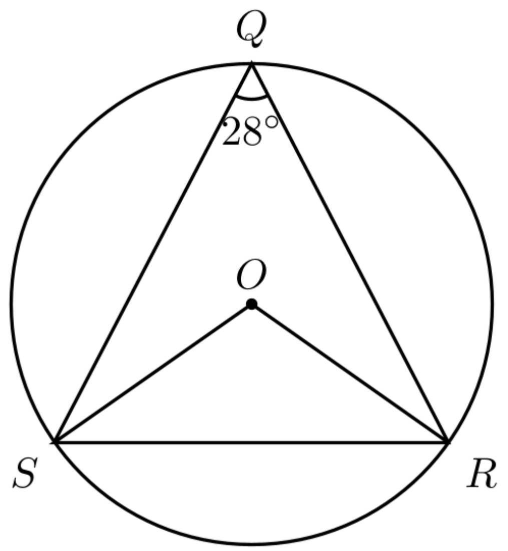
1 Solutions to problems public WAEC math 2023 By cheetahwaec.com

Problem 1



In the diagram, O is the center of the circle and $\angle$SQR = 28°. Find $\angle$ORS.
Possible answers:
A. 56°
B. 28°
C. 76°
D. 62°
Correct answer: D
Solution. Given the diagram where O is the center of the circle and $\angle \text{SQR} = 28°$, we need to find $\angle$ORS.
1. **Inscribed Angle Theorem**: The inscribed angle $\angle$SQR intercepts arc SR. According to the theorem, the measure of the inscribed angle is half the measure of its intercepted arc. Therefore, the measure of arc SR is $2 \times 28° = 56°$.
2. **Central Angle**: The central angle subtended by arc SR is $\angle$SOR. Since the central angle is equal to the measure of its arc, $\angle \text{SOR} = 56°$.
3. **Isosceles Triangle**: In triangle SOR, sides OS and OR are radii of the circle, making it isosceles. The base angles $\angle$OSR and $\angle$ORS are equal. The sum of the angles in a triangle is 180°, so:

$$\angle \text{OSR} + \angle \text{ORS} + \angle \text{SOR} = 180°$$

Substituting the known values:

$$\angle \text{OSR} + \angle \text{ORS} + 56° = 180°$$

Since $\angle \text{OSR} = \angle \text{ORS}$:

$$2 \times \angle \text{ORS} + 56^\circ = 180^\circ$$

Solving for $\angle$ORS:

$$2 \times \angle \text{ORS} = 124^\circ$$

$$\angle \text{ORS} = 62^\circ$$

Thus, the measure of $\angle$ORS is $\boxed{D}$.

Problem 2

Solve $1 + \sqrt[3]{x-3} = 4$.

Possible answers:

A. 30

B. 6

C. 12

D. 66

Correct answer: A

Solution. To solve the equation $1 + \sqrt[3]{x-3} = 4$, we start by isolating the cube root term. First, subtract 1 from both sides:

$$1 + \sqrt[3]{x-3} - 1 = 4 - 1$$

This simplifies to:

$$\sqrt[3]{x-3} = 3$$

Next, we cube both sides to eliminate the cube root:

$$(\sqrt[3]{x-3})^3 = 3^3$$

This simplifies to:

$$x - 3 = 27$$

Finally, we solve for x by adding 3 to both sides:

$$x - 3 + 3 = 27 + 3$$

Which simplifies to:

$$x = 30$$

Verification by substituting $x = 30$ back into the original equation confirms the solution is correct. Therefore, the answer is $\boxed{A}$.

Problem 3

The angle of a sector of a circle of radius 3.4 cm is $115°$. Find the area of the sector. (Take $\pi = \frac{22}{7}$)

Possible answers:

A. 11.6 cm^2

B. 12.7 cm^2

C. 10.2 cm^2

D. 9.4 cm^2

Correct answer: A

Solution. To find the area of the sector of a circle with radius $3.4\,\text{cm}$ and central angle $115°$, we use the formula for the area of a sector:

$$\text{Area} = \left(\frac{\theta}{360}\right) \times \pi r^2$$

Given:
- Radius $r = 3.4\,\text{cm}$
- Central angle $\theta = 115°$
- $\pi = \frac{22}{7}$

First, calculate the fraction of the circle represented by the sector:

$$\frac{115}{360} = \frac{23}{72}$$

Next, calculate the area of the entire circle:

$$\pi r^2 = \frac{22}{7} \times (3.4)^2$$

Compute 3.4^2:

$$3.4 \times 3.4 = 11.56$$

Now, calculate the area of the circle:

$$\frac{22}{7} \times 11.56 = \frac{22 \times 11.56}{7} = \frac{254.32}{7} \approx 36.3314$$

Finally, calculate the area of the sector:

$$\left(\frac{23}{72}\right) \times 36.3314 \approx 11.6057\,\text{cm}^2$$

Rounding to one decimal place, the area of the sector is approximately $11.6\,\text{cm}^2$.

$$\boxed{A}$$

Problem 4

There are 30 students in a class; 15 study woodwork and 13 study metal work; 6 study neither of the 2 subjects. How many student study woodwork but not metal work?

Correct answer:

B

Solution. Given there are 30 students in a class, where 15 study woodwork, 13 study metalwork, and 6 study neither subject, we need to find the number of students who study woodwork but not metalwork.

First, we calculate the number of students who study at least one of the subjects:

$$30 - 6 = 24$$

Using the principle of inclusion-exclusion for sets, we denote:
- $|W|$ as the number of students studying woodwork (15),
- $|M|$ as the number of students studying metalwork (13),
- $|W \cup M|$ as the number of students studying at least one subject (24),
- $|W \cap M|$ as the number of students studying both subjects.

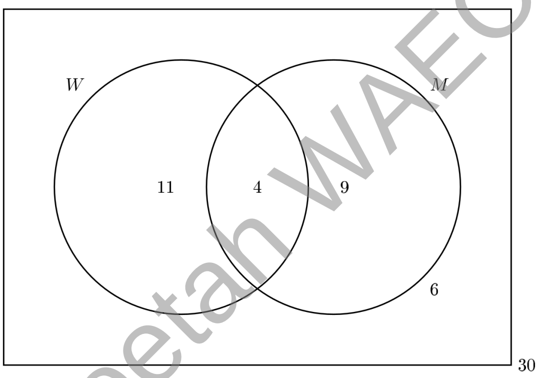


Figure 1:

The formula for the union of two sets is:

$$|W \cup M| = |W| + |M| - |W \cap M|$$

Plugging in the known values:

$$24 = 15 + 13 - |W \cap M|$$

Solving for $|W \cap M|$:

$$24 = 28 - |W \cap M| \implies |W \cap M| = 28 - 24 = 4$$

The number of students studying woodwork but not metalwork is the total number of woodwork students minus those studying both:

$$|W| - |W \cap M| = 15 - 4 = 11$$

Thus, the number of students who study woodwork but not metalwork is $\boxed{11}$.

Problem 5

Mr Manu is 4 times as old as his son, Adu; 7 years ago the sum of their ages was 76. How old is Adu?

Possible answers:

A. 22 years

B. 12 years

C. 18 years

D. 15 years

Correct answer: C

Solution. Let x be Adu's current age. According to the problem, Mr. Manu is 4 times as old as Adu, so Mr. Manu's current age is $4x$.

Seven years ago, Adu's age was $x - 7$ and Mr. Manu's age was $4x - 7$. The sum of their ages 7 years ago is given as 76. We can set up the equation:

$$(x - 7) + (4x - 7) = 76$$

Simplifying the equation:

$$x + 4x - 7 - 7 = 76$$

$$5x - 14 = 76$$

Adding 14 to both sides:

$$5x = 90$$

Dividing both sides by 5:

$$x = 18$$

Thus, Adu is currently 18 years old. Verifying, if Adu is 18, Mr. Manu is $4 \times 18 = 72$. Seven years ago, Adu was $18 - 7 = 11$ and Mr. Manu was $72 - 7 = 65$. The sum of their ages then is $11 + 65 = 76$, which matches the given information.

Therefore, the correct answer is $\boxed{C}$.

Problem 6

The angle of elevation of the top of a building from a point Z on the ground is $50°$. If the height of the building is 124 m, find the distance from Z to the foot of the building.

Possible answers:

A. 147.78 m

B. 104.05 m

C. 161.87 m

D. 192.91 m

Correct answer: B

Solution. To find the distance from point Z to the foot of the building, we use the tangent of the angle of elevation. The height of the building is the opposite side of the right triangle, and the distance we need to find is the adjacent side.

Given:

- Angle of elevation: $50°$
- Height of the building (opposite side): 124 meters

We use the tangent function:

$$\tan(50°) = \frac{\text{opposite}}{\text{adjacent}} = \frac{124}{\text{distance}}$$

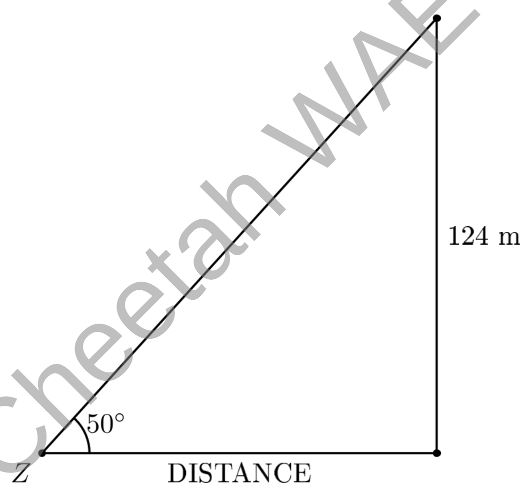


Figure 2:

Rearranging to solve for the distance:

$$\text{distance} = \frac{124}{\tan(50°)}$$

Calculating $\tan(50°) \approx 1.191753592$:

$$\text{distance} = \frac{124}{1.191753592} \approx 104.05$$

Thus, the distance from Z to the foot of the building is $\boxed{B}$.

Problem 7
One-third of the sum of two numbers is 12, twice their difference is 12. Find the numbers.
Possible answers:
A. 22 and 14
B. 20 and 16
C. 21 and 15
D. 23 and 13
Correct answer: C
Solution. Let the two numbers be x and y.
The first condition states that one-third of the sum of the two numbers is 12:

$$\frac{1}{3}(x + y) = 12$$

Multiplying both sides by 3, we get:

$$x + y = 36$$

The second condition states that twice their difference is 12:

$$2(x - y) = 12$$

Dividing both sides by 2, we get:

$$x - y = 6$$

We now have the system of equations:
1. $x + y = 36$
2. $x - y = 6$
Adding these two equations:

$$(x + y) + (x - y) = 36 + 6$$

$$2x = 42$$

$$x = 21$$

Substituting $x = 21$ back into the first equation:

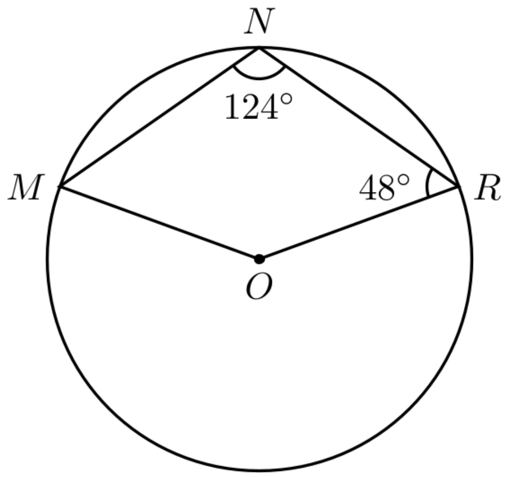
$$21 + y = 36$$

$$y = 15$$

Thus, the numbers are 21 and 15. Verifying the conditions:
- The sum is $21 + 15 = 36$, and one-third of 36 is 12.
- The difference is $21 - 15 = 6$, and twice 6 is 12.

Both conditions are satisfied, so the answer is $\boxed{C}$.

Problem 8



In the diagram above, M, N, R are points on the circle centre $O; \angle ORN = 48°$ and $\angle RNM = 124°$. Find $\angle OMN$.
Possible answers:
A. $58°$
B. $64°$
C. $48°$
D. $76°$
Correct answer: D
Solution. Given the diagram with points M, N, and R on a circle centered at O, we need to find $\angle OMN$. The given angles are $\angle ORN = 48°$ and $\angle RNM = 124°$.
1. **Analyzing $\angle ORN$**:
- Triangle ORN is isosceles with $OR = ON$ (radii).
- Therefore, $\angle ORN = \angle ONR = 48°$.
- The central angle $\angle RON = 180° - 48° - 48° = 84°$, which is the measure of arc RN.
2. **Analyzing $\angle RNM$**:
- $\angle RNM = 124°$ is an inscribed angle intercepting arc RM.
- The measure of arc RM is $2 \times 124° = 248°$ (major arc).
3. **Finding arc NM**:
- The total circumference is $360°$.
- The major arc RM is $248°$, so the minor arc RM is $360° - 248° = 112°$.
- Using the known arcs: arc $RN = 84°$ and minor arc $RM = 112°$.
- The remaining arc NM is calculated as $360° - 84° - 112° = 164°$. However, since $\angle RNM$ intercepts the major arc RM, we need to consider the major arc:
- Arc $NM = 360° - 84° - 248° = 28°$.
4. **Calculating $\angle OMN$**:
- The central angle $\angle MON$ is equal to arc $NM = 28°$.
- In triangle OMN, which is isosceles with $OM = ON$, the base angles $\angle OMN$ and $\angle ONM$ are equal.
- Therefore, $\angle OMN = \frac{180° - 28°}{2} = 76°$.
The final answer is $\boxed{D}$.

Problem 9

Simplify $3\sqrt{12} + 10\sqrt{3} - \frac{6}{\sqrt{3}}$.

Possible answers:

A. $10\sqrt{3}$

B. $18\sqrt{3}$

C. $14\sqrt{3}$

$7\sqrt{3}$

Correct answer: C

Solution. To simplify $3\sqrt{12} + 10\sqrt{3} - \frac{6}{\sqrt{3}}$:

1. Simplify $3\sqrt{12}$:

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \implies 3\sqrt{12} = 3 \times 2\sqrt{3} = 6\sqrt{3}$$

2. The term $10\sqrt{3}$ is already simplified.

3. Simplify $-\frac{6}{\sqrt{3}}$ by rationalizing the denominator:

$$-\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$$

4. Combine all terms:

$$6\sqrt{3} + 10\sqrt{3} - 2\sqrt{3} = (6 + 10 - 2)\sqrt{3} = 14\sqrt{3}$$

Thus, the simplified expression is $\boxed{C}$.

Problem 10

For what value of x is $\frac{x^2+2}{10x^2-13x-3}$ is undefined?

Possible answers:

A. $\frac{1}{5}, \frac{3}{2}$

B. $-\frac{1}{5}, \frac{3}{2}$

C. $\frac{1}{5}, -\frac{3}{2}$

D. $-\frac{1}{5}, -\frac{3}{2}$

Correct answer: B

Solution. To determine for what value of x the expression $\frac{x^2+2}{10x^2-13x-3}$ is undefined, we need to find the values of x that make the denominator zero.

The denominator is $10x^2 - 13x - 3$. We solve the quadratic equation $10x^2 - 13x - 3 = 0$ using the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, where $a = 10$, $b = -13$, and $c = -3$.

1. Calculate the discriminant:

$$\sqrt{(-13)^2 - 4 \cdot 10 \cdot (-3)} = \sqrt{169 + 120} = \sqrt{289} = 17$$

2. Substitute into the quadratic formula:

$$x = \frac{-(-13) \pm 17}{2 \cdot 10} = \frac{13 \pm 17}{20}$$

3. Find the two solutions:

$$x = \frac{13 + 17}{20} = \frac{30}{20} = \frac{3}{2}$$

$$x = \frac{13 - 17}{20} = \frac{-4}{20} = -\frac{1}{5}$$

Thus, the values of x that make the denominator zero are $\frac{3}{2}$ and $-\frac{1}{5}$. These correspond to option B.

$$\boxed{B}$$

Problem 11

Express 413_7 to base 5
Possible answers:
A. 2311_5
B. 1131_5
C. 1311_5
D. 2132_5
Correct answer: C
Solution. To convert 413_7 to base 5, we first convert it to base 10 (decimal):

$$413_7 = 4 \cdot 7^2 + 1 \cdot 7^1 + 3 \cdot 7^0$$

Calculating each term:
- $4 \cdot 49 = 196$
- $1 \cdot 7 = 7$
- $3 \cdot 1 = 3$

Adding these together:

$$196 + 7 + 3 = 206_{10}$$

Next, we convert 206_{10} to base 5. We determine the coefficients for powers of 5:
- $206 \div 125 = 1$ (remainder 81)
- $81 \div 25 = 3$ (remainder 6)
- $6 \div 5 = 1$ (remainder 1)
- $1 \div 1 = 1$ (remainder 0)

Thus, the coefficients are 1, 3, 1, and 1, giving the number 1311_5.
Verification by converting 1311_5 back to decimal:

$$1 \cdot 5^3 + 3 \cdot 5^2 + 1 \cdot 5^1 + 1 \cdot 5^0 = 125 + 75 + 5 + 1 = 206$$

The final answer is $\boxed{C}$.

Problem 12

Evaluate, correct to three decimal place $\frac{4.314 \times 0.000056}{0.0067}$.
Possible answers:
A. 0.037
B. 0.004
C. 0.361
D. 0.036
Correct answer: D

Solution. To evaluate the expression $\frac{4.314 \times 0.000056}{0.0067}$ and round it to three decimal places, we can follow these steps:
1. **Multiply 4.314 by 0.000056**:
- Convert the numbers to scientific notation: 4.314×10^0 and 5.6×10^{-5}.
- Multiply the coefficients: $4.314 \times 5.6 = 24.1584$.
- Combine the powers of 10: $10^0 \times 10^{-5} = 10^{-5}$.
- Result: $24.1584 \times 10^{-5} = 0.000241584$.
2. **Divide the result by 0.0067**:
- Convert the numbers to scientific notation: 2.41584×10^{-4} and 6.7×10^{-3}.
- Divide the coefficients: $2.41584 \div 6.7 \approx 0.360573$.
- Combine the powers of 10: $10^{-4} \div 10^{-3} = 10^{-1}$.
- Result: $0.360573 \times 10^{-1} = 0.0360573$.
3. **Round to three decimal places**:
- The result 0.0360573 rounded to three decimal places is 0.036 because the fourth decimal place is 0, which is less than 5.
Thus, the final answer is $\boxed{D}$.

Problem 13

The interior angle of a regular polygon is 6 times its exterior angle, find the number of sides of the polygon.

Possible answers:

A. 12

B. 15

C. 10

D. 14

Correct answer: D

Solution. Let the measure of the exterior angle be x degrees. Then the interior angle would be $6x$ degrees. Since the interior angle and the exterior angle are supplementary, we have:

$$6x + x = 180^\circ$$

Solving for x:

$$7x = 180^\circ \implies x = \frac{180^\circ}{7} \approx 25.714^\circ$$

The number of sides n of a regular polygon is given by dividing the sum of all exterior angles (360 degrees) by the measure of one exterior angle:

$$n = \frac{360^\circ}{x} = \frac{360^\circ}{\frac{180^\circ}{7}} = 360^\circ \times \frac{7}{180^\circ} = 2 \times 7 = 14$$

We can also verify this using the formula for the interior angle of a regular polygon. The interior angle is given by:

$$\frac{(n-2) \times 180^\circ}{n}$$

Since the interior angle is 6 times the exterior angle:

$$\frac{(n-2) \times 180^\circ}{n} = 6 \times \frac{360^\circ}{n}$$

Multiplying both sides by n:

$$(n-2) \times 180^\circ = 6 \times 360^\circ$$

Simplifying the right side:

$$(n-2) \times 180^\circ = 2160^\circ$$

Dividing both sides by 180:

$$n - 2 = \frac{2160^\circ}{180^\circ} = 12$$

Thus:

$$n = 12 + 2 = 14$$

Both methods confirm that the number of sides is 14. Therefore, the final answer is:

$$\boxed{D}$$

Problem 14

Solve $2^{5x} \div 2^x = \sqrt[5]{2^{10}}$.

Possible answers:

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{5}{3}$

Correct answer: B

Solution. To solve the equation $2^{5x} \div 2^x = \sqrt[5]{2^{10}}$, follow these steps:

1. **Simplify the left side** using exponent division rules:
$2^{5x} \div 2^x = 2^{5x-x} = 2^{4x}$.

2. **Simplify the right side** by converting the fifth root to an exponent:
$\sqrt[5]{2^{10}} = (2^{10})^{\frac{1}{5}} = 2^{10 \cdot \frac{1}{5}} = 2^2 = 4$.

3. **Set the exponents equal** since the bases are the same:
$4x = 2$.

4. **Solve for x**:
$x = \frac{2}{4} = \frac{1}{2}$.

Verification: Substitute $x = \frac{1}{2}$ back into the original equation:

- Left side: $2^{5 \cdot \frac{1}{2}} \div 2^{\frac{1}{2}} = 2^{\frac{5}{2} - \frac{1}{2}} = 2^2 = 4$.
- Right side: $\sqrt[5]{2^{10}} = 4$.

Both sides equal 4, confirming the solution is correct.

Answer: $\boxed{B}$

Problem 15

If $\log_a 3 = m$ and $\log_a 5 = p$, find $\log_a 75$.

Possible answers:

A. $m^2 + p$

B. $2m + p$

C. $m + 2p$

D. $m + p^2$

Correct answer: C

Solution. To find $\log_a 75$ given $\log_a 3 = m$ and $\log_a 5 = p$:

1. **Factorize 75**:

$75 = 3 \times 5^2$.

2. **Apply logarithm product rule**:

$$\log_a 75 = \log_a(3 \times 5^2) = \log_a 3 + \log_a 5^2$$

3. **Use the power rule for $\log_a 5^2$**:

$$\log_a 5^2 = 2 \log_a 5 = 2p$$

4. **Substitute given values**:

$$\log_a 75 = \log_a 3 + 2p = m + 2p$$

Answer: $\boxed{C}$

Problem 16

Find the roots of the equations: $3m^2 - 2m - 65 = 0$.

Possible answers:

A. $\left(-5, \frac{-13}{3}\right)$

B. $\left(5, \frac{-13}{3}\right)$

C. $\left(5, \frac{13}{3}\right)$

D. $\left(-5, \frac{13}{3}\right)$

Correct answer: B

Solution. To find the roots of the quadratic equation $3m^2 - 2m - 65 = 0$, we start by identifying the coefficients: $a = 3$, $b = -2$, and $c = -65$.

We attempt to factor the quadratic equation. We look for two numbers that multiply to $3 \times (-65) = -195$ and add up to -2. The numbers -15 and 13 satisfy these conditions. We rewrite the middle term using these numbers:

$$3m^2 - 15m + 13m - 65 = 0$$

Grouping the terms, we get:

$$(3m^2 - 15m) + (13m - 65) = 0$$

Factoring out the common factors from each group:

$$3m(m - 5) + 13(m - 5) = 0$$

Factoring out the common term $(m - 5)$:

$$(3m + 13)(m - 5) = 0$$

Setting each factor equal to zero gives the roots:

$$3m + 13 = 0 \implies m = -\frac{13}{3}$$

$$m - 5 = 0 \implies m = 5$$

We verify the roots using the quadratic formula $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$\text{Discriminant} = (-2)^2 - 4 \cdot 3 \cdot (-65) = 4 + 780 = 784$$

$$\sqrt{784} = 28$$

$$m = \frac{2 \pm 28}{6}$$

This gives the solutions:

$$m = \frac{2+28}{6} = 5 \quad \text{and} \quad m = \frac{2-28}{6} = -\frac{13}{3}$$

Both methods confirm the roots are 5 and $-\frac{13}{3}$. The correct answer is $\boxed{B}$.

Problem 17

A student measured the height of a pole as 5.98 m which is less than the actual height. If the percentage error is 5%, find correct to two decimal places the actual height of the pole.
Possible answers:
A. 6.29 m
B. 7.67 m
C. 7.18 m
D. 6.65 m
Correct answer: A

Solution. The student measured the height of a pole as 5.98 m, which is less than the actual height. The percentage error is given as 5%. To find the actual height, we use the formula for percentage error:

$$\text{Percentage Error} = \left(\frac{\text{Actual Height} - \text{Measured Height}}{\text{Actual Height}} \right) \times 100$$

Given the percentage error is 5%, we set up the equation:

$$5 = \left(\frac{H - 5.98}{H} \right) \times 100$$

Solving for H:

$$0.05 = \frac{H - 5.98}{H}$$

Multiplying both sides by H:

$$0.05H = H - 5.98$$

Rearranging terms:

$$H - 0.05H = 5.98$$

$$0.95H = 5.98$$

$$H = \frac{5.98}{0.95}$$

Performing the division:

$$H \approx 6.2947$$

Rounding to two decimal places, the actual height is 6.29 m.
Thus, the correct answer is $\boxed{A}$.

Problem 18

The radius and height of a solid cylinder is 8 cm and 14 cm respectively. Find, correct to two decimal places the total surface area. (Take $\pi = \frac{22}{7}$)

Possible answers:

A. $1,106.29\ \text{cm}^2$

B. $1,016.29\ \text{cm}^2$

C. $1,106.89\ \text{cm}^2$

D. $1,206.27\ \text{cm}^2$

Correct answer: A

Solution. To find the total surface area of a cylinder with radius 8 cm and height 14 cm, we use the formula $2\pi r(r+h)$.

1. Calculate the radius squared:

$$r^2 = 8^2 = 64$$

2. Calculate the area of the two circles (top and bottom):

$$2\pi r^2 = 2 \times \frac{22}{7} \times 64 = 2 \times \frac{1408}{7} = 2 \times 201.142857 = 402.285714\,\text{cm}^2$$

3. Calculate the lateral surface area:

$$2\pi rh = 2 \times \frac{22}{7} \times 8 \times 14 = 2 \times \frac{2464}{7} = 2 \times 352 = 704\,\text{cm}^2$$

4. Add the areas together and round to two decimal places:

$$402.285714 + 704 = 1106.285714\,\text{cm}^2$$

Rounding 1106.285714 to two decimal places gives 1106.29.

Thus, the total surface area is $\boxed{A}$.

Problem 19

Find the value of a in the equation: $\cos(a+14)^\circ = \sin(4a+6)^\circ$.

Possible answers:

A. 15

B. 17

C. 14

D. 21

Correct answer: C

Solution. To find the value of a in the equation $\cos(a+14)^\circ = \sin(4a+6)^\circ$, we use the co-function identity $\cos(\theta) = \sin(90^\circ - \theta)$. This allows us to rewrite the equation as:

$$\sin(76^\circ - a) = \sin(4a + 6^\circ)$$

For the sines of two angles to be equal, their measures must either be equal or supplementary. We consider both cases:

Case 1:

$$76^\circ - a = 4a + 6^\circ + 360^\circ k$$

Solving for a:

$$76 - a = 4a + 6 \implies 70 = 5a \implies a = 14$$

Case 2:

$$76^\circ - a = 180^\circ - (4a + 6^\circ) + 360^\circ k$$

Solving for a:

$$76 - a = 174 - 4a \implies -98 = -3a \implies a = \frac{98}{3} \approx 32.67$$

Since $a \approx 32.67$ is not an integer and not among the given options, we discard this solution. Checking other values of k (positive or negative) results in values outside the given options.

Verifying the solution $a = 14$:

- Left side: $\cos(14+14)^\circ = \cos(28^\circ) \approx 0.8829$
- Right side: $\sin(4 \cdot 14 + 6)^\circ = \sin(62^\circ) \approx 0.8829$

Both sides are equal, confirming $a = 14$ is correct. Checking other options (15, 17, 21) results in unequal values.

Thus, the correct answer is $\boxed{C}$.

Problem 20

The radius of a sphere is 3 cm. Find, in terms of π, its volume.
Possible answers:
A. 30π cm^3
B. 108π cm^3
C. 27π cm^3
D. 36π cm^3
Correct answer: D

Solution. To find the volume of a sphere with a radius of 3 cm, we use the formula for the volume of a sphere, which is $V = \frac{4}{3}\pi r^3$.
1. Substitute the given radius $r = 3$ cm into the formula:

$$V = \frac{4}{3}\pi(3)^3$$

2. Calculate the cube of the radius:

$$(3)^3 = 27$$

3. Multiply this result by $\frac{4}{3}$:

$$\frac{4}{3} \times 27 = \frac{4 \times 27}{3} = \frac{108}{3} = 36$$

4. Therefore, the volume of the sphere is:

$$V = 36\pi \text{ cm}^3$$

The correct answer is $\boxed{D}$.

Problem 21

If m : n = $2\frac{1}{3} : 1\frac{1}{5}$ and n : q = $1\frac{1}{2} : 1\frac{1}{3}$, find q : m.

Possible answers:

A. 35 : 18

B. 16 : 35

C. 18 : 35

D. 35 : 16

Correct answer: B

Solution. First, we convert the mixed numbers in the given ratios to improper fractions:

1. For the ratio $m : n = 2\frac{1}{3} : 1\frac{1}{5}$:

- $2\frac{1}{3} = \frac{7}{3}$
- $1\frac{1}{5} = \frac{6}{5}$
- The ratio becomes $\frac{7}{3} : \frac{6}{5}$
- Simplifying, $\frac{7}{3} \div \frac{6}{5} = \frac{7}{3} \times \frac{5}{6} = \frac{35}{18}$
- Therefore, $m : n = 35 : 18$

2. For the ratio $n : q = 1\frac{1}{2} : 1\frac{1}{3}$:

- $1\frac{1}{2} = \frac{3}{2}$
- $1\frac{1}{3} = \frac{4}{3}$
- The ratio becomes $\frac{3}{2} : \frac{4}{3}$
- Simplifying, $\frac{3}{2} \div \frac{4}{3} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$
- Therefore, $n : q = 9 : 8$

Next, we combine the two ratios by making the common term n the same:

- The first ratio $m : n = 35 : 18$
- The second ratio $n : q = 9 : 8$ is scaled to $18 : 16$ by multiplying both parts by 2

Combining these, we get the ratio $m : n : q = 35 : 18 : 16$. Therefore, the ratio $q : m$ is $16 : 35$.

The final answer is $\boxed{B}$.

Problem 22
Make x the subject of the relation $y = \frac{ax^3-b}{3z}$.
Possible answers:
A. $x = \sqrt[3]{\frac{ax^3-b}{3z}}$
B. $x = \sqrt[3]{\frac{3yz-b}{a}}$
C. $x = \sqrt[3]{\frac{3yz+b}{a}}$
D. $x = \sqrt[3]{\frac{3yzb}{a}}$
Correct answer: C
Solution. To make x the subject of the relation $y = \frac{ax^3-b}{3z}$:
1. **Multiply both sides by $3z$:**

$$3zy = ax^3 - b$$

2. **Add b to both sides:**

$$3zy + b = ax^3$$

3. **Divide by a:**

$$x^3 = \frac{3zy + b}{a}$$

4. **Take the cube root of both sides:**

$$x = \sqrt[3]{\frac{3zy + b}{a}}$$

Verification: Substitute $x = \sqrt[3]{\frac{3zy+b}{a}}$ back into the original equation to confirm it simplifies to y, confirming correctness.
Correct Answer:
$\boxed{C}$ $x = \sqrt[3]{\frac{3yz+b}{a}}$

Problem 23

An empty cylindrical tank is 140 cm in diameter. If 200 litres of water was poured into the tank. Calculate, correct to the nearest centimeter, the height of the water in the tank. (Take $\pi = \frac{22}{7}$)
Possible answers:
A. 91 cm
B. 7 cm
C. 13 cm
D. 57 cm
Correct answer: C

Solution. The diameter of the cylindrical tank is 140 cm, so the radius is 70 cm. The volume of water poured into the tank is 200 liters, which converts to $200,000$ cubic centimeters (since 1 liter = 1000 cubic centimeters).
The formula for the volume of a cylinder is $V = \pi r^2 h$. Substituting the known values:

$$200,000 = \left(\frac{22}{7}\right) \times (70)^2 \times h$$

First, calculate 70^2:

$$70^2 = 4900$$

Next, substitute this back into the equation:

$$200,000 = \left(\frac{22}{7}\right) \times 4900 \times h$$

Simplify $\left(\frac{22}{7}\right) \times 4900$:

$$\frac{22}{7} \times 4900 = 22 \times 700 = 15,400$$

Now the equation is:

$$200,000 = 15,400 \times h$$

Solving for h:

$$h = \frac{200,000}{15,400} \approx 12.987$$

Rounding to the nearest centimeter gives $h \approx 13$ cm.
Thus, the height of the water in the tank is $\boxed{C}$.

Problem 24

Arrange the following in ascending order of magnitude 110_{two} , 31_{eight} , 42_{five} .
Possible answers:
A. 110_{two} , 31_{five} , 42_{eight}
B. 42_{five} , 110_{two} , 31_{eight}
C. 42_{five} , 31_{eight} , 110_{two}
D. 110_{two} , 42_{five} , 31_{eight}
Correct answer: D

Solution. To arrange the numbers 110_{two}, 31_{eight}, and 42_{five} in ascending order, we first convert them to base ten:

1. 110_{two}:

$$1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 4 + 2 + 0 = 6_{10}$$

2. 31_{eight}:

$$3 \cdot 8^1 + 1 \cdot 8^0 = 24 + 1 = 25_{10}$$

3. 42_{five}:

$$4 \cdot 5^1 + 2 \cdot 5^0 = 20 + 2 = 22_{10}$$

Converting these to decimal, we get 6, 25, and 22 respectively. Arranging these values in ascending order gives 6, 22, 25, which corresponds to 110_{two}, 42_{five}, 31_{eight}.
Thus, the correct answer is $\boxed{D}$.

Problem 25

A number is chosen at random from 40 and 50 inclusive. Find the probability that the number is prime.

Possible answers:

A. $\frac{8}{11}$

B. $\frac{3}{11}$

C. $\frac{4}{11}$

D. $\frac{5}{11}$

Correct answer: B

Solution. To find the probability that a number chosen at random from 40 to 50 inclusive is prime, we first determine the total number of numbers in this range. The numbers are 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, which gives us a total of 11 numbers.

Next, we identify the prime numbers within this range. A prime number is a number greater than 1 that has no positive divisors other than 1 and itself. Checking each number:

- 40: Not prime (divisible by 2, 4, 5, etc.)
- 41: Prime (no divisors other than 1 and 41)
- 42: Not prime (divisible by 2)
- 43: Prime (no divisors other than 1 and 43)
- 44: Not prime (divisible by 2)
- 45: Not prime (divisible by 5 and 9)
- 46: Not prime (divisible by 2)
- 47: Prime (no divisors other than 1 and 47)
- 48: Not prime (divisible by 2)
- 49: Not prime (divisible by 7)
- 50: Not prime (divisible by 2 and 5)

The prime numbers are 41, 43, and 47, giving us 3 prime numbers.

The probability is the number of prime numbers divided by the total number of numbers, which is $\frac{3}{11}$.

Thus, the final answer is $\boxed{B}$.

Problem 26

John was facing $S35^\circ$. If he turned 90° in the anticlockwise direction, find his new direction.
Possible answers:
A.S55°E
B. S35°W
C. N55°E
D. N35°W
Correct answer: C

Solution. John was initially facing S35, which is interpreted as S35E (35 degrees east of south). When he turns 90 anticlockwise (to his left), this direction is subtracted from his current bearing.
1. **Initial Bearing**: S35E corresponds to 145 in standard compass notation (180 - 35 = 145).
2. **Turn**: Subtracting 90 anticlockwise: $145 - 90 = 55$.
3. **New Direction**: 55 from north towards east, which is **N55E**.
Answer: C. N55°E

Problem 27

A line L passing through the point $(6, -13)$ is parallel to the line which passes through $(7, 4)$ and $(-3, 9)$. Find the equation of the line L.

Possible answers:

A. $y = \frac{1}{2}x - 10$

B. y = $\frac{-1}{2}x + 10$

C. $y = \frac{-1}{2}x - 10$

D. $y = \frac{1}{2}x + 10$

Correct answer: C

Solution. To find the equation of the line L passing through the point $(6, -13)$ and parallel to the line through $(7, 4)$ and $(-3, 9)$:

1. **Calculate the slope of the line passing through $(7, 4)$ and $(-3, 9)$**:

$$\text{slope} = \frac{9-4}{-3-7} = \frac{5}{-10} = -\frac{1}{2}$$

Since parallel lines have the same slope, the slope of line L is also $-\frac{1}{2}$.

2. **Use the point-slope form to find the equation of line L**:

The point-slope formula is $y - y_1 = m(x - x_1)$. Substituting $m = -\frac{1}{2}$, $x_1 = 6$, and $y_1 = -13$:

$$y - (-13) = -\frac{1}{2}(x - 6)$$

Simplifying this:

$$y + 13 = -\frac{1}{2}x + 3$$

Subtract 13 from both sides:

$$y = -\frac{1}{2}x + 3 - 13$$

$$y = -\frac{1}{2}x - 10$$

3. **Verify the solution**:

- The slope $-\frac{1}{2}$ is correct.

- Checking the point $(6, -13)$ in the equation $y = -\frac{1}{2}x - 10$:

$$-\frac{1}{2}(6) - 10 = -3 - 10 = -13$$

Thus, the equation of line L is $y = -\frac{1}{2}x - 10$, which corresponds to option C.

$$\boxed{C}$$

Problem 28

A bag contains 4 white marbles and 3 blue marbles. Another bag contains 5 red marbles and 6 blue marbles. If a marble is picked at random from each bag, find the probability that they are of the same colour.

Possible answers:

A. $\frac{9}{11}$

B. $\frac{18}{77}$

C. $\frac{1}{2}$

D. $\frac{11}{12}$

Correct answer: B

Solution. To find the probability that two marbles picked at random from each bag are of the same color, we need to consider the overlapping color between the two bags.

The first bag contains 4 white marbles and 3 blue marbles, totaling 7 marbles. The probability of picking a blue marble from the first bag is:

$$\frac{3}{7}$$

The second bag contains 5 red marbles and 6 blue marbles, totaling 11 marbles. The probability of picking a blue marble from the second bag is:

$$\frac{6}{11}$$

Since the only common color between the two bags is blue, the probability that both marbles picked are blue is the product of the individual probabilities:

$$\frac{3}{7} \times \frac{6}{11} = \frac{18}{77}$$

Thus, the probability that the marbles picked are of the same color is:

$$\boxed{B}$$

Problem 29

The truth set of $8 + 2x - x^2 = 0$ is $\{p, q\}$. Evaluate $p + q$.
Possible answers:
A. 4
B. 2
C. -6
D. -2
Correct answer: B

Solution. To find the truth set of the equation $8 + 2x - x^2 = 0$, we first rewrite it in standard quadratic form. Multiplying both sides by -1, we get:

$$x^2 - 2x - 8 = 0$$

Next, we factor the quadratic equation. We look for two numbers that multiply to -8 and add to -2. These numbers are 2 and -4. Thus, the equation factors as:

$$(x + 2)(x - 4) = 0$$

The solutions are $x = -2$ and $x = 4$. Therefore, the truth set is $\{-2, 4\}$. Adding these roots together:

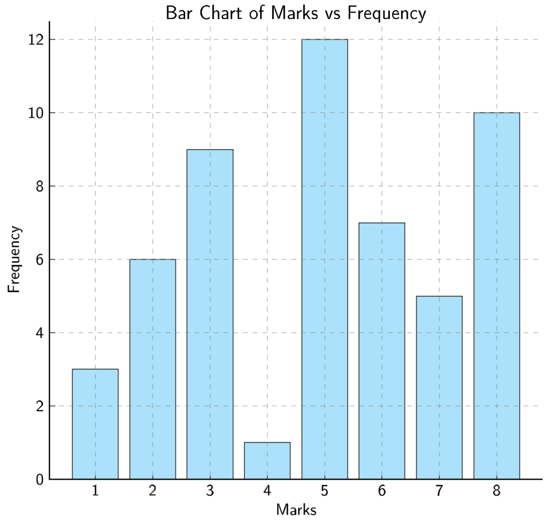
$$-2 + 4 = 2$$

We can also use Vieta's formulas, which state that the sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ is $-b/a$. Here, $a = 1$ and $b = -2$, so the sum of the roots is:

$$-\left(\frac{-2}{1}\right) = 2$$

Thus, the final answer is $\boxed{B}$.

Problem 30



The bar chart below represents the distribution of marks scored by students in an economics examination. If the failed mark was 4, what is the probability that a student selected at random passed?
Possible answers:
A. 0.36
B. 0.74
C. 0.52
D. 0.64
Correct answer: D
Solution. To determine the probability that a randomly selected student passed the economics examination, we need to consider the given data distribution and the definition of the failed mark. The failed mark is 4, which means students who scored 4 or below failed, while those who scored above 4 passed.
The frequencies of the marks are as follows:
- Mark 1: 3 students
- Mark 2: 6 students
- Mark 3: 9 students
- Mark 4: 1 student
- Mark 5: 12 students
- Mark 6: 7 students
- Mark 7: 5 students
- Mark 8: 10 students
First, we calculate the number of students who failed (marks 1 to 4):

$$3 + 6 + 9 + 1 = 19$$

Next, we calculate the number of students who passed (marks 5 to 8):

$$12 + 7 + 5 + 10 = 34$$

The total number of students is the sum of all frequencies:

$$3 + 6 + 9 + 1 + 12 + 7 + 5 + 10 = 53$$

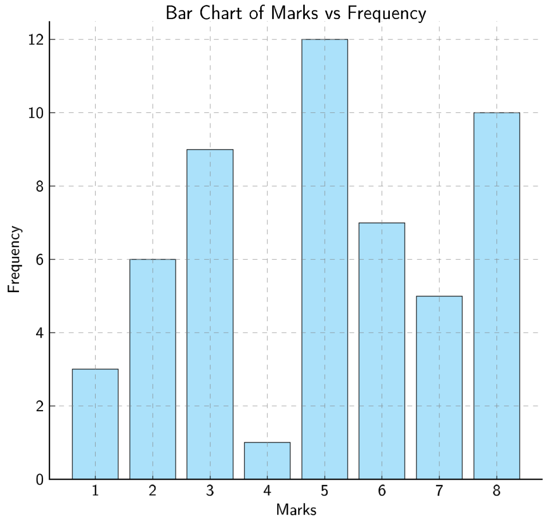
The probability that a randomly selected student passed is the ratio of the number of passing students to the total number of students:

$$\frac{34}{53} \approx 0.6415$$

Given the closest answer option, the probability is approximately 0.64.

$$\boxed{D}$$

Problem 31



What percentage of the students scored at most 5 marks? See the bar chart below.
Possible answers:
A. 58.5%
B. 63.2%
C. 38.3%
D. 41.5%
Correct answer: A
Solution. To determine the percentage of students who scored at most 5 marks, we need to sum the frequencies for marks 1 through 5 and then find what percentage this total is of the overall number of students.
Given data:
- Mark 1: Frequency 3
- Mark 2: Frequency 6
- Mark 3: Frequency 9
- Mark 4: Frequency 1
- Mark 5: Frequency 12
- Mark 6: Frequency 7
- Mark 7: Frequency 5
- Mark 8: Frequency 10

First, calculate the sum of frequencies for marks 1 to 5:

$$3 + 6 + 9 + 1 + 12 = 31$$

Next, calculate the total number of students by summing all frequencies from marks 1 to 8:

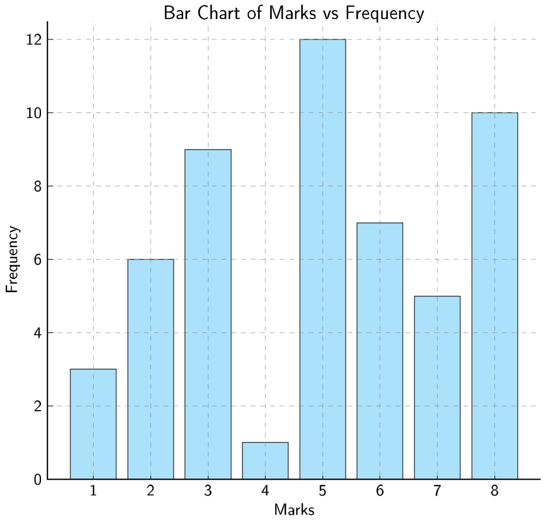
$$3 + 6 + 9 + 1 + 12 + 7 + 5 + 10 = 53$$

To find the percentage of students who scored at most 5 marks:

$$\left(\frac{31}{53}\right) \times 100 \approx 58.5\%$$

Thus, the correct answer is $\boxed{A}$.

Problem 32



How many students scored at least 3 marks? See the bar chart below.
Possible answers:
A. 44
B. 52
C. 38
D. 18
Correct answer: A
Solution. To determine how many students scored at least 3 marks, we need to sum the frequencies of the bars corresponding to marks 3 through 8. The given frequencies are as follows:
- Mark 3: 9
- Mark 4: 1
- Mark 5: 12
- Mark 6: 7
- Mark 7: 5
- Mark 8: 10

Adding these frequencies together step by step:

$$9 \text{ (Mark 3)} + 1 \text{ (Mark 4)} = 10$$

$$10 + 12 \text{ (Mark 5)} = 22$$

$$22 + 7 \text{ (Mark 6)} = 29$$

$$29 + 5 \text{ (Mark 7)} = 34$$

$$34 + 10 \text{ (Mark 8)} = 44$$

Thus, the total number of students who scored at least 3 marks is $\boxed{A}$.

Problem 33

Factorize completely: $x^2 - (y+z)^2$.

Possible answers:

A. $(x-y-z)(x-y-z)$

B. $(x+y+z)(x-y-z)$

C. $(x+y+z)(x+y-z)$

D. $(x+y-z)(x-y+z)$

Correct answer: B

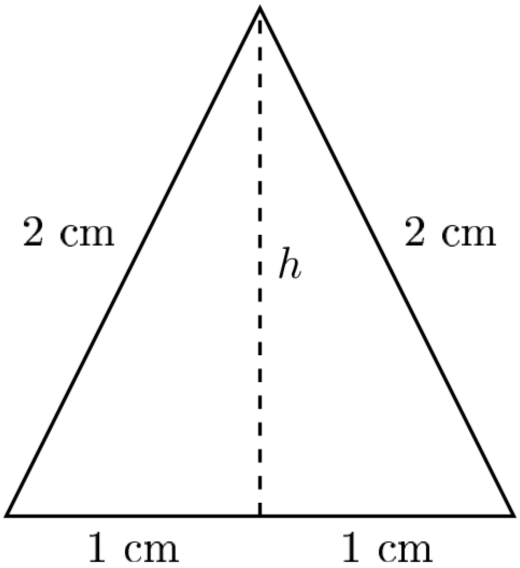
Solution. The expression $x^2 - (y+z)^2$ is a difference of squares, which factors as $(a-b)(a+b)$ where $a = x$ and $b = y+z$. Applying this:

$$x^2 - (y+z)^2 = (x-(y+z))(x+(y+z)) = (x-y-z)(x+y+z).$$

This matches option **B**, as the order of factors does not affect the product.

Answer: B. $(x+y+z)(x-y-z)$

Problem 34



An equilateral triangle has a side of 2 *cm*. Calculate in cm the height of the triangle.
Possible answers:
A. 5
B. $\sqrt{5}$
C. 3
D. $\sqrt{3}$
Correct answer: D

Solution. The height of an equilateral triangle with a side length of 2 cm can be calculated using the Pythagorean theorem. Splitting the triangle into two right-angled triangles, the base of each is 1 cm (half of 2 cm), the hypotenuse is 2 cm, and the height h is the unknown. Applying the theorem:

$$2^2 = 1^2 + h^2 \implies 4 = 1 + h^2 \implies h^2 = 3 \implies h = \sqrt{3}\,\text{cm}.$$

Answer: D. $\sqrt{3}$

Problem 35

Mrs Kebeh stands at a distance of 110 m away from a building of vertical height 58 m. If Kebeh is 2 m tall, find the angle of elevation of the top of the building from her eye.

Possible answers:

A. 27°

B. 28°

C. 20°

D. 26°

Correct answer: A

Solution. Mrs. Kebeh is standing 110 meters away from a building that is 58 meters tall. Since she is 2 meters tall, the vertical height from her eye level to the top of the building is $58 - 2 = 56$ meters. To find the angle of elevation, we use the tangent function in the right triangle formed by her horizontal distance to the building and the vertical height difference.

The tangent of the angle of elevation θ is given by:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{56}{110} \approx 0.5091$$

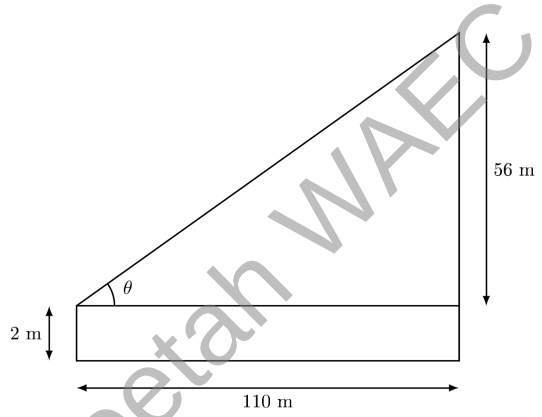


Figure 3:

Using a calculator to find the arctangent of 0.5091, we find that it is approximately 27 degrees. The closest answer from the given options is 27, which corresponds to option A.

$$\boxed{A}$$

Problem 36

Consider the statements: p: Siah is from Foya. q: Foya is in Lofa. Write in symbolic for the statement: If Siah is from Foya, then Foya is in Lofa.

Possible answers:

A. $\sim q \Leftrightarrow p$

B. $q \Rightarrow p$

C. $p \Rightarrow q$

D. $p \Leftrightarrow q$

Correct answer: C

Solution. The statement If Siah is from Foya, then Foya is in Lofa translates symbolically as $p \Rightarrow q$, where p: Siah is from Foya and q: Foya is in Lofa.

- **Option C** ($p \Rightarrow q$) directly represents the implication If p, then q.

- Other options either reverse the implication (B), use a biconditional (A, D), or introduce a negation (A), which do not match the original statement.

Answer: C. $p \Rightarrow q$

Problem 37

Write the name of a triangle with the vertices $(1, -3)$, $(6, 2)$ and $(0, 4)$.
Possible answers:
A. Scalene triangle
B. Isosceles triangle
C. Right-angle triangle
D. Equilateral triangle
Correct answer: B

Solution. To determine the type of triangle formed by the vertices $A(1, -3)$, $B(6, 2)$, and $C(0, 4)$, we calculate the lengths of its sides using the distance formula:
1. **Length of AB:**

$$AB = \sqrt{(6-1)^2 + (2-(-3))^2} = \sqrt{5^2 + 5^2} = \sqrt{50}$$

2. **Length of BC:**

$$BC = \sqrt{(0-6)^2 + (4-2)^2} = \sqrt{(-6)^2 + 2^2} = \sqrt{40}$$

3. **Length of CA:**

$$CA = \sqrt{(1-0)^2 + (-3-4)^2} = \sqrt{1^2 + (-7)^2} = \sqrt{50}$$

The side lengths are $AB = \sqrt{50}$, $BC = \sqrt{40}$, and $CA = \sqrt{50}$. Since $AB = CA$, two sides are equal, making the triangle **isosceles**.

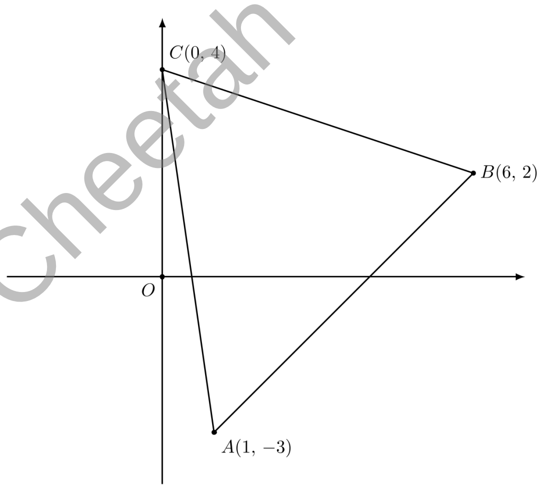


Figure 4:

To check for a right angle, we verify the Pythagorean theorem for all combinations of sides. None of the sums of squares of the shorter sides equal the square of the longest side, confirming it is not a right-angled triangle. Additionally, slopes of the sides do not multiply to -1, further ruling out a right angle.

Answer: B. Isosceles triangle.

Problem 38

The price of a shoe was decreased by 22%. If the new price is \$27.3. What is the original price?
Possible answers:
A. \$62.30
B. \$42.30
C. \$72.00
D. \$35.00
Correct answer: D

Solution. The original price of the shoe can be found by recognizing that after a 22% decrease, the new price (\$27.3) represents 78% of the original price. Setting up the equation:

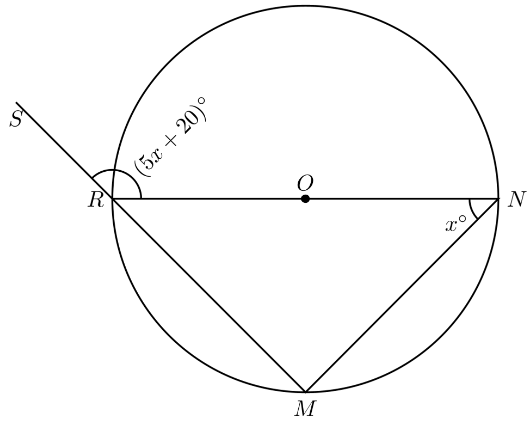
$$\text{Original Price} \times 0.78 = 27.3$$

Solving for the original price:

$$\text{Original Price} = \frac{27.3}{0.78} = 35$$

Verification: A 22% decrease on \$35 is \$7.7, leading to a new price of \$35 - \$7.7 = \$27.3, which matches the given information.
Answer: D. \$35.00

Problem 39



In the diagram, NR is a diameter, $\angle \mathrm{MNR} = \mathrm{x}^\circ$ and, $\angle \mathrm{SRN} = (5\mathrm{x}+20)^\circ$. Find the value of $2x^\circ$.
Possible answers:
A. 42°
B. 35°
C. 20°
D. 90°
Correct answer: B

Solution. Given the diagram where NR is a diameter of the circle with center O, $\angle MNR = x^\circ$, and $\angle SRN = (5x + 20)^\circ$, we need to find the value of $2x^\circ$.
1. **Thales' Theorem**: Since NR is the diameter, $\angle NMR = 90^\circ$ (right angle at M).
2. **Triangle MNR**: In triangle MNR, the sum of angles is 180°. Therefore, $\angle MRN = 90^\circ - x^\circ$.
3. **External Angle Theorem**: The external angle at R (i.e., $\angle SRN$) is equal to the sum of the non-adjacent internal angles of triangle MNR. Thus, $\angle SRN = \angle NMR + \angle MNR = 90^\circ + x^\circ$.
4. **Setting Up the Equation**: Given $\angle SRN = (5x+20)^\circ$, we equate this to the external angle:

$$5x + 20 = 90 + x$$

5. **Solving for x**:

$$5x + 20 = 90 + x, 4x = 70, x = 17.5$$

6. **Finding $2x$**:

$$2x = 2 \times 17.5 = 35$$

Thus, the value of $2x^\circ$ is $\boxed{B}$.

Problem 40
Solve: $\log_3 x + \log_3(x-8) = 2$.
Possible answers:
A. 8
B. 6
C. 9
D. 7
Correct answer: C
Solution. To solve the equation $\log_3 x + \log_3(x-8) = 2$:
1. **Combine the logarithms** using the product rule:

$$\log_3[x(x-8)] = 2$$

Simplifies to:

$$\log_3(x^2 - 8x) = 2$$

2. **Convert to exponential form**:

$$3^2 = x^2 - 8x \implies 9 = x^2 - 8x$$

3. **Form a quadratic equation**:

$$x^2 - 8x - 9 = 0$$

4. **Factor the quadratic**:

$$(x+1)(x-9) = 0$$

Solutions: $x = -1$ or $x = 9$.
5. **Check validity**:
- $x = -1$ is invalid (logarithm of a negative number).
- $x = 9$ is valid since $\log_3 9 = 2$ and $\log_3(9-8) = \log_3 1 = 0$, satisfying $2 + 0 = 2$.
6. **Verify answer choices**:
Only $x = 9$ (Option C) is valid, as $x > 8$ is required for the logarithms to be defined.
Answer: $\boxed{C}$

Problem 41
Find the quadratic equation whose roots are $\frac{2}{3}$ and $\frac{-3}{4}$.
Possible answers:
A. $12y^2 - y - 6 = 0$
B. $12y^2 - y + 6 = 0$
C. $12y^2 + y - 6 = 0$
D. $y^2 + y - 6 = 0$
Correct answer: C
Solution. To find the quadratic equation whose roots are $\frac{2}{3}$ and $\frac{-3}{4}$, we start by using the fact that a quadratic equation with roots r_1 and r_2 can be written as $y^2 - (r_1 + r_2)y + (r_1 \cdot r_2) = 0$.
1. **Calculate the sum of the roots**:

$$r_1 + r_2 = \frac{2}{3} + \left(-\frac{3}{4}\right) = \frac{8}{12} - \frac{9}{12} = -\frac{1}{12}$$

2. **Calculate the product of the roots**:

$$r_1 \cdot r_2 = \frac{2}{3} \cdot \left(-\frac{3}{4}\right) = -\frac{6}{12} = -\frac{1}{2}$$

3. **Form the quadratic equation**:

$$y^2 - \left(-\frac{1}{12}\right)y + \left(-\frac{1}{2}\right) = 0 \implies y^2 + \frac{1}{12}y - \frac{1}{2} = 0$$

4. **Clear the fractions by multiplying through by 12**:

$$12y^2 + y - 6 = 0$$

Thus, the quadratic equation is $12y^2 + y - 6 = 0$, which corresponds to option C.

$$\boxed{C}$$

Problem 42

The length of the diagonal of a square is 12 cm. Calculate the area of the square.

Possible answers:

A. 36 cm^2

B. 48 cm^2

C. 72 cm^2

D. 18 cm^2

Correct answer: C

Solution. To calculate the area of the square when given the diagonal length of 12 cm, follow these steps:

1. **Relate diagonal to side length**:

For a square with side length a, the diagonal d is $d = a\sqrt{2}$.

Given $d = 12$, solve for a:

$$a = \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}\,\text{cm}.$$

2. **Calculate the area**:

The area A of the square is a^2:

$$A = (6\sqrt{2})^2 = 6^2 \cdot (\sqrt{2})^2 = 36 \cdot 2 = 72\,\text{cm}^2.$$

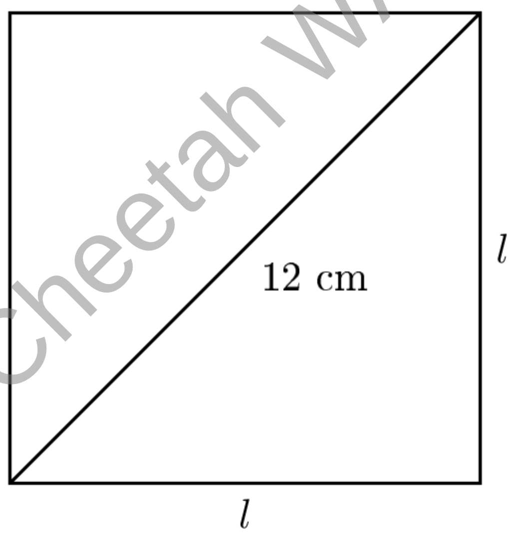


Figure 5:

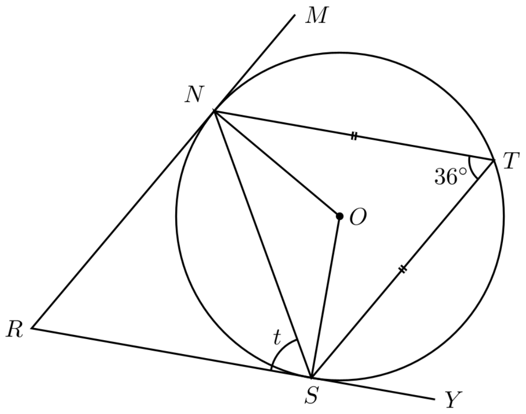
Alternative method:

Use the direct formula for the area in terms of the diagonal:

$$A = \frac{d^2}{2} = \frac{12^2}{2} = \frac{144}{2} = 72\,\text{cm}^2.$$

Answer: C. 72 cm^2

Problem 43



In the diagram above, O is the centre of a circle; $|NT| = |ST|$ and $\angle NTS = 36°$. Find the measure of the angle marked t.

Possible answers:

A. 72°

B. 54°

C. 36°

D. 108°

Correct answer: C

Solution. Given a circle with center O, point R is outside the circle, and tangents RM and RY touch the circle at points N and S respectively. Point T is on the circle such that $\angle NTS = 36°$ and $NT = ST$. We need to find the angle $\angle NSR$.

1. **Properties of Tangents**: Since RM and RY are tangents from point R, $RN = RS$. Triangle RNS is isosceles with $RN = RS$.

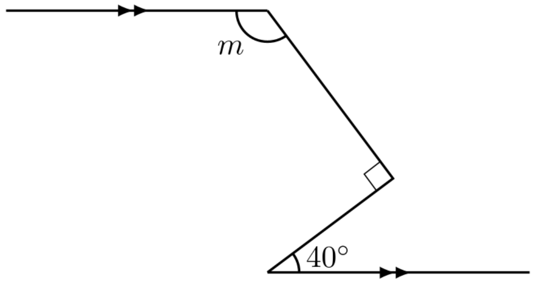
2. **Alternate Segment Theorem**: The angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment. For tangent RS and chord SN, the angle $\angle NSR$ is equal to the angle subtended by SN in the alternate segment.

3. **Angle in the Alternate Segment**: Given $\angle NTS = 36°$ and $NT = ST$, triangle NTS is isosceles with $\angle NTS = 36°$. The angle subtended by chord SN at point T in the alternate segment is 36°.

4. **Applying the Theorem**: By the Alternate Segment Theorem, $\angle NSR$ (the angle between tangent RS and chord SN) is equal to $\angle NTS$, which is 36°.

Thus, the angle $\angle NSR$ is $\boxed{C}$.

Problem 44



Find the value of m in the diagram below.
Possible answers:
A. 40°
B. 50°
C. 130°
D. 140°
Correct answer: C

Solution. Given points A, B, C, D, E with the following conditions:
- The angle between AB and BC is m (obtuse).
- C is below line AB.
- Line DE is below C and parallel to AB.
- The angle between BC and CD is 90 degrees.
- The angle between CD and DE is 40 degrees.

1. **Coordinate Setup**:
- Place B at the origin (0,0).
- AB is horizontal, with A to the left of B.
- C is below AB, forming an obtuse angle at B.
- DE is parallel to AB and below C.

2. **Vector Analysis**:
- Vector BC is from B(0,0) to C(-b, -c).
- Vector CD is perpendicular to BC, with slope -b/c (negative reciprocal of BC's slope c/b).

3. **Angle Calculation**:
- The angle between CD and DE is 40 degrees, implying the slope of CD is -tan(40).
- This slope relationship gives $b = c\tan(40)$.

4. **Dot Product and Cosine Law**:
- Vectors BA (from B to A) and BC (from B to C) form the angle m.
- Using the dot product, we find $\cos(m) = -\sin(40)$.

5. **Result**:
- $\cos(m) = -\sin(40)$ implies $m = 130$ (since $\cos(130) = -\sin(40)$).

Thus, the answer is $\boxed{C}$.

Problem 45

Find the gradient of the line passing through the points $(\frac{1}{2}, -\frac{1}{3})$ and $(3, \frac{2}{3})$.
Possible answers:
A. $\frac{2}{5}$
B. $\frac{5}{2}$
C. $\frac{2}{7}$
D. $\frac{7}{2}$
Correct answer: A

Solution. To find the gradient of the line passing through the points $\left(\frac{1}{2}, -\frac{1}{3}\right)$ and $\left(3, \frac{2}{3}\right)$, we use the formula for the gradient m between two points (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Labeling the points, we have $(x_1, y_1) = \left(\frac{1}{2}, -\frac{1}{3}\right)$ and $(x_2, y_2) = \left(3, \frac{2}{3}\right)$.
First, calculate the change in y:

$$y_2 - y_1 = \frac{2}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

Next, calculate the change in x:

$$x_2 - x_1 = 3 - \frac{1}{2} = \frac{6}{2} - \frac{1}{2} = \frac{5}{2}$$

Now, divide the change in y by the change in x:

$$m = \frac{1}{\frac{5}{2}} = 1 \times \frac{2}{5} = \frac{2}{5}$$

Thus, the gradient of the line is $\boxed{A}$.

Problem 46
A notebook of length 15 cm was measured to be 16.8 cm, calculate, correct to two decimal places, the percentage error in the measurement.
Possible answers:
A. 12.00%
B. 11.71%
C. 10.71%
D. 11.21%
Correct answer: A
Solution. To calculate the percentage error in the measurement of the notebook's length, we use the formula:

$$\text{Percentage Error} = \left(\frac{|\text{Measured Value} - \text{Actual Value}|}{\text{Actual Value}}\right) \times 100$$

Given:
- Actual length = 15 cm
- Measured length = 16.8 cm
First, find the absolute error:

$$|16.8 - 15| = 1.8 \text{ cm}$$

Next, divide the absolute error by the actual value:

$$\frac{1.8}{15} = 0.12$$

Convert this to a percentage by multiplying by 100:

$$0.12 \times 100 = 12.00\%$$

Thus, the percentage error, correct to two decimal places, is 12.00%. Therefore, the correct answer is:

$$\boxed{A}$$

Problem 47
If $2x - 3y = -11$ and $3x + 2y = 3$, evaluate $(y - x)^2$.
Possible answers:
A. 16
B. 25
C. 9
D. 4
Correct answer: A
Solution. To solve the system of equations $2x - 3y = -11$ and $3x + 2y = 3$, we use the elimination method.
First, we multiply the equations to align the coefficients of x:
1. Multiply the first equation by 3: $6x - 9y = -33$
2. Multiply the second equation by 2: $6x + 4y = 6$
Subtract the first modified equation from the second modified equation:

$$(6x + 4y) - (6x - 9y) = 6 - (-33)$$

Simplifying the left side:

$$6x + 4y - 6x + 9y = 13y$$

Simplifying the right side:

$$6 + 33 = 39$$

This gives:

$$13y = 39 \implies y = 3$$

Substitute $y = 3$ back into the second original equation $3x + 2y = 3$:

$$3x + 2(3) = 3 \implies 3x + 6 = 3 \implies 3x = -3 \implies x = -1$$

Thus, the solutions are $x = -1$ and $y = 3$. Evaluating $(y - x)^2$:

$$(3 - (-1))^2 = (3 + 1)^2 = 4^2 = 16$$

The final answer is $\boxed{A}$.

Problem 48

The diagonals of a rhombus are 16 cm and 12 cm, find the length of the side.
Possible answers:
A. 20 cm
C. 8 cm
C. 14 cm
D. 10 cm
Correct answer: D

Solution. To find the length of the side of a rhombus with diagonals 16 cm and 12 cm:
1. **Halve the diagonals**:
- Longer diagonal: $\frac{16}{2} = 8\,\text{cm}$
- Shorter diagonal: $\frac{12}{2} = 6\,\text{cm}$
2. **Apply the Pythagorean theorem**:
The diagonals are perpendicular bisectors, forming a right triangle with legs 8 cm and 6 cm. The side of the rhombus is the hypotenuse.

$$s^2 = 8^2 + 6^2 = 64 + 36 = 100 \implies s = \sqrt{100} = 10\,\text{cm}$$

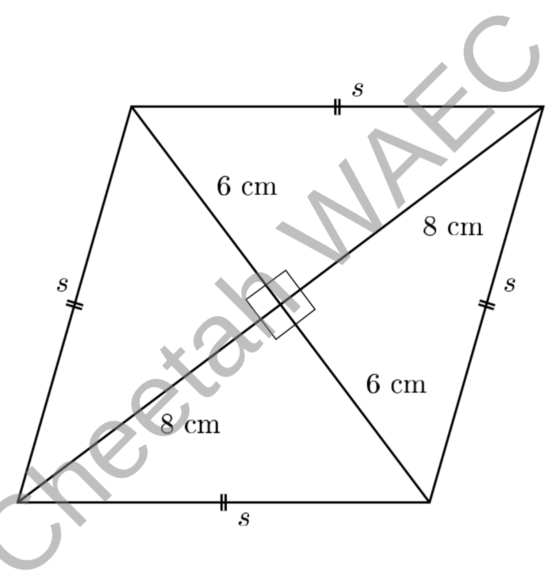


Figure 6:

Answer: $\boxed{D}$

Problem 49

M varies jointly as the square of n and square root of q. If $M = 24$ when $n = 2$ and $q = 4$, find M when $n = 5, q = 9$.

Possible answers:

A. 288

B. 400

C. 300

D. 225

Correct answer: D

Solution. Given that M varies jointly as the square of n and the square root of q, we can express this relationship as:

$$M = k \cdot n^2 \cdot \sqrt{q}$$

where k is the constant of proportionality.

We are given that $M = 24$ when $n = 2$ and $q = 4$. Substituting these values into the equation to find k:

$$24 = k \cdot (2)^2 \cdot \sqrt{4}$$

$$24 = k \cdot 4 \cdot 2$$

$$24 = 8k$$

$$k = \frac{24}{8} = 3$$

Now that we have determined $k = 3$, the equation becomes:

$$M = 3 \cdot n^2 \cdot \sqrt{q}$$

We need to find M when $n = 5$ and $q = 9$. Substituting these values into the equation:

$$M = 3 \cdot (5)^2 \cdot \sqrt{9}$$

$$M = 3 \cdot 25 \cdot 3$$

$$M = 3 \cdot 75$$

$$M = 225$$

Thus, the final answer is $\boxed{D}$.

Problem 50

Find the mean deviation of numbers: $14, 15, 16, 17, 18, 19$.
Possible answers:
A. 2.5
B. 1.7
C. 1.5
D. 3.5
Correct answer: C

Solution. To find the mean deviation of the numbers $14, 15, 16, 17, 18, 19$:
1. **Calculate the mean**:

$$\text{Mean} = \frac{14 + 15 + 16 + 17 + 18 + 19}{6} = \frac{99}{6} = 16.5$$

2. **Find the absolute deviations from the mean**:

$$|14 - 16.5| = 2.5,$$

$$|15 - 16.5| = 1.5,$$

$$|16 - 16.5| = 0.5,$$

$$|17 - 16.5| = 0.5,$$

$$|18 - 16.5| = 1.5,$$

$$|19 - 16.5| = 2.5.$$

3. **Calculate the mean of these absolute deviations**:

$$\text{Mean Deviation} = \frac{2.5 + 1.5 + 0.5 + 0.5 + 1.5 + 2.5}{6} = \frac{9}{6} = 1.5$$

Thus, the mean deviation is $\boxed{C}$.

Problem 51

A. A car travels a distance of 112 km at an average speed of 70 km/h. It then travels a further 60 km at an average speed of 50 km/h. Calculate, for the entire journey, the total time taken.
B. If $\frac{x}{y} = 2$ and $\frac{y}{z} = 3$, find the value of $\frac{x+y}{y+z}$. Represent the answer as $\frac{a}{b}$, where a, b are integers, write down the answer as a, b

Correct answer:
2.8
Correct answer:
9,4

Solution. A. To calculate the total time taken for the entire journey, we use the formula time $= \frac{\text{distance}}{\text{speed}}$.
For the first part of the journey:
- Distance = 112 km
- Speed = 70 km/h
- Time = $\frac{112}{70} = \frac{8}{5}$ hours

For the second part of the journey:
- Distance = 60 km
- Speed = 50 km/h
- Time = $\frac{60}{50} = \frac{6}{5}$ hours

Total time taken:

$$\frac{8}{5} + \frac{6}{5} = \frac{14}{5} \text{ hours}$$

B. Given $\frac{x}{y} = 2$ and $\frac{y}{z} = 3$, we express x and z in terms of y:
- $x = 2y$
- $y = 3z$ implies $z = \frac{y}{3}$

Substitute into $\frac{x+y}{y+z}$:
- Numerator: $x + y = 2y + y = 3y$
- Denominator: $y + z = y + \frac{y}{3} = \frac{4y}{3}$

Thus, the value is:

$$\frac{3y}{\frac{4y}{3}} = \frac{3y \cdot 3}{4y} = \frac{9}{4}$$

Final answers:
A. $\boxed{\frac{14}{5}}$
B. $\boxed{9, 4}$

Problem 52

A. In a football match, tickets for children and adults were sold at $D3.00$ and $D5.00$ respectively. If 400 people attended a football match and $D1700.00$ was collected in ticket sales. How many tickets were sold to adults?

B. Mr Johnson sold 250 tickets. If 175 of the tickets were for adults, how much sales did he make altogether?

Correct answer:
250
Correct answer:
1100

Solution. To solve problem A, we need to determine the number of adult tickets sold. Let C be the number of child tickets and A be the number of adult tickets. We have the following system of equations:

$$\begin{cases} C + A = 400 \\ 3C + 5A = 1700 \end{cases}$$

Solving the first equation for C:

$$C = 400 - A$$

Substituting this into the second equation:

$$3(400 - A) + 5A = 1700$$

Simplifying:

$$1200 - 3A + 5A = 17001200 + 2A = 17002A = 500A = 250$$

Thus, the number of adult tickets sold is $\boxed{250}$.

For problem B, Mr. Johnson sold 250 tickets with 175 being adult tickets. The number of child tickets sold is:

$$250 - 175 = 75$$

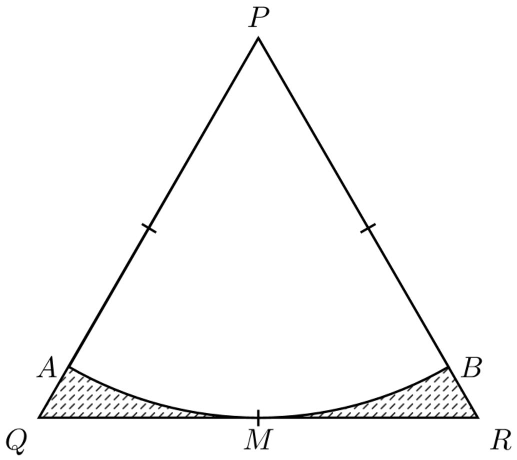
Calculating the total sales:
- Sales from adult tickets: $175 \times 5 = 875$
- Sales from child tickets: $75 \times 3 = 225$

Adding these together:

$$875 + 225 = 1100$$

Thus, the total sales made is $\boxed{1100}$.

Problem 53



a. In the diagram above, PQR is an equilateral triangle of side 18 cm; M is the midpoint of QR. An arc of a circle with center P touches QR at M and meets PQ at A and PR at B. Calculate, correct to two decimal places, the area of the shaded region. (Take $\pi = \frac{22}{7}$)

Correct answer:
13.01

Solution. To solve the problem, we need to calculate the area of the shaded region, which is the area of the equilateral triangle PQR minus the area of the sector PAB.
1. **Area of the Equilateral Triangle PQR**:
- The side length of the triangle is 18 cm.
- The height h of the triangle is calculated as:

$$h = \sqrt{18^2 - 9^2} = \sqrt{324 - 81} = \sqrt{243} = 9\sqrt{3} \text{ cm}$$

- The area of the triangle is:

$$\text{Area}_{\triangle PQR} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 18 \times 9\sqrt{3} = 81\sqrt{3} \text{ cm}^2$$

2. **Radius of the Circle**:
- The radius of the circle centered at P and passing through M (the midpoint of QR) is the distance PM, which is equal to the height of the triangle:

$$\text{Radius} = 9\sqrt{3} \text{ cm}$$

3. **Area of the Sector PAB**:
- The angle $\angle APB$ is found to be 60 degrees using the dot product method and symmetry considerations.
- The area of the sector with radius $9\sqrt{3}$ cm and central angle 60 degrees is:

$$\text{Area}_{\text{sector}} = \frac{60}{360} \times \pi \times (9\sqrt{3})^2 = \frac{1}{6} \times \pi \times 243 = \frac{81}{2}\pi \text{ cm}^2$$

4. **Shaded Area Calculation**:
- Using $\pi = \frac{22}{7}$:

$$\text{Area}_{\text{shaded}} = 81\sqrt{3} - \frac{81}{2} \times \frac{22}{7}$$

- Converting to numerical values:

$$81\sqrt{3} \approx 81 \times 1.732 \approx 140.292$$

$$\frac{81}{2} \times \frac{22}{7} \approx 40.5 \times 3.142857 \approx 127.2857$$

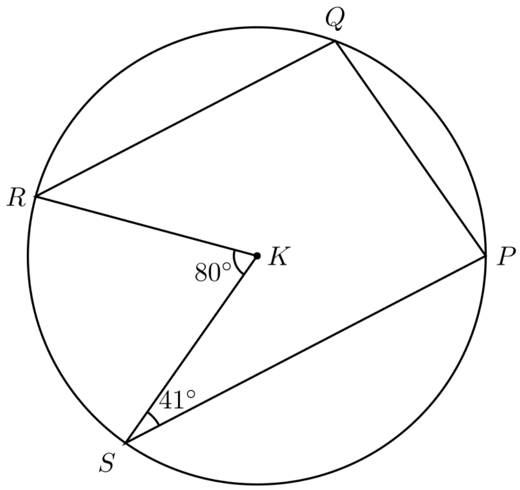
- Subtracting the sector area from the triangle area:

$$140.292 - 127.2857 \approx 13.0063$$

5. **Final Answer**:
- Rounded to two decimal places, the area of the shaded region is:

$$\boxed{13.01} \text{ square centimeters}$$

Problem 54



A. In the diagram above, P, Q, R, and S are points on the circle with centre K; KR is a bisector of angle $\angle SRQ, \angle KSP = 41°$ and $\angle SKR = 80°$. Find:$\angle RQP$;
B. Find $\angle SPQ$.

Correct answer:
89
Correct answer:
80
Solution. Given the diagram with points P, Q, R, and S on the circle with center K, where KR bisects $\angle SRQ$, $\angle KSP = 41°$, and $\angle SKR = 80°$, we need to find $\angle RQP$ and $\angle SPQ$.
1. **Finding $\angle RQP$:**
- $\angle SKR = 80°$ is the central angle for arc SR, so arc $SR = 80°$.
- $\angle KSP = 41°$ in triangle KSP (isosceles with $KS = KP$) gives $\angle SKP = 98°$, so arc $SP = 98°$.
- Since KR bisects $\angle SRQ$, $\angle SRK = \angle KRQ = 50°$. Thus, $\angle SRQ = 100°$, which is an inscribed angle subtending major arc $SQ = 200°$.
- The central angle for arc RQ is $80°$ (from triangle KRQ), so arc $RQ = 80°$.
- Arc QP is calculated as $360° - 80° - 98° - 80° = 102°$.
- Arc RP is the sum of arcs RQ and QP, which is $80° + 102° = 182°$. The minor arc RP is $360° - 182° = 178°$.
- $\angle RQP$ is an inscribed angle subtending minor arc RP, so $\angle RQP = \frac{1}{2} \times 178° = 89°$.

2. **Finding $\angle SPQ$:**
- In cyclic quadrilateral $SPQR$, opposite angles sum to $180°$. Given $\angle SRQ = 100°$, $\angle SPQ = 180° - 100° = 80°$.
Thus, the final answers are:

$$\text{A. } \boxed{89°}$$

$$\text{B. } \boxed{80°}$$

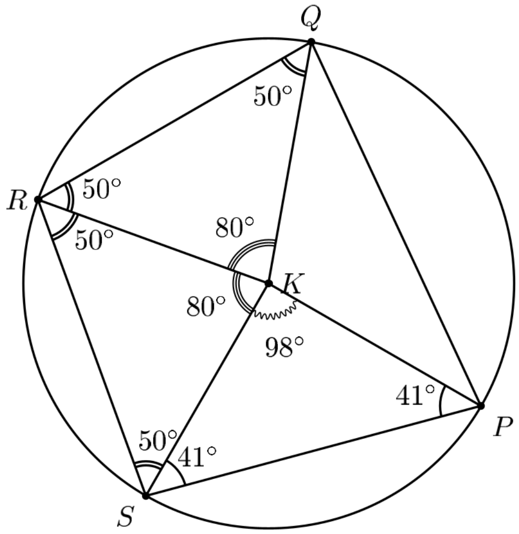


Figure 7:

Problem 55

a. A boy stands at the point M on the same horizontal level as the foot, T of a vertical building. He observes an object on the top, P of the building at an angle of elevation of $66°$. He moves directly backward to a new point C and observes the same object at an angle of $53°$. If $|MT| = 50\ m$:
Illustrate the information in a diagram;
bi. Calculate and correct to one decimal place: the height of the building;
bii. Calculate and correct to one decimal place: $|MC|$.

Correct answer:
graph
Correct answer:
112.3
Correct answer:
34.6

Solution. To solve the problem, we start by noting that the boy observes the top of the building from two different points, M and C, with angles of elevation 66 and 53 respectively. The distance from M to T is 50 meters. We need to find the height of the building (PT) and the distance from M to C (MC).

1. **Height of the building (PT):**
- From point M, the angle of elevation to the top of the building P is 66. Using the tangent function in triangle PMT:

$$\tan(66°) = \frac{PT}{MT}$$

Substituting $MT = 50$ meters:

$$PT = 50 \times \tan(66°)$$

Using the value $\tan(66°) \approx 2.2460$:

$$PT \approx 50 \times 2.2460 \approx 112.3 \text{ meters}$$

2. **Distance from M to C (MC):**
- From point C, the angle of elevation to the top of the building P is 53. Using the tangent function in triangle PCT:

$$\tan(53°) = \frac{PT}{CT}$$

Here, $CT = MT + MC = 50 + MC$. Substituting PT from the previous calculation:

$$PT = (50 + MC) \times \tan(53°)$$

Equating the two expressions for PT:

$$50 \times \tan(66°) = (50 + MC) \times \tan(53°)$$

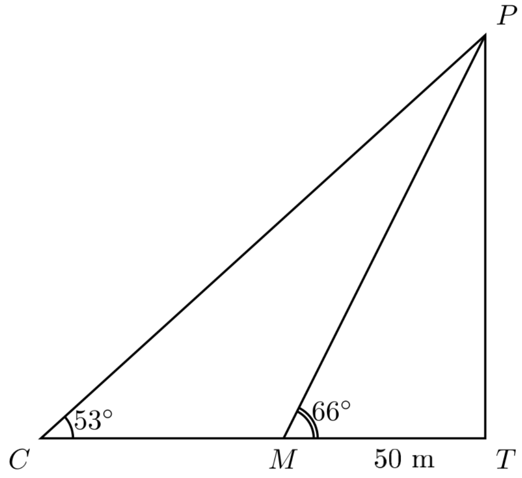


Figure 8:

Solving for MC:

$$MC = \frac{50 \times \tan(66^\circ)}{\tan(53^\circ)} - 50$$

Using the values $\tan(66^\circ) \approx 2.2460$ and $\tan(53^\circ) \approx 1.3270$:

$$MC \approx \frac{50 \times 2.2460}{1.3270} - 50 \approx \frac{112.3}{1.3270} - 50 \approx 84.624 - 50 \approx 34.6 \text{ meters}$$

Final Answer

The height of the building is $\boxed{112.3}$ meters and the distance $|MC|$ is $\boxed{34.6}$ meters.

Problem 56

A. Let $M = \{n : 2n - 3 \leq 37\}$, where n is a counting number.
Write down all the elements in M separetd by the comma and without blank spaces.
If a number is selected at random from M what is the probability that it is a multiple of 3? If a number is selected at random from M what is the probability that it is a factor of 10? Represent the answers as $\frac{a}{b}$ and write the answers as a, b, where a, b are integers.
B. A shop owner gave an end-of-year bonus to two of his attendees, Kontor and Gapson in the ratio of their ages. Gapson's age is one and a half times that of Kontor who is 20 years old. Kontor received Le $200,000.00$. (i) Find he total amount shared. (ii) Find Gapson's amount.

Correct answer:
1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20
Correct answer:
3,10
Correct answer:
1,5
Correct answer:
500000
Correct answer:
300000
Solution. To solve part A, we start with the inequality $2n - 3 \leq 37$. Solving for n:

$$2n - 3 \leq 37 \implies 2n \leq 40 \implies n \leq 20$$

Thus, the set M contains all counting numbers from 1 to 20:

$$M = \{1, 2, 3, \ldots, 20\}$$

Next, we determine the probability that a randomly selected number from M is a multiple of 3. The multiples of 3 in M are 3, 6, 9, 12, 15, and 18, which gives 6 numbers. The probability is:

$$\frac{6}{20} = \frac{3}{10} \implies \boxed{3, 10}$$

For the probability that a randomly selected number from M is a factor of 10, the factors of 10 in M are 1, 2, 5, and 10, which gives 4 numbers. The probability is:

$$\frac{4}{20} = \frac{1}{5} \implies \boxed{1, 5}$$

For part B, Kontor is 20 years old and Gapson is 1.5 times older, so Gapson is 30 years old. The ratio of their ages is 20:30, which simplifies to 2:3. Kontor received 200,000 Le, which corresponds to 2 parts of the total amount. Each part is:

$$\frac{200,000}{2} = 100,000$$

The total amount shared is:

$$5 \times 100,000 = 500,000 \implies \boxed{500000}$$

Gapson's amount is:

$$3 \times 100,000 = 300,000 \implies \boxed{300000}$$

Final Answer
A. Elements of M: $\boxed{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}$
Probability of multiple of 3: $\boxed{3, 10}$
Probability of factor of 10: $\boxed{1, 5}$
B. (i) Total amount: $\boxed{500000}$
B. (ii) Gapson's amount: $\boxed{300000}$

Problem 57

A. The sum of three numbers is 81. The second number is twice the first, the third number is 6 more than the second. Find the numbers. Write down the answer as a, b, c, where a is the first number, b is the second one, c is the third one.

B. There are points $P(3,5)$ and $Q(-5,7)$ on the Cartesian plane such that $R(x,y)$ is the midpoint of PQ. Find the equation of the line that passes through R and perpendicular to line PQ. Represent the answer as $y = ax + b$, write down the answer as a, b

Correct answer:
15,30,36
Correct answer:
4,10

Solution. A. The three numbers are found by setting up equations based on their relationships. Let the first number be a. Then the second is $2a$, and the third is $2a+6$. Their sum is $a + 2a + (2a+6) = 81$, leading to $5a + 6 = 81$. Solving, $5a = 75$ so $a = 15$. The numbers are $15, 30, 36$.
Answer: 15,30,36
B. The midpoint R of PQ is calculated as $\left(\frac{3+(-5)}{2}, \frac{5+7}{2}\right) = (-1,6)$. The slope of PQ is $\frac{7-5}{-5-3} = -\frac{1}{4}$. The perpendicular slope is 4. Using point-slope form with $R(-1,6)$: $y-6 = 4(x+1)$, simplifying to $y = 4x + 10$.
Answer: 4,10

Problem 58

A. Complete the table of values of y = $2x^2 - x - 4$ for $-3 \leq$ x ≤ 3, write the answer as a sequence of missing values for y coresponding to the values of x from -2 to 3.

x	-3	-2	-1	0	1	2	3
y	17			-4			

B. Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 unit on the y-axis, draw the graph of y = $2x^2 - x - 4$ for $-3 \leq$ x ≤ 3.

C. i. Use the graph to find: the roots of the equation $2x^2 - x - 4$.

ii. Use the graph to find the: values of x for which y increases as x increases. Represent the asnwer as $x > \frac{a}{b}$, where a, b are integers, write down the answer as a, b.

iii. Use the graph to find, correct to one decimal place, the minimum point of y.

Correct answer:
6,-1,-3,2,11
Correct answer:
graph
Correct answer:
-1.2::1.7
Correct answer:
1,4
Correct answer:
-4.1

Solution. A. The missing y values for $x = -2, -1, 1, 2, 3$ are calculated as follows:
- For $x = -2$: $y = 2(-2)^2 - (-2) - 4 = 6$
- For $x = -1$: $y = 2(-1)^2 - (-1) - 4 = -1$
- For $x = 1$: $y = 2(1)^2 - 1 - 4 = -3$
- For $x = 2$: $y = 2(2)^2 - 2 - 4 = 2$
- For $x = 3$: $y = 2(3)^2 - 3 - 4 = 11$

Answer: $6, -1, -3, 2, 11$

B. For the graph we use y values for $x = -3, -2, -1, 1, 2, 3$. Value for -3 is $y = 2(-3)^2 - (-3) - 4 = 17$. Other values were computed in Part A.

Scaling:
x-axis: 2 cm = 1 unit,
y-axis: 2 cm = 2 units (equivalent to 1 cm = 1 unit).
For the graph see Figure 1 below.

C.

i. The roots of the equation $2x^2 - x - 4 = 0$ are approximately where the graph crosses the x-axis. Using the quadratic formula, the exact roots are $\frac{1 \pm \sqrt{33}}{4} \approx -1.2$ and 1.7.

Answer: $x \approx -1.2$ and $x \approx 1.7$

ii. The function increases for $x > \frac{1}{4}$.

Answer: $1, 4$

iii. The minimum point occurs at the vertex $\left(\frac{1}{4}, -\frac{33}{8}\right)$.

Answer: $\left(\frac{1}{4}, -\frac{33}{8}\right)$

Final Answer

A. $\boxed{6}$, $\boxed{-1}$, $\boxed{-3}$, $\boxed{2}$, $\boxed{11}$

C.i. Roots: $\boxed{-1.2}$ and $\boxed{1.7}$

C.ii. $\boxed{1}$, $\boxed{4}$

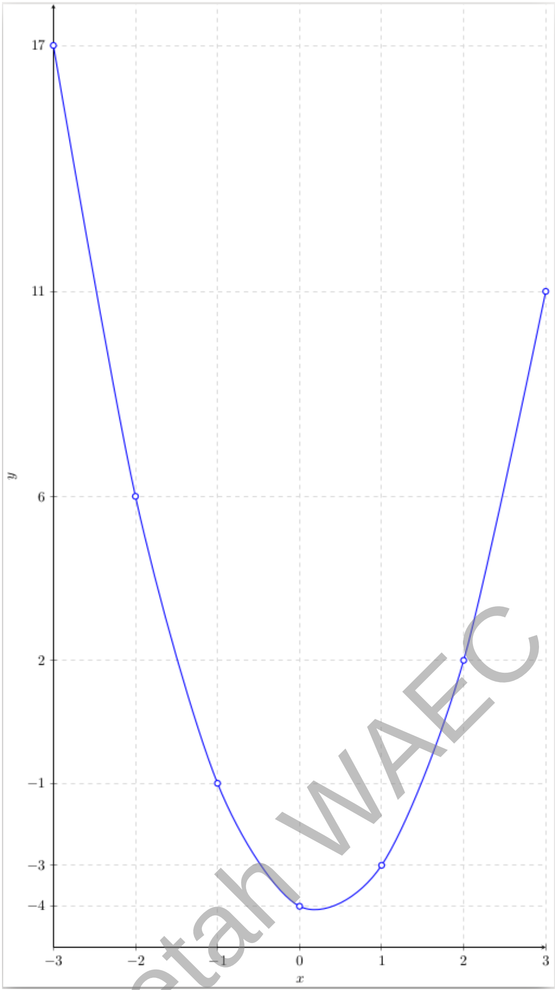


Figure 9: Part B answer

C.iii. Minimum point: $\boxed{\left(\frac{1}{4}, -\frac{33}{8}\right)}$

Problem 59
A. The table below shows the height of teak trees harvested by a farmer. Find the median height.

Height(m)	3	4	5	6	7	8
Number of trees	4	6	4	5	6	2

B. Calculate and correct to one decimal place the mean.
C. Calculate and correct to one decimal place the standard deviation.

Correct answer:
5
Correct answer:
5.3
Correct answer:
1.6
Solution. To find the median height of the teak trees, we first calculate the total number of trees:

$$4 + 6 + 4 + 5 + 6 + 2 = 27$$

Since there are 27 trees, the median is the 14th value. We determine the cumulative frequency distribution:
- Height 3m: 4 trees (positions 1-4)
- Height 4m: 6 trees (positions 5-10)
- Height 5m: 4 trees (positions 11-14)
- Height 6m: 5 trees (positions 15-19)
- Height 7m: 6 trees (positions 20-25)
- Height 8m: 2 trees (positions 26-27)

The 14th tree falls in the 5m height group, so the median height is 5 meters.
To calculate the mean height, we find the total height and divide by the number of trees:

$$\text{Total height} = (3\times4)+(4\times6)+(5\times4)+(6\times5)+(7\times6)+(8\times2) = 12+24+20+30+42+16 = 144$$

$$\text{Mean} = \frac{144}{27} \approx 5.333\ldots \approx 5.3 \text{ (correct to one decimal place)}$$

To calculate the standard deviation, we first find the variance. The deviations from the mean (5.333) are squared, multiplied by the number of trees, summed, and divided by the total number of trees:

$$\text{Variance} = \frac{1}{27}\left[4(3-5.333)^2 + 6(4-5.333)^2 + 4(5-5.333)^2 + 5(6-5.333)^2 + 6(7-5.333)^2 + 2(8-5.333)^2\right]$$

Calculating each term:
- $4 \times (-2.333)^2 \approx 21.776$
- $6 \times (-1.333)^2 \approx 10.662$
- $4 \times (-0.333)^2 \approx 0.444$
- $5 \times (0.667)^2 \approx 2.220$
- $6 \times (1.667)^2 \approx 16.668$

- $2 \times (2.667)^2 \approx 14.222$
Summing these values:

$$21.776 + 10.662 + 0.444 + 2.220 + 16.668 + 14.222 = 65.992$$

$$\text{Variance} = \frac{65.992}{27} \approx 2.444$$

$$\text{Standard deviation} = \sqrt{2.444} \approx 1.563 \approx 1.6 \text{ (correct to one decimal place)}$$

Final answers:
A. The median height is $\boxed{5}$ meters.
B. The mean height is $\boxed{5.3}$ meters.
C. The standard deviation is $\boxed{1.6}$.

Problem 60

a. In a town, Chief X resides 60 m away on a bearing of 057° from Palace P, while Chief Y resides on a bearing of 150° from the same Palace P. The residence of X and Y are 180 m apart. Illustrate the information in a diagram.

b. Find and correct to three significant figures, the: i. bearing of X from Y; ii. distance between P and Y.

Correct answer:
graph
Correct answer:
349
Correct answer:
167
Solution.

Bearings Problem via Sine & Cosine Laws

Given:

- Palace P is our reference point.
- Chief X is 60 m from P on a bearing of 057°.
- Chief Y is on a bearing of 150° from P, at unknown distance $PY = d$.
- The distance XY between X and Y is 180 m.

From the bearing angles 057° and 150°, the angle at P in triangle PXY is

$$\angle P \;=\; 150^\circ \;-\; 57^\circ \;=\; 93^\circ.$$

Label the sides so that $PX = 60$ is opposite $\angle Y$, $PY = d$ is opposite $\angle X$, and $XY = 180$ is opposite $\angle P$.

(a) Diagram.
A sketch of $\triangle PXY$ with:

$$PX = 60, \quad XY = 180, \quad PY = d,$$

$$\angle P = 93^\circ, \quad \angle X + \angle Y + 93^\circ = 180^\circ.$$

Bearing lines from P make 57° to PX and 150° to PY.

(b i) Bearing of X from Y (Law of Sines)

First, we find the *interior* angle at Y in $\triangle PXY$. By the Sine Rule,

$$\frac{\sin(\angle Y)}{PX} = \frac{\sin(\angle P)}{XY} \implies \sin(\angle Y) = \frac{PX}{XY}\sin(\angle P) = \frac{60}{180}\sin(93^\circ).$$

Numerically,

$$\sin(93^\circ) \approx 0.9945,\ \sin(\angle Y) \approx \frac{60}{180} \times 0.9945 = 0.3315,$$

$$\angle Y = \sin^{-1}(0.3315) \approx 19.3^\circ.$$

Next, recall that from Y's point of view, the line YP has *reverse* bearing $150^\circ + 180^\circ = 330^\circ$.
Inside the triangle, moving from YP to YX sweeps out $\angle Y \approx 19.3^\circ$.
Hence the bearing of X *from* Y is

$$\boxed{330^\circ + 19.3^\circ = 349^\circ} \quad \text{(to 3 s.f.)}.$$

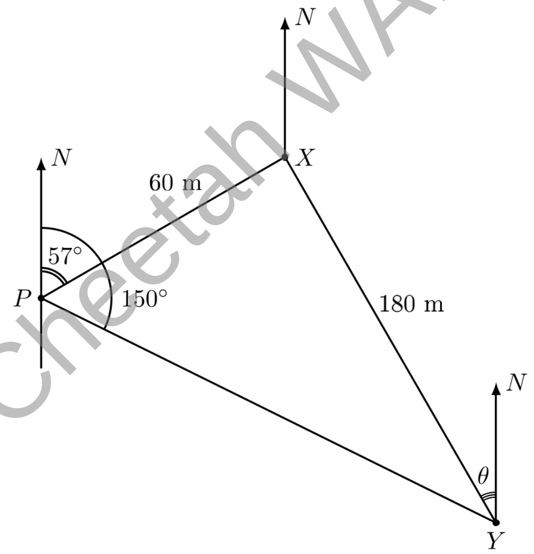


Figure 10:

(b ii) Distance PY (Law of Cosines)

We now find $d = PY$. By the Cosine Rule on side $XY = 180$ opposite $\angle P = 93°$,

$$XY^2 = PX^2 + PY^2 - 2\,(PX)(PY)\,\cos(\angle P).$$

Hence

$$180^2 = 60^2 + d^2 - 2 \times 60 \times d\,\cos(93°).$$

That is

$$32400 = 3600 + d^2 - 120\,d\,\cos(93°).$$

Since $\cos(93°) \approx -0.05234$, the equation becomes

$$32400 = 3600 + d^2 + (120\,d) \times 0.05234,$$

leading to a quadratic in d. Solving yields

$$d \approx 166.6\,\text{m}.$$

Rounded to three significant figures,

$$\boxed{PY \approx 167\,\text{m}}.$$

Final Answers (3 s.f.):

(i) Bearing of X from $Y \approx 349°$.

(ii) Distance P–$Y \approx 167\,\text{m}$.

Problem 61

A. Two regular polygons P and Q are such that the number of sides of P is twice the number of sides of Q. The difference between the exterior angle of P and Q is $45°$. Find the number of sides of P.
B. The area of a semi-circle is 32π cm^2. Find, in terms of π, the circumference of the semi-circle. Represent the answer as $a\pi + b$, where a, b are integers, write down the answer as a, b.

Correct answer:
8
Correct answer:
8,16
Solution. Given two regular polygons P and Q such that the number of sides of P is twice the number of sides of Q, and the difference between their exterior angles is $45°$:
- Let the number of sides of Q be n. Then, the number of sides of P is $2n$.
- The exterior angle of Q is $\frac{360°}{n}$ and the exterior angle of P is $\frac{360°}{2n}$.
- The difference between these exterior angles is given by:

$$\frac{360}{n} - \frac{360}{2n} = 45$$

Simplifying this equation:

$$\frac{360 - 180}{n} = 45 \implies \frac{180}{n} = 45 \implies n = \frac{180}{45} = 4$$

- Therefore, the number of sides of Q is 4, and the number of sides of P is $2 \times 4 = 8$.
The number of sides of P is $\boxed{8}$.
Given the area of a semi-circle is $32\pi\,\text{cm}^2$:
- The area of a semi-circle is $\frac{1}{2}\pi r^2$. Setting up the equation:

$$\frac{1}{2}\pi r^2 = 32\pi$$

Solving for r:

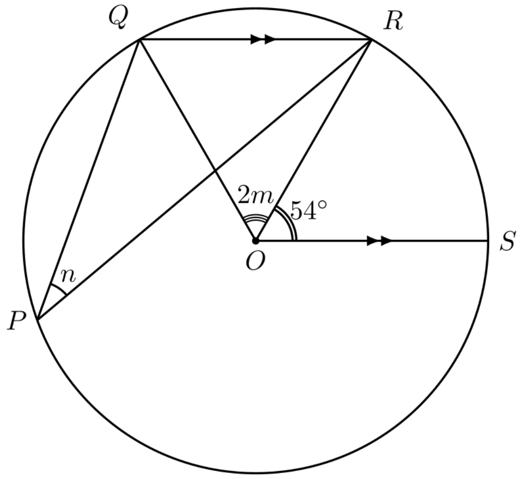
$$\frac{1}{2}r^2 = 32 \implies r^2 = 64 \implies r = 8\,\text{cm}$$

- The circumference of the semi-circle is the sum of half the circumference of a full circle and the diameter:

$$\pi r + 2r = \pi \times 8 + 2 \times 8 = 8\pi + 16$$

- Representing this as $a\pi + b$, the values are $a = 8$ and $b = 16$.
The answer is $\boxed{8, 16}$.

Problem 62

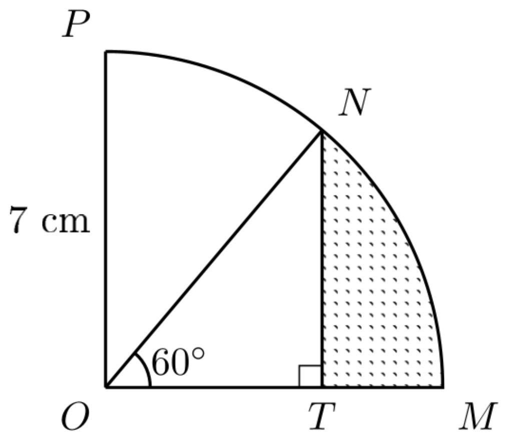


A. In the diagram above P, Q, R, and S are points on the circle with centre O; $\mathrm{QR}||\mathrm{OS}, \angle\mathrm{QOR} = 2\ \mathrm{m}, \angle\mathrm{QPR} = \mathrm{n}$ and $\angle\mathrm{SOR} = 54°$. Find the values of m and n.
B. The length of a rectangle is 4 cm more than the width. If the perimeter is 40 cm, find the area.

Correct answer:
36
Correct answer:
36
Correct answer:
96
Solution. Given the diagram with points P, Q, R, and S on the circle with center O, we need to find the values of m and n.
Part A:
1. **Finding m:**
- QR is parallel to OS.
- Angle $\angle SOR = 54°$.
- Since QR is parallel to OS, the corresponding angle $\angle ORQ = 54°$.
- In triangle $\triangle ORQ$, which is isosceles with $OQ = OR$, the angles at Q and R are equal.
- The angle at O (i.e., $\angle QOR$) is calculated as $180° - 2 \times 54° = 72°$.
- Given $\angle QOR = 2m$, we find $2m = 72° \Rightarrow m = 36°$.
2. **Finding n:**
- Angle $\angle QPR$ is an inscribed angle subtended by arc QR.
- The central angle subtended by arc QR is 72°.
- The inscribed angle $\angle QPR$ is half the central angle, so $n = \frac{1}{2} \times 72° = 36°$.
Part B:
1. **Given:**
- The length of a rectangle is 4 cm more than the width.
- The perimeter of the rectangle is 40 cm.
2. **Finding the dimensions:**
- Let the width be w. Then the length is $w + 4$.
- Perimeter formula: $2 \times (\text{length} + \text{width}) = 40$.
- Substituting the values: $2 \times (w + w + 4) = 40 \Rightarrow 2 \times (2w + 4) = 40 \Rightarrow 4w + 8 = 40 \Rightarrow 4w =$

$32 \Rightarrow w = 8$.
- Length: $8 + 4 = 12$.
3. **Calculating the area:**
- Area: length $\times$ width $= 12 \times 8 = 96$.
Final Answer
A. $m = \boxed{36}$ and $n = \boxed{36}$
B. The area is $\boxed{96}$ cm.

Problem 63



A. In the diagram above, the radius of the sector of circle centre O is 7 cm and $\angle MON$ is $60°$. Find, correct to one decimal place, the area of the shaded portion. (Take $\pi = \frac{22}{7}$)
B. The x and y intercepts of a straight line are $\frac{-3}{4}$ and $\frac{2}{7}$ respectively. Find the equation of the line. Represent the answer as $y = \frac{a}{b}x + \frac{c}{d}$, where a, b, c, d are integers, write down the answer as a, b, c, d.

Correct answer:
15.1
Correct answer:
8,21,2,7
Solution. To solve part A, we need to find the area of the shaded portion in the diagram. The radius of the sector is 7 cm, and the angle $\angle MON$ is $60°$. The shaded area is the region bounded by the points T, M, the arc MN, and N.
1. **Calculate the area of the sector MON**:

$$\text{Area of sector} = \frac{60}{360} \times \pi \times 7^2 = \frac{1}{6} \times \frac{22}{7} \times 49 = \frac{154}{6} \approx 25.6667\,\text{cm}^2$$

2. **Calculate the area of triangle OTN**:
- OT is half of OM because T is the midpoint (since $\triangle MON$ is equilateral):

$$OT = \frac{7}{2} = 3.5\,\text{cm}$$

- The height NT in the equilateral triangle $\triangle MON$:

$$NT = \frac{\sqrt{3}}{2} \times 7 \approx 6.062\,\text{cm}$$

- Area of $\triangle OTN$:

$$\text{Area} = \frac{1}{2} \times OT \times NT = \frac{1}{2} \times 3.5 \times 6.062 \approx 10.6085\,\text{cm}^2$$

3. **Subtract the area of triangle OTN from the area of the sector MON**:

$$\text{Shaded area} = 25.6667 - 10.6085 \approx 15.0582 \approx 15.1\,\text{cm}^2$$

For part B, we need to find the equation of the line with x-intercept $\frac{-3}{4}$ and y-intercept $\frac{2}{7}$.
1. **Use the intercept form of the line equation**:

$$\frac{x}{-\frac{3}{4}} + \frac{y}{\frac{2}{7}} = 1$$

2. **Convert to slope-intercept form**:
- Multiply through by 6 to clear the denominators:

$$-8x + 21y = 6$$

- Solve for y:

$$21y = 8x + 6 \implies y = \frac{8}{21}x + \frac{2}{7}$$

The final answers are:
- **A.** The area of the shaded portion is $\boxed{15.1}$.
- **B.** The equation of the line is $y = \frac{8}{21}x + \frac{2}{7}$, represented as $\boxed{8, 21, 2, 7}$.

Problem 64
The x and y intercepts of a straight line are $\frac{-3}{4}$ and $\frac{2}{7}$ respectively. Find the equation of the line. Represent the answer as $y = \frac{a}{b}x+\frac{c}{d}$, where a, b, c, d are integers, write down the answer as a, b, c, d.

Correct answer:
8,21,2,7
Solution. Given the x-intercept $\frac{-3}{4}$ and y-intercept $\frac{2}{7}$ of a straight line, we need to find its equation in the form $y = \frac{a}{b}x + \frac{c}{d}$.
1. **Using the intercept form of a line**: The intercept form is $\frac{x}{a} + \frac{y}{b} = 1$, where a is the x-intercept and b is the y-intercept. Substituting the given intercepts:

$$\frac{x}{-\frac{3}{4}} + \frac{y}{\frac{2}{7}} = 1$$

Simplifying the denominators by multiplying by their reciprocals:

$$-\frac{4}{3}x + \frac{7}{2}y = 1$$

2. **Solving for y**:

$$\frac{7}{2}y = \frac{4}{3}x + 1$$

Multiplying both sides by $\frac{2}{7}$:

$$y = \left(\frac{4}{3} \times \frac{2}{7}\right)x + \left(1 \times \frac{2}{7}\right)$$

Simplifying the terms:

$$y = \frac{8}{21}x + \frac{2}{7}$$

3. **Verification using slope-intercept form**:
- The slope m between points $\left(-\frac{3}{4}, 0\right)$ and $\left(0, \frac{2}{7}\right)$ is:

$$m = \frac{\frac{2}{7} - 0}{0 - \left(-\frac{3}{4}\right)} = \frac{\frac{2}{7}}{\frac{3}{4}} = \frac{8}{21}$$

- Using the y-intercept $\frac{2}{7}$, the equation is:

$$y = \frac{8}{21}x + \frac{2}{7}$$

4. **Checking intercepts**:
- For the x-intercept ($y = 0$):

$$0 = \frac{8}{21}x + \frac{2}{7} \implies x = -\frac{3}{4}$$

- For the y-intercept ($x = 0$):

$$y = \frac{2}{7}$$

Thus, the equation of the line is $y = \frac{8}{21}x + \frac{2}{7}$. The integers a, b, c, d are $8, 21, 2, 7$.

$$\boxed{8, 21, 2, 7}$$