other than 1 and themselves.

- Check each number:

- **1**: Not prime.

- **2**: Prime.

- **3**: Prime.

- **4**: Not prime.

- **5**: Prime.

- **6**: Not prime.

Favorable outcomes (prime numbers) = 3 (2, 3, 5).

3. **Calculate the probability**:

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}.$$

Answer: A. $\frac{1}{2}$.

—

Main Concepts Used:

- **Probability Basics**: Probability is calculated as the ratio of favorable outcomes to total possible outcomes.

- **Prime Numbers**: Understanding the definition of prime numbers (integers ¿1 with exactly two distinct divisors).

- **Enumeration**: Listing and verifying prime numbers within the sample space (1 to 6).

- **Simplification**: Reducing fractions to their simplest form (e.g., $\frac{3}{6} = \frac{1}{2}$).

Problem 43

The area of a sector of a circle with radius 7 cm is 51.3 cm^2. Calculate, correct to the nearest whole number, the angle of the sector. [Take $\pi = \frac{22}{7}$]

Possible answers:

A. 60°

B. 120°

C. 150°

D. 150°

Correct answer: B

Solution. To calculate the angle of the sector, we use the formula for the area of a sector:

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$

Given:

- Area $= 51.3\,\text{cm}^2$
- Radius $r = 7\,\text{cm}$
- $\pi = \frac{22}{7}$

Substitute the values into the formula:

$$51.3 = \frac{\theta}{360} \times \frac{22}{7} \times 7^2$$

Simplify $7^2 = 49$ and $\frac{22}{7} \times 49 = 154$:

$$51.3 = \frac{\theta}{360} \times 154$$

Solve for θ:

$$\frac{\theta}{360} = \frac{51.3}{154} \approx 0.3331$$

$$\theta \approx 0.3331 \times 360 \approx 119.9^\circ$$

Rounded to the nearest whole number, $\theta \approx 120^\circ$.

Answer: B. 120°

Main Concepts:

1. **Sector Area Formula**: Connecting angle, radius, and area.
2. **Algebraic Rearrangement**: Isolating the angle variable.
3. **Numerical Substitution**: Plugging in values and simplifying.
4. **Unit Consistency**: Ensuring the angle is in degrees and properly rounded.

Problem 44

A cliff on the bank of a river 87 m high. A boat on the river is 22 m away from the cliff. Calculate, correct to the nearest degree, the angle of depression of the boat from the top of the cliff.

Possible answers:

A. 76°

B. 64°

C. 36°

D. 24°

Correct answer: A

Solution. To solve the problem of finding the angle of depression from the top of the cliff to the boat, we use trigonometry in a right triangle. The cliff height (87 m) is the vertical side opposite the angle, and the horizontal distance (22 m) is the adjacent side. The tangent of the angle of depression is calculated as:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{87}{22} \approx 3.9545$$

Using the arctangent function, we find:

$$\theta = \arctan(3.9545) \approx 75.7^\circ$$

Rounded to the nearest degree, the angle of depression is **76**.

Answer: A. 76°

—

Main Concepts:

1. **Angle of Depression:** The angle formed between the horizontal line from the observer and the line of sight to the object below, congruent to the angle of elevation due to alternate interior angles.

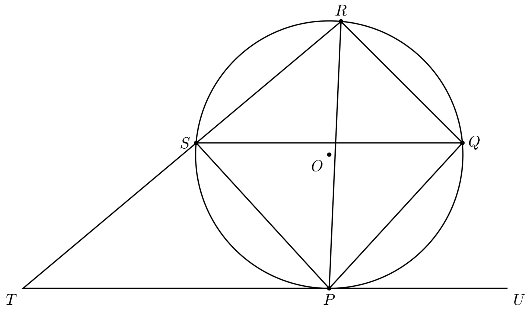
2. **Right Triangle Trigonometry:** The problem forms a right triangle with vertical leg (87 m) and horizontal leg (22 m).

3. **Tangent Function:** Relates the angle to the opposite and adjacent sides: $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$.

4. **Inverse Trigonometric Calculation:** Using arctan to solve for the angle from the tangent ratio.

5. **Rounding:** Approximating the result (75.7) to the nearest degree (76).

Problem 45



In the diagram, TU is a tangent to the circle at P. If $\angle\text{PTS} = 44°, \angle\text{SQP} = 35°$, find $\angle\text{PST}$.
Possible answers:
A. 101°
B. 125°
C. 130°
D. 135°
Correct answer: A
Solution. Given:

- $\angle SPT = 35°$ (by the Alternate Segment Theorem)
- $\angle PTS = 44°$

In triangle PST, the sum of angles is 180°:

$$\angle PTS + \angle SPT + \angle PST = 180°$$

Substituting the known values:

$$44° + 35° + \angle PST = 180°$$

Solving for $\angle PST$:

$$\angle PST = 180° - 44° - 35° = 101°$$

Final Answer:

$$\boxed{101°}$$

Problem 46

The probability that Amaka will pass an examination is $\frac{3}{7}$ and that Bala will pass is $\frac{4}{9}$. Find the probability that both will pass the examination.

Possible answers:

A. $\frac{2}{21}$

B. $\frac{4}{21}$

C. $\frac{5}{21}$

D. $\frac{9}{21}$

Correct answer: B

Solution. To find the probability that both Amaka and Bala will pass the examination, we use the **multiplication rule for independent events** since their outcomes do not affect each other.

1. **Amaka's probability of passing**: $\frac{3}{7}$

2. **Bala's probability of passing**: $\frac{4}{9}$

Multiplying these probabilities gives:

$$\frac{3}{7} \times \frac{4}{9} = \frac{12}{63}$$

Simplifying $\frac{12}{63}$ by dividing both numerator and denominator by their GCD (3):

$$\frac{12 \div 3}{63 \div 3} = \frac{4}{21}$$

Answer: B. $\frac{4}{21}$

—

Main Concepts:

1. **Independent Events**: Amaka's and Bala's exam results are independent, as no dependency is stated.

2. **Multiplication Rule**: For independent events A and B, $P(A \text{ and } B) = P(A) \times P(B)$.

3. **Fraction Simplification**: Reducing $\frac{12}{63}$ to $\frac{4}{21}$ by dividing by the GCD (3).

Problem 47
Which of the following points lies on the line $3x - 8y = 11$?
Possible answers:
A. $(1, 1)$
B. $(1, -1)$
C. $(-1, 1)$
D. $(-1, -1)$
Correct answer: B
Solution. To determine which point lies on the line $3x - 8y = 11$, substitute the coordinates of each option into the equation and check if the left-hand side equals 11:
- **Option A $(1, 1)$:**
$3(1) - 8(1) = 3 - 8 = -5 \neq 11.$
Not a solution.
- **Option B $(1, -1)$:**
$3(1) - 8(-1) = 3 + 8 = 11.$
Valid solution.
- **Option C $(-1, 1)$:**
$3(-1) - 8(1) = -3 - 8 = -11 \neq 11.$
Not a solution.
- **Option D $(-1, -1)$:**
$3(-1) - 8(-1) = -3 + 8 = 5 \neq 11.$
Not a solution.
Answer: $\boxed{B}$
—
Main Concepts:
1. **Substitution:** Plugging the x- and y-coordinates of each point into the linear equation to verify if they satisfy it.
2. **Sign Handling:** Carefully managing negative values, such as recognizing that $-8(-1)$ becomes $+8$.
3. **Equation Validation:** Confirming equality between the left- and right-hand sides of the equation after substitution.
These steps ensure systematic verification of solutions for linear equations.

Problem 48

Find the range of the following set of numbers: 28, 29, 39, 38, 33, 37, 26, 20, 15, and 25.
Possible answers:
A. 22
B. 24
C. 25
D. 27
Correct answer: B

Solution. The range of a set of numbers is calculated by subtracting the smallest value from the largest value. For the given set:
Numbers: 28, 29, 39, 38, 33, 37, 26, 20, 15, 25
1. **Identify the maximum**: The largest number is **39**.
2. **Identify the minimum**: The smallest number is **15**.
3. **Calculate the range**: $39 - 15 = 24$.
Answer: **B. 24**
Main Concepts:
- **Range**: Defined as the difference between the highest and lowest values in a dataset.
- **Sorting**: Organizing data in ascending or descending order helps easily identify the extremes.
- **Data Accuracy**: Ensuring no values are overlooked when determining the maximum and minimum.

Problem 49

The fourth and eighth terms of an arithmetic progression are 16 and 40 respectively. Find the common difference.

Possible answers:

A. -6

B. 6

C. -2

D. 2

Correct answer: B

Solution. The common difference of the arithmetic progression is found by using the nth term formula $a_n = a_1 + (n-1)d$. Given the 4th term $a_4 = 16$ and the 8th term $a_8 = 40$, we set up the equations:

$$\begin{cases} a_1 + 3d = 16 \\ a_1 + 7d = 40 \end{cases}$$

Subtracting the first equation from the second eliminates a_1:

$$4d = 24 \implies d = 6.$$

Verifying by substituting $d = 6$ confirms the terms match the given values. The common difference is **6**.

Main Concepts:

1. **Arithmetic Progression (AP):** A sequence where each term increases by a constant difference.

2. **Term Formula:** $a_n = a_1 + (n-1)d$ relates the nth term to the first term and common difference.

3. **System of Equations:** Derived from given terms to solve for unknowns.

4. **Elimination Method:** Subtracting equations to isolate and solve for d.

5. **Consistency Check:** Substituting back ensures the solution satisfies the original problem.

Answer: B. 6

Problem 50

For what values of y is $\frac{y+2}{8y^2-10y+3}$ not defined?
Possible answers:
A. $\frac{-3}{4}, \frac{1}{2}$
B. $\frac{-3}{4}, \frac{-1}{2}$
C. $\frac{3}{4}, \frac{1}{2}$
D. $\frac{3}{4}, \frac{-1}{2}$
Correct answer: C

Solution. To determine the values of y for which the expression $\frac{y+2}{8y^2-10y+3}$ is undefined, we find where the denominator equals zero:

$$8y^2 - 10y + 3 = 0.$$

Step 1: Solve the quadratic equation
Using the quadratic formula $y = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ with $a = 8$, $b = -10$, and $c = 3$:

$$y = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 8 \cdot 3}}{2 \cdot 8} = \frac{10 \pm \sqrt{100 - 96}}{16} = \frac{10 \pm 2}{16}.$$

This gives two solutions:

$$y = \frac{10+2}{16} = \frac{12}{16} = \frac{3}{4}, \quad y = \frac{10-2}{16} = \frac{8}{16} = \frac{1}{2}.$$

Step 2: Verify via factoring
The denominator factors as $(4y - 3)(2y - 1)$, confirming the roots $y = \frac{3}{4}$ and $y = \frac{1}{2}$.
Final Answer
The expression is undefined at $y = \frac{3}{4}$ and $y = \frac{1}{2}$.
Correct choice: C. $\frac{3}{4}, \frac{1}{2}$.
Main Concepts:
1. **Undefined Expressions:** A rational expression is undefined where its denominator is zero.
2. **Quadratic Equations:** Solving $8y^2 - 10y + 3 = 0$ via the quadratic formula or factoring identifies these critical y-values.
3. **Factoring Quadratics:** Breaking down the quadratic into binomials confirms the roots efficiently.

Problem 51

The time (t) taken to buy fuel at a filling station varies directly as the number of vehicles (V) in a queue and varies inversely as the number of pumps (P), available at the station. In a station with 5 pumps, it took 10 minutes to fuel 20 vehicles. Find the:
a) relationship between t, P, and V, represent solution as $\frac{aV}{2P}$, where a is an integer, write the answer as a;
b) time it takes to fuel 50 vehicles at a station with 2 pumps;
c) number of pumps required to fuel 40 vehicles in 20 minutes.

Correct answer:
5
Correct answer:
62.5
Correct answer:
5
Solution. **a)** The relationship between time t, number of vehicles V, and number of pumps P is derived as follows:
Given $t \propto \frac{V}{P}$, we write $t = \frac{aV}{2P}$. Using the values $t = 10$, $V = 20$, and $P = 5$:

$$10 = \frac{a \cdot 20}{2 \cdot 5} \implies 10 = \frac{20a}{10} \implies a = 5.$$

Answer: $\boxed{5}$.
b) Using $t = \frac{5V}{2P}$, substitute $V = 50$ and $P = 2$:

$$t = \frac{5 \cdot 50}{2 \cdot 2} = \frac{250}{4} = 62.5 \text{ minutes}.$$

Answer: $\boxed{62.5}$.
c) Rearrange $t = \frac{5V}{2P}$ to solve for P:

$$P = \frac{5V}{2t}.$$

Substitute $V = 40$ and $t = 20$:

$$P = \frac{5 \cdot 40}{2 \cdot 20} = \frac{200}{40} = 5.$$

Answer: $\boxed{5}$.
Main Concepts:
1. **Direct and Inverse Variation:** Time t varies directly with V and inversely with P, leading to the formula $t = k\frac{V}{P}$.
2. **Proportionality Constant:** Solving for k using given values establishes the relationship $t = \frac{5V}{2P}$.
3. **Algebraic Manipulation:** Rearranging the formula to solve for unknowns (e.g., P) demonstrates flexibility in applying the derived relationship.
4. **Unit Consistency:** Ensuring units for time, vehicles, and pumps align correctly in calculations.

Problem 52

A car travelled at a distance of $(2x + 13)$ km at 67.5 km/h and $(5x - 20)$ km at 72 km/h. If the total time for the entire journey was 90 minutes, find the value of x.

Correct answer:
16

Solution. To solve the problem, we start by noting the total time for the journey is 90 minutes (1.5 hours). Using the formula Time $= \frac{\text{Distance}}{\text{Speed}}$, we express the time for each segment of the trip:
- Time for the first distance: $\frac{2x+13}{67.5}$ hours
- Time for the second distance: $\frac{5x-20}{72}$ hours

Adding these times gives the equation:

$$\frac{2x + 13}{67.5} + \frac{5x - 20}{72} = 1.5$$

Converting denominators to fractions and simplifying:

$$\frac{4x + 26}{135} + \frac{5x - 20}{72} = 1.5$$

Multiplying through by the common denominator (1080) to eliminate fractions:

$$8(4x + 26) + 15(5x - 20) = 1620$$

Simplifying further:

$$32x + 208 + 75x - 300 = 1620 \implies 107x - 92 = 1620$$

Solving for x:

$$107x = 1712 \implies x = 16$$

Verification:
- First distance: $2(16) + 13 = 45\,\text{km} \implies \text{Time} = \frac{45}{67.5} = 0.666\,\text{hours}$ (40 minutes)
- Second distance: $5(16) - 20 = 60\,\text{km} \implies \text{Time} = \frac{60}{72} = 0.833\,\text{hours}$ (50 minutes)
- Total time: $40 + 50 = 90\,\text{minutes}$, confirming $x = 16$.

Main Concepts:
1. **Time-Distance-Speed Relationship**: Applied to calculate time for each journey segment.
2. **Unit Conversion**: Ensured consistency between minutes and hours.
3. **Algebraic Manipulation**: Formulated and simplified the equation.
4. **Linear Equation Solving**: Solved for x through isolation.
5. **Verification**: Validated the solution by substitution.

Final Answer: $\boxed{16}$

Problem 53

A circular floor of a building is to be tiled with ceramic tiles each of side 40 cm. If the perimeter of the floor is 66 m, calculate, correct to the nearest whole number, the number of tiles required to completely tile the floor. [Take $\pi = \frac{22}{7}$]

Correct answer:
2166

Solution. To calculate the number of tiles required to tile a circular floor with a perimeter (circumference) of 66 meters, we start by determining the radius of the floor using the circumference formula $C = 2\pi r$:

$$66 = 2 \times \frac{22}{7} \times r \implies r = \frac{66 \times 7}{2 \times 22} = 10.5 \text{ meters}$$

Next, we find the area of the circular floor using the radius $r = 10.5$ meters:

$$\text{Area} = \pi r^2 = \frac{22}{7} \times (10.5)^2 = \frac{22}{7} \times 110.25 = 346.5 \text{ square meters}$$

Convert the area to square centimeters (1 square meter = 10,000 square centimeters):

$$346.5 \text{ square meters} = 346.5 \times 10,000 = 3,465,000 \text{ square centimeters}$$

Each tile has a side length of 40 cm, so the area of one tile is:

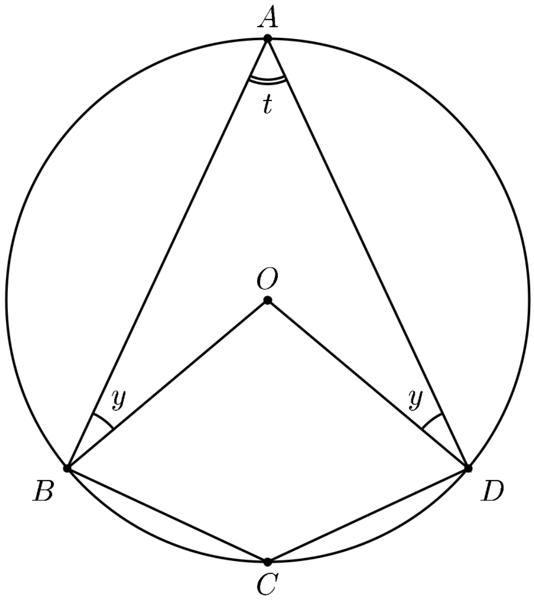
$$\text{Tile area} = 40 \times 40 = 1600 \text{ square centimeters}$$

The number of tiles required is the total floor area divided by the tile area:

$$\text{Number of tiles} = \frac{3,465,000}{1600} = 2165.625$$

Rounding 2165.625 to the nearest whole number gives **2166**.
Main Concepts:
1. **Circumference of a Circle**: Used to find the radius from the given perimeter.
2. **Area of a Circle**: Calculated using the radius to determine the floor area.
3. **Unit Conversion**: Converted square meters to square centimeters for consistency with tile dimensions.
4. **Area Division**: Divided total floor area by tile area to find the number of tiles.
5. **Rounding**: Adjusted the result to the nearest whole number to account for complete tiles.
$\boxed{2166}$

Problem 54



In the diagram,there is a circle with center O. The quadratic OBCD is a rhombus such that ∠ADO = ∠OBA = y and ∠BAD = t. Find:
a) the value of t;
b) the value of y;
c) ∠ADC.

Correct answer:
60
Correct answer:
30
Correct answer:
90

Solution. To find $\angle ADC$, we proceed with the following steps:

$$\angle OBD + \angle ODB + \angle BOD = 180^\circ$$

$$2\angle ODB = 180^\circ - \angle BOD$$

$$2\angle ODB = 180^\circ - 2t$$

$$2\angle ODB = 180^\circ - 120^\circ$$

$$2\angle ODB = 60^\circ$$

$$\angle ODB = \frac{60^\circ}{2} = 30^\circ$$

Now, $\angle ADC$ is the sum of three angles:

$$\angle ADC = \angle ADO + \angle ODB + \angle BDC$$

Given that $\angle ADO = y = 30^\circ$, $\angle ODB = 30^\circ$, and $\angle BDC = 30^\circ$ (since $\triangle BDC$ is isosceles with $BD = BC$ and $\angle BDC = \angle BCD$),

$$\angle ADC = 30^\circ + 30^\circ + 30^\circ = 90^\circ$$

Therefore, the measure of $\angle ADC$ is $\boxed{90^\circ}$.

Problem 55

A basket contains 3 gold-plated marbles, 4 diamond marbles, and some silver marbles, all of the same size and shape. Two marbles were drawn from the basket at random one after the other without replacement. If the probability that the two marbles were all silver is $\frac{1}{15}$, find the number of silver marbles.

Correct answer:
3

Solution. To solve the problem, let's denote the number of silver marbles as S. The total number of marbles in the basket is $3 + 4 + S = 7 + S$. The probability of drawing two silver marbles without replacement is given by:

$$\frac{S}{7+S} \times \frac{S-1}{6+S} = \frac{1}{15}.$$

Simplifying this equation:
1. Cross-multiply to eliminate fractions:

$$15S(S-1) = (7+S)(6+S).$$

2. Expand both sides:

$$15S^2 - 15S = S^2 + 13S + 42.$$

3. Rearrange terms to form a quadratic equation:

$$14S^2 - 28S - 42 = 0.$$

4. Simplify by dividing by 14:

$$S^2 - 2S - 3 = 0.$$

5. Solve using the quadratic formula $S = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$:

$$S = \frac{2 \pm \sqrt{16}}{2} \implies S = 3 \text{ or } S = -1.$$

Since S must be positive, the solution is $S = 3$. Verification confirms this satisfies the original probability condition.
Answer: The number of silver marbles is $\boxed{3}$.
—
Main Concepts:
1. **Probability Without Replacement:** Adjusting probabilities after each draw to account for reduced quantities.
2. **Combining Dependent Probabilities:** Multiplying sequential probabilities for dependent events.
3. **Equation Setup:** Translating a probability condition into an algebraic equation.
4. **Quadratic Equation Solving:** Expanding, simplifying, and solving quadratic equations.

5. **Validation:** Ensuring solutions are physically meaningful (positive integers) and satisfy the original problem.

Problem 56

Given that $(x+2), (4x+3)$, and $(7x+24)$ are consecutive terms of a geometric progression (G.P.), find the:

a) values of x, represent the answer as $\frac{a}{b}, \frac{c}{d}$, where a is a negative integer and b, c, d are positive integers, write the answer as a, b, c, d;

b) common ratio.

Correct answer:
-13,9,3,1
Correct answer:
-5::3

Solution. Given that $(x+2)$, $(4x+3)$, and $(7x+24)$ are consecutive terms of a geometric progression (G.P.), we need to find the values of x and the common ratio.

1. **Setting Up the Equation:**
In a G.P., the ratio between consecutive terms is constant. Therefore, we set up the equation:

$$\frac{4x+3}{x+2} = \frac{7x+24}{4x+3}$$

2. **Cross-Multiplying and Simplifying:**
Cross-multiplying gives:

$$(4x+3)^2 = (x+2)(7x+24)$$

Expanding both sides:

$$16x^2 + 24x + 9 = 7x^2 + 38x + 48$$

Simplifying:

$$9x^2 - 14x - 39 = 0$$

3. **Solving the Quadratic Equation:**
Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ with $a = 9$, $b = -14$, and $c = -39$:

$$x = \frac{14 \pm 40}{18}$$

This gives the solutions:

$$x = 3 \quad \text{and} \quad x = -\frac{13}{9}$$

4. **Validating the Solutions:**
- For $x = 3$, the terms are 5, 15, and 45 with a common ratio of 3.
- For $x = -\frac{13}{9}$, the terms are $\frac{5}{9}$, $-\frac{25}{9}$, and $\frac{125}{9}$ with a common ratio of -5.
5. **Common Ratio Calculation:**
- For $x = 3$, the common ratio is 3.
- For $x = -\frac{13}{9}$, the common ratio is -5.

Final Answer
a) The values of x are $\boxed{-13, 9, 3, 1}$.
b) The common ratios are $\boxed{-5}$ and $\boxed{3}$.
Main Concepts
1. **Geometric Progression (G.P.):** The ratio between consecutive terms is constant.
2. **Setting Up the Equation:** Using the property of G.P. to equate the ratios of consecutive terms.
3. **Solving Quadratic Equations:** Cross-multiplying and simplifying to form a quadratic equation, then solving using the quadratic formula.
4. **Validation of Solutions:** Checking solutions to ensure valid G.P. terms and common ratios.
5. **Common Ratio Calculation:** Determining the ratio for each valid x.

Problem 57

AGE	5	6	7	8	9	10
FREQUENCY	2	2x − 1	y + 2	4	2	y − 1

The table shows the ages of 20 children in a household.
Given that $x : y = 1 : 2$.
a) values of x and y.
b) mean ages of the children, give the answer as a decimal number.

Correct answer:
2,4
Correct answer:
7.5
Solution.

Question:

The table below shows the ages of 20 children in a household:

Age	5 | 6 | 7 | 8 | 9 | 10 |
Frequency	2 | $2x - 1$ | $y + 2$ | 4 | 2 | $y - 1$ |

Given: $\frac{x}{y} = \frac{1}{2}$

Solution

(a) Find the values of x and y

We are told the total frequency is 20. So:

$$2 + (2x - 1) + (y + 2) + 4 + 2 + (y - 1) = 20$$

Simplify:

$$(2x - 1) + (y + 2) + (y - 1) + 2 + 4 + 2 = 20$$

$$2x + 2y + (2 + 4 + 2 - 1 + 2 - 1) = 20 \Rightarrow 2x + 2y + 8 = 20$$

$$2x + 2y = 12 \Rightarrow x + y = 6 \tag{1}$$

Also, $\frac{x}{y} = \frac{1}{2} \Rightarrow x = \frac{1}{2}y$
Substitute equation (2) into equation (1):

$$\frac{1}{2}y + y = 6 \Rightarrow \frac{3}{2}y = 6 \Rightarrow y = 4$$

Now from equation (2):

$$x = \frac{1}{2} \cdot 4 = 2$$

Answer: $x = 2, \quad y = 4$

(b) Find the mean age of the children

Now substitute the values of $x = 2, y = 4$ into the frequency table:

Age	Frequency
5	2
6	$2(2) - 1 = 3$
7	$4 + 2 = 6$
8	4
9	2
10	$4 - 1 = 3$

Now compute total sum of ages:

$$\text{Total age sum} = 5 \cdot 2 + 6 \cdot 3 + 7 \cdot 6 + 8 \cdot 4 + 9 \cdot 2 + 10 \cdot 3$$

$$= 10 + 18 + 42 + 32 + 18 + 30 = 150$$

$$\text{Total children} = 20 \Rightarrow \text{Mean age} = \frac{150}{20} = 7.5$$

Answer: Mean age = $\boxed{7.5}$

Problem 58

Two observers Abu and Badu, 46 m apart, observe a bird on a vertical pole from the same side of the bird. The angles of elevation of the bird from Abu's and Badu's eye are 40^o and 48^o respectively. If at the foot of the pole Abu and Badu are on same horizontal;
a) illustrate the information in a diagram;
b) calculate, correct to one decimal place, the height of the pole.

Correct answer:
graph
Correct answer:
157.9
Solution. **a) Diagram Illustration:**
The diagram consists of a vertical pole with a bird at point P. The foot of the pole is labeled C. Observers Abu (A) and Badu (B) are 46 meters apart on the same horizontal line as C. Badu is closer to the pole, so $BC = x$ and $AC = x + 46$. Right triangles $\triangle APC$ and $\triangle BPC$ are formed with angles of elevation $40°$ and $48°$ at A and B, respectively. The height $h = PC$ is the height of the pole.
b) Height Calculation:
1. **Define Variables**:
- Let $BC = x$ (Badu's horizontal distance to the pole).
- Then $AC = x + 46$ (Abu's horizontal distance to the pole).
2. **Set Up Equations**:
- For Badu: $\tan(48°) = \frac{h}{x} \Rightarrow h = x\tan(48°)$.
- For Abu: $\tan(40°) = \frac{h}{x+46} \Rightarrow h = (x + 46)\tan(40°)$.
3. **Equate and Solve for x**:

$$x \tan(48°) = (x + 46) \tan(40°)$$

Substitute $\tan(48°) \approx 1.1106$ and $\tan(40°) \approx 0.8391$:

$$1.1106x = 0.8391x + 38.5986 \Rightarrow 0.2715x = 38.5986 \Rightarrow x \approx 142.17\,\text{m}$$

4. **Calculate Height h**:

$$h = x \tan(48°) \approx 142.17 \times 1.1106 \approx 157.9\,\text{m}$$

Final Answer:
The height of the pole is $\boxed{157.9\,\text{m}}$.
Main Concepts:
1. **Trigonometric Ratios**: Tangent function relates height and horizontal distance.
2. **Angle of Elevation**: Determines the slope of the line of sight.
3. **System of Equations**: Derived from two observers' perspectives.
4. **Linear Equation Solving**: Algebraic manipulation to find the unknown distance.
5. **Position Analysis**: Closer observer has a steeper angle of elevation.

Problem 59

The diameter of a circle centre O is 26 cm . If a chord PQ is drawn such that it is 5 cm from O to the centre of the chord, calculate, correct to the nearest whole number:
a) $\angle$POQ;
b) the area of the minor segment formed by the chord PQ. [Take $\pi = \frac{22}{7}$]

Correct answer:
135
Correct answer:
139
Solution. **Solution:**
Given:
- Diameter of the circle = 26 cm Radius $r = 13\,\text{cm}$.
- Distance from center O to the midpoint of chord PQ = 5 cm.
a) Calculate $\angle$POQ:
1. The perpendicular distance from O to chord PQ forms a right triangle $\triangle OMP$, where M is the midpoint of PQ.
2. In $\triangle OMP$:
- $OM = 5\,\text{cm}$ (given),
- $OP = 13\,\text{cm}$ (radius).
3. Use trigonometry to find $\angle$MOP:

$$\cos(\angle\text{MOP}) = \frac{OM}{OP} = \frac{5}{13} \implies \angle\text{MOP} = \arccos\left(\frac{5}{13}\right) \approx 67.38^\circ.$$

4. Since $\angle\text{POQ} = 2 \times \angle\text{MOP}$:

$$\angle\text{POQ} \approx 2 \times 67.38^\circ = 134.76^\circ \approx 135^\circ \text{ (nearest whole number).}$$

b) Calculate the area of the minor segment:
1. **Area of sector OPQ:**

$$\text{Sector Area} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{135^\circ}{360^\circ} \times \frac{22}{7} \times 13^2 \approx 199.18\,\text{cm}^2.$$

2. **Area of $\triangle OPQ$:**

$$\text{Triangle Area} = \frac{1}{2} \times OP \times OQ \times \sin(\theta) = \frac{1}{2} \times 13 \times 13 \times \sin(135^\circ) \approx 59.75\,\text{cm}^2.$$

3. **Area of minor segment:**

$$\text{Segment Area} = \text{Sector Area} - \text{Triangle Area} \approx 199.18 - 59.75 = 139.43\,\text{cm}^2 \approx 139\,\text{cm}^2 \text{ (nearest whole number).}$$

Final Answers:
a) $\boxed{135^\circ}$
b) $\boxed{139\,\text{cm}^2}$
Main Concepts:
1. **Circle Geometry:** Perpendicular distance from the center to a chord bisects the chord.

2. **Right Triangle Trigonometry:** Used to find the central angle.
3. **Law of Cosines:** Verified the central angle in an isosceles triangle.
4. **Sector and Triangle Areas:** Calculated using central angle and radius.
5. **Segment Area:** Derived by subtracting triangle area from sector area.
6. **Approximation:** Rounded results to the nearest whole number as required.

Problem 60

(a) In a man's will, he gave $\frac{2}{5}$ of the total acres of the farm to the wife and $\frac{1}{3}$ of what is left to the family. The rest of the farm was to be shared amongst his three children in the ratio $3 : 5 : 2$. Given that, the child who had the least share received 8 acres, calculate the:
(i) total acres the man left.
(ii) number of acres the wife received.
(b) The price of a Television set is $\$1,600.00$. It can be purchased by a deposit of $\$400.00$ and the rest of the amount paid by 12 monthly installments at 25% per annum simple interest. If the Television set is purchased by installment, find the total cost.

Correct answer:
100
Correct answer:
40
Correct answer:
1900
Solution. **Solution:**
(a) Calculating the Total Acres and Wife's Share
1. **Total Acres (T):**
- Wife receives $\frac{2}{5}T$, leaving $\frac{3}{5}T$.
- Family receives $\frac{1}{3} \times \frac{3}{5}T = \frac{1}{5}T$, leaving $\frac{2}{5}T$.
- The remaining $\frac{2}{5}T$ is split among three children in the ratio $3 : 5 : 2$. The smallest share (2 parts) equals 8 acres.
- Let x represent one part: $2x = 8 \Rightarrow x = 4$.
- Total children's share: $3x + 5x + 2x = 10x = 40$ acres.
- Solve $\frac{2}{5}T = 40 \Rightarrow T = 100$ acres.
2. **Wife's Share:**
- Wife received $\frac{2}{5} \times 100 = 40$ acres.
(b) Total Cost of Television via Installments
- Deposit: $400.
- Amount financed: $\$1,600 - \$400 = \$1,200$.
- Simple Interest: $I = 1,200 \times 0.25 \times 1 = \$300$.
- Total repayment: $\$1,200 + \$300 = \$1,500$.
- Total cost: $\$400 + \$1,500 = \$1,900$.
Final Answers:
- (i) Total acres: **100 acres**
- (ii) Wife's share: **40 acres**
- (b) Total installment cost: **$1,900.00**
Main Concepts:
- **Part (a):** Fractional distribution, successive deductions, ratio interpretation, and solving linear equations to backtrack the total value.
- **Part (b):** Simple interest calculation, understanding deposits, and summing total costs including interest.

Problem 61

a) Find the equation of the line that passes through the origin and the point of intersection of the lines $x+2y=7$ and $x-y=4$. Represent solution as $x+ay=0$, where a is an integer, write the answer as a.

b) The ratio of an interior angle to an exterior angle of a regular polygon is $4:1$. Find the: (i) value of the exterior angle; (ii) number of sides; and (iii) sum of the interior angles of the polygon.

Correct answer:
-5
Correct answer:
36
Correct answer:
10
Correct answer:
1440

Solution. To solve part (a), we first find the intersection of the lines $x+2y=7$ and $x-y=4$. Solving these equations by substitution, we get $x=5$ and $y=1$. The line passing through the origin $(0,0)$ and the intersection point $(5,1)$ has a slope of $\frac{1}{5}$. Using the slope-intercept form, the equation is $y=\frac{1}{5}x$. Converting this to the form $x+ay=0$, we get $x-5y=0$, so $a=-5$.
For part (b), given the ratio of an interior angle to an exterior angle is 4:1, we let the exterior angle be x. The interior angle is $4x$, and since they are supplementary, $4x+x=180°$. Solving gives $x=36°$ (exterior angle). The number of sides n is $\frac{360°}{36°}=10$. The sum of the interior angles is $(10-2)\times 180°=1440°$.

Final Answer

a) $\boxed{-5}$

b) (i) $\boxed{36°}$

(ii) $\boxed{10}$

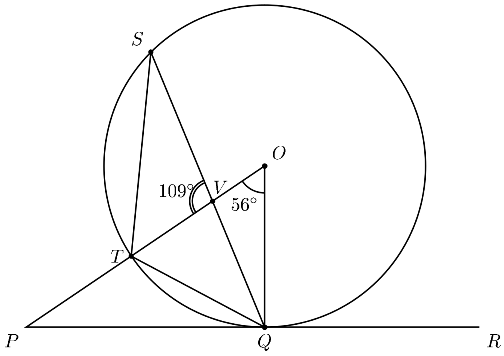
(iii) $\boxed{1440°}$

Main Concepts

a) **Intersection of Lines and Line Equation**: Solving a system of linear equations to find the intersection point, then using the two-point form to determine the line equation through the origin.

b) **Regular Polygon Angles**: Using the supplementary property of interior and exterior angles, relating the exterior angle to the number of sides, and calculating the sum of interior angles.

Problem 62



a) In the diagram, PR is a tangent to the circle O at Q, $\angle$POQ $= 56°$ and PO intersects SO at V such that $\angle$SVP $= 109°$. Calculate: (i) $\angle$TQP (ii) $\angle$QTS.
b) Simplify $\frac{2n^2-3n-2}{2n^2+3n+1} \times \frac{n^2-1}{n^2-4}$. Represent the answer as $\frac{n+a}{n+b}$, where a, b are integers, write the answer as a, b.

Correct answer:
28
Correct answer:
105
Correct answer:
-1,2
Solution. To solve the given problems, we utilized several geometric and algebraic concepts:
Part a)
Part b) Simplification of the algebraic expression:
1. **Factoring Quadratics**: Each quadratic expression was factored into its components.
2. **Simplifying Rational Expressions**: Common factors were canceled across numerators and denominators, leading to the simplified form $\frac{n-1}{n+2}$, expressed as $\frac{n+(-1)}{n+2}$.

Geometric Problem Solution with Explanations

Given Information

- $\overline{PR}$ is tangent to circle O at point Q
- Central angle $\angle POQ = 56°$
- Lines $\overline{PO}$ and $\overline{SO}$ intersect at V with $\angle SVP = 109°$

Part (i): Finding $\angle TQP$

$$\text{By the circle theorem: } 2 \times \angle TQP = \angle POQ$$

$$\text{(Angle at center is twice angle at circumference)}$$

$$\Rightarrow \angle TQP = \frac{\angle POQ}{2} = \frac{56^\circ}{2} = 28^\circ$$

Explanation: The angle $\angle TQP$ is an inscribed angle subtended by arc TP, while $\angle POQ$ is the central angle subtended by the same arc. According to the circle theorem, the central angle is always twice the measure of the inscribed angle subtending the same arc.

Part (ii): Finding $\angle QTS$

1. First, consider triangle SVP:

$$\angle SVP = 109^\circ \text{ (given)}$$

$$\angle VST = \angle TQP = 28^\circ \text{ (from part (i) as angles subtended by the same arc)}$$

$$\angle STV = 180^\circ - 109^\circ - 28^\circ = 43^\circ$$

   Explanation: The sum of angles in triangle SVP must be 180°. We know two angles ($\angle SVP$ and $\angle VSP$), so we can find the third.

2. Now examine triangle OQT:

$$\text{It is isosceles}$$
$$\angle OTQ = \frac{180^\circ - \angle TOQ}{2} = \frac{180 - 56}{2} = 62^\circ$$

3. Finally, for $\angle QTS$:

$$\angle QTS = \angle STO + \angle OTQ = 43 + 62 = 105^\circ$$

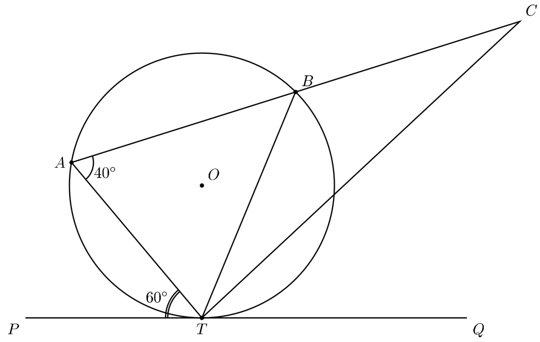
Final Answers

(i) $\angle TQP = \boxed{28^\circ}$

(ii) $\angle QTS = \boxed{105^\circ}$

Final Answers:
a) (i) $\boxed{62^\circ}$
a) (ii) $\boxed{75^\circ}$
b) $\boxed{-1, 2}$
Main Concepts:
- **Alternate Segment Theorem** and **Isosceles Triangle Properties** for part a(i).
- **Central and Inscribed Angles**, **Law of Sines**, and **Trigonometric Identities** for part a(ii).
- **Factoring Quadratics** and **Simplifying Rational Expressions** for part b.

Problem 63



a) In the diagram, there is a circle with centre O, PQ is a tangent to the circle at T and ABC is a straight line, TC bisects $\angle$BTQ, $\angle$BAT $= 44°$ and $\angle$PTA $= 60°$. Find $\angle$ACT.
b) The circumference of the base of a cylindrical tank is 11 m . The height of the tank is 3 m more than 6 times the base radius. Calculate the:
(i) radius, write the answer as a decimal number; (ii) height, write the answer as a decimal number; (iii) volume of the tank.

Correct answer:
38
Correct answer:
1.75
Correct answer:
13.5
Correct answer:
130
Solution.

Geometric Angle Solution

Given Information and Diagram Analysis

- BT is a line with point Q
- $\angle BTQ$ is bisected by line TC
- $\angle BAT = 44°$ (angles in alternate segments)
- $\angle ABT = \angle PTA = 60°$ (alternate segment angles)

Angle Calculations

1. **Bisected Angle Calculation:**

$$
\begin{aligned}
BTC &= \frac{\angle BTQ}{2} \quad \text{(since } TC \text{ bisects } \angle BTQ) \\
&= \frac{44^\circ}{2} \\
&= 22^\circ
\end{aligned}
$$

2. **Alternate Segment Angles:**

$$
\begin{aligned}
\angle BTCQ = \angle BAT = 44^\circ \quad \text{(angles in alternate segments)} \\
\angle ABT = \angle PTA = 60^\circ \quad \text{(alternate segment theorem)}
\end{aligned}
$$

3. **Exterior Angle Theorem Application:**

$$
\begin{aligned}
\angle ACT + \angle BTC = \angle ABT \quad \text{(exterior angle equals sum of opposite interior angles)} \\
\angle ACT + 22^\circ = 60^\circ \\
\angle ACT = 60^\circ - 22^\circ \\
= 38^\circ
\end{aligned}
$$

Final Answer

$$\boxed{\angle ACT = 38^\circ}$$

For part (b), the calculations are as follows:
1. **Circumference to Radius**: Given circumference $C = 11\,\text{m}$, solve for radius r using $C = 2\pi r$. This gives $r = \frac{11}{2\pi} \approx 1.75\,\text{m}$.
2. **Height Calculation**: Height h is $3\,\text{m} + 6r \approx 3 + 6 \times 1.75 = 13.5\,\text{m}$.
3. **Volume Calculation**: Volume $V = \pi r^2 h \approx \pi \times (1.75)^2 \times 13.5 \approx 129.9\,\text{m}^3$.
Final Answer
a) $\boxed{38^\circ}$
b) (i) $\boxed{1.75}$ m
(ii) $\boxed{13.5}$ m
(iii) $\boxed{129.9}$ m
Main Concepts
a) **Alternate Segment Theorem**, **Triangle Angle Sum**, **Angle Bisector Theorem**, and **Supplementary Angles** were used to determine the required angle through geometric properties and triangle angle calculations.
b) **Circumference formula**, **linear relationships**, and **volume formula for a cylinder** were applied to find the radius, height, and volume of the tank.