1 Solutions to problems public WAEC math 2022 By cheetahwaec.com

Problem 1
Evaluate, correct to four significant figures, 573.06×184.25.
Possible answers:
A. 105600.00
B. 105622.00
C. 105500.00
D. 105632.00
Correct answer: A
Solution. To evaluate 573.06×184.25 and correct it to four significant figures, we start by performing the exact multiplication:
1. Break down the multiplication using the distributive property:

$$573.06 \times 184.25 = (57306 \times 18425) \times 10^{-4}$$

2. Calculate the product of the integers 57306 and 18425:

$$57306 \times 18425 = 1,055,863,050$$

3. Adjust for the decimal places:

$$1,055,863,050 \times 10^{-4} = 105,586.305$$

Next, we round $105,586.305$ to four significant figures. The first four significant figures are 1, 0, 5, and 5. The fifth digit is 8, which is greater than 5, so we round up the fourth digit (5) to 6. This results in $105,600$.
The options provided are:
- A. 105600.00
- B. 105622.00
- C. 105500.00
- D. 105632.00

The correct answer, rounded to four significant figures, is 105600.00, which corresponds to option A.

$$\boxed{A}$$

Problem 2

Change 432_{five} to a number in base three.

Possible answers:

A. 10100_{three}

B. 11100_{three}

C. 11101_{three}

D. 10110_{three}

Correct answer: B

Solution. To convert 432_{five} to a number in base three, we first convert it to base ten:

$$432_{\text{five}} = 4 \times 5^2 + 3 \times 5^1 + 2 \times 5^0 = 4 \times 25 + 3 \times 5 + 2 \times 1 = 100 + 15 + 2 = 117_{10}$$

Next, we convert 117_{10} to base three by dividing by 3 and keeping track of the remainders:

$$117 \div 3 = 39 \quad \text{remainder } 0 39 \div 3 = 13 \quad \text{remainder } 0 13 \div 3 = 4 \quad \text{remainder } 1 4 \div 3 = 1 \quad \text{remainder } 1 1 \div 3 = 0 \quad \text{remainde}$$

Reading the remainders from last to first, we get 11100_{three}. Converting 11100_{three} back to base ten confirms it is correct:

$$1 \times 3^4 + 1 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 0 \times 3^0 = 81 + 27 + 9 + 0 + 0 = 117_{10}$$

Checking the answer options, only 11100_{three} (option B) converts to 117 in base ten. Thus, the final answer is:

$$\boxed{B}$$

Problem 3

Given that A and B are sets such that $n(A) = 8$, $n(B) = 12$ and $n(A \cap B) = 3$, find $n(A \cup B)$.
Possible answers:
A. 15
B. 17
C. 20
D. 23
Correct answer: B

Solution. To find the number of elements in the union of sets A and B, we use the principle of inclusion-exclusion:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Given:
- $n(A) = 8$
- $n(B) = 12$
- $n(A \cap B) = 3$

Substitute the values into the formula:

$$n(A \cup B) = 8 + 12 - 3 = 17$$

Answer: B. 17

Problem 4
If $\sqrt{24}+\sqrt{96}-\sqrt{600}=y\sqrt{6}$, find the value of y.
Possible answers:
A. 4
B. 2
C. -2
D. -4
Correct answer: D
Solution. To solve the equation $\sqrt{24}+\sqrt{96}-\sqrt{600}=y\sqrt{6}$ and find the value of y, we start by simplifying each square root term.
First, simplify $\sqrt{24}$:

$$\sqrt{24}=\sqrt{4\times 6}=\sqrt{4}\times\sqrt{6}=2\sqrt{6}$$

Next, simplify $\sqrt{96}$:

$$\sqrt{96}=\sqrt{16\times 6}=\sqrt{16}\times\sqrt{6}=4\sqrt{6}$$

Then, simplify $\sqrt{600}$:

$$\sqrt{600}=\sqrt{100\times 6}=\sqrt{100}\times\sqrt{6}=10\sqrt{6}$$

Substituting these simplified terms back into the original equation, we get:

$$2\sqrt{6}+4\sqrt{6}-10\sqrt{6}=y\sqrt{6}$$

Combining the coefficients of $\sqrt{6}$ on the left side:

$$(2+4-10)\sqrt{6}=y\sqrt{6}$$

This simplifies to:

$$-4\sqrt{6}=y\sqrt{6}$$

Dividing both sides by $\sqrt{6}$ gives:

$$y=-4$$

Thus, the value of y is $\boxed{D}$.

Problem 5

Evaluate $23 \times 54 (\text{mod} 7)$.

Possible answers:

A. 2

B. 3

C. 5

D. 6

Correct answer: B

Solution. To evaluate $23 \times 54 \pmod 7$, we can simplify each number modulo 7 first.

1. Calculate $23 \mod 7$:

$$23 \div 7 = 3 \text{ remainder } 2 \implies 23 \equiv 2 \pmod 7$$

2. Calculate $54 \mod 7$:

$$54 \div 7 = 7 \text{ remainder } 5 \implies 54 \equiv 5 \pmod 7$$

3. Multiply the results and take modulo 7:

$$2 \times 5 = 10 \implies 10 \div 7 = 1 \text{ remainder } 3 \implies 10 \equiv 3 \pmod 7$$

To verify, we can also compute 23×54 first and then take modulo 7:

$$23 \times 54 = 1242 \implies 1242 \div 7 = 177 \text{ remainder } 3 \implies 1242 \equiv 3 \pmod 7$$

Both methods confirm the result is 3. Therefore, the answer is $\boxed{B}$.

Problem 6

If $4^{3x} = 16^{x+1}$, find the value of x.
Possible answers:
A. 2
B. 3
C. 4
D. 5
Correct answer: A

Solution. To solve the equation $4^{3x} = 16^{x+1}$, express both sides with the same base. Since $4 = 2^2$ and $16 = 2^4$, rewrite the equation:

$$(2^2)^{3x} = (2^4)^{x+1}$$

Apply the exponent rule $(a^b)^c = a^{bc}$:

$$2^{6x} = 2^{4(x+1)}$$

Simplify the exponents:

$$2^{6x} = 2^{4x+4}$$

Since the bases are equal, set the exponents equal:

$$6x = 4x + 4$$

Solve for x:

$$6x - 4x = 4 \implies 2x = 4 \implies x = 2$$

Verification confirms both sides equal 4096 when $x = 2$.
Answer: $\boxed{A}$

Problem 7

A weaver bought a bundle of grass for \$50.00 from which he made 8 mats. If each mat was sold for \$15.00, find the percentage profit.

Possible answers:
A. 240%
B. 140%
C. 120%
D. 40%
Correct answer: B

Solution. The weaver's total cost price (CP) is \$50.00. By selling 8 mats at \$15.00 each, the total selling price (SP) is $8 \times 15 = \$120.00$. The profit is $SP - CP = 120 - 50 = \$70.00$. The percentage profit is calculated as:

$$\text{Percentage Profit} = \left(\frac{\text{Profit}}{\text{CP}}\right) \times 100 = \left(\frac{70}{50}\right) \times 100 = 140\%.$$

Answer: B. 140%

Problem 8

Find the 17th term of the arithmetic progression, given that first three terms are $-6, -1, 4$.
Possible answers:
A. -91
B. -86
C. 74
D. 79
Correct answer: C

Solution. Given the first three terms of the arithmetic progression: -6, -1, and 4, we need to find the 17th term.
First, we determine the common difference d by subtracting the first term from the second term:

$$d = -1 - (-6) = -1 + 6 = 5$$

We confirm the common difference by subtracting the second term from the third term:

$$d = 4 - (-1) = 4 + 1 = 5$$

The common difference d is consistently 5.
The formula for the n-th term of an arithmetic progression is:

$$a_n = a_1 + (n - 1)d$$

where $a_1 = -6$, $d = 5$, and $n = 17$. Plugging in these values, we get:

$$a_{17} = -6 + (17 - 1) \cdot 5$$

Calculating the expression inside the parentheses:

$$17 - 1 = 16$$

Then multiplying by the common difference:

$$16 \cdot 5 = 80$$

Adding this result to the first term:

$$a_{17} = -6 + 80 = 74$$

Thus, the 17th term of the arithmetic progression is $\boxed{C}$.

Problem 9

M varies directly as n and inversely as the square of p. If $M = 3$ when $n = 2$ and $p = 1$, find M in terms of n and p.

Possible answers:
A. $\frac{3n}{2p^2}$
B. $\frac{2n}{3p^2}$
C. $\frac{2n}{3p}$
D. $\frac{3n^2}{2p^2}$
Correct answer: A

Solution. Given that M varies directly as n and inversely as the square of p, we start with the relationship:

$$M = \frac{k \cdot n}{p^2}$$

where k is a constant. We are given $M = 3$, $n = 2$, and $p = 1$. Substituting these values into the equation to find k:

$$3 = \frac{k \cdot 2}{1^2}$$

Simplifying, we get:

$$3 = 2k$$

Solving for k:

$$k = \frac{3}{2}$$

Substituting k back into the equation, we get:

$$M = \frac{3}{2} \cdot \frac{n}{p^2} = \frac{3n}{2p^2}$$

Thus, the correct answer is:

$$\boxed{A}$$

Problem 10

If a = 3 and b = −7, find the value of $\frac{5b+(a+b)^2}{(a-b)^2}$.

Possible answers:

A. 0.51

B. 0.91

C. −0.19

D. −0.51

Correct answer: C

Solution. To solve the expression $\frac{5b+(a+b)^2}{(a-b)^2}$ with $a = 3$ and $b = -7$:

1. **Numerator Calculation:**
- Compute $5b$: $5 \times (-7) = -35$.
- Compute $(a + b)^2$: $3 + (-7) = -4$, then $(-4)^2 = 16$.
- Add the results: $-35 + 16 = -19$.
2. **Denominator Calculation:**
- Compute $(a - b)^2$: $3 - (-7) = 10$, then $10^2 = 100$.
3. **Final Value:**
- Divide numerator by denominator: $\frac{-19}{100} = -0.19$.

Answer: $\boxed{C}$

Problem 11

Three boys shared D $10,500.00$ in the ratio $6:7:8$. Find the largest share.
Possible answers:
A. 4000
B. 5000
C. 4500
D. 3500
Correct answer: A

Solution. To find the largest share when 10,500 is divided in the ratio $6:7:8$:
1. **Calculate the total number of parts**:
$6+7+8=21$ parts.
2. **Determine the value of one part**:
$\frac{10{,}500}{21}=500$.
3. **Compute the largest share (corresponding to 8 parts)**:
$8\times 500=4,000$.
Answer: A. 4000

Problem 12

The length of a piece of stick is 1.75 m. A boy measured it as 1.80 m. Find the percentage error.
Possible answers:
A. $4\frac{4}{7}$
B. $2\frac{6}{7}$
C. $2\frac{7}{9}$
D. $4\frac{7}{9}$
Correct answer: B

Solution. The problem is about finding the percentage error when a boy measured a stick that is actually 1.75 meters long as 1.80 meters.
First, we find the absolute error:

$$|1.80\,\text{m} - 1.75\,\text{m}| = 0.05\,\text{m}$$

Next, we divide the absolute error by the actual value:

$$\frac{0.05\,\text{m}}{1.75\,\text{m}} = \frac{0.05}{1.75}$$

Converting the values into fractions for easier calculation:

$$0.05 = \frac{1}{20} \quad \text{and} \quad 1.75 = \frac{7}{4}$$

Dividing these fractions:

$$\frac{1/20}{7/4} = \frac{1}{20} \times \frac{4}{7} = \frac{4}{140} = \frac{1}{35}$$

Converting the fractional error to a percentage:

$$\frac{1}{35} \times 100\% = \frac{100}{35}\% = \frac{20}{7}\%$$

Converting the improper fraction to a mixed number:

$$\frac{20}{7} = 2\frac{6}{7}$$

Thus, the percentage error is $2\frac{6}{7}\%$, which corresponds to option B.

$$\boxed{B}$$

Problem 13

If $5x+3y=4$ and $5x-3y=2$, what is the value of $\left(25x^2-9y^2\right)$?
Possible answers:
A. 20
B. 16
C. 2
D. 8
Correct answer: D

Solution. Given the equations $5x+3y=4$ and $5x-3y=2$, we need to find the value of $25x^2-9y^2$.

First, we add the two equations to eliminate y:

$$(5x+3y)+(5x-3y)=4+2$$

Simplifying, we get:

$$10x=6 \implies x=\frac{3}{5}$$

Next, we substitute $x=\frac{3}{5}$ back into one of the original equations, for example $5x+3y=4$:

$$5\left(\frac{3}{5}\right)+3y=4 \implies 3+3y=4 \implies 3y=1 \implies y=\frac{1}{3}$$

Now, we compute $25x^2-9y^2$:

$$25x^2=25\left(\frac{3}{5}\right)^2=25\left(\frac{9}{25}\right)=9$$

$$9y^2=9\left(\frac{1}{3}\right)^2=9\left(\frac{1}{9}\right)=1$$

$$25x^2-9y^2=9-1=8$$

Alternatively, recognizing that $25x^2-9y^2$ is a difference of squares:

$$25x^2-9y^2=(5x)^2-(3y)^2=(5x-3y)(5x+3y)$$

Using the given equations:

$$(5x+3y)(5x-3y)=4\times 2=8$$

Thus, the value of $25x^2-9y^2$ is $\boxed{D}$.

Problem 14

Mary has \$3.00 more than Ben but \$5.00 less than Jane. If Mary has \$$x$, how much does Jane and Ben have altogether?

Possible answers:

A. \$$(2x - 8)$
B. \$$(2x + 8)$
C. \$$(2x - 2)$
D. \$$(2x + 2)$

Correct answer: D

Solution. Mary has \$x.
- **Ben's amount**: Mary has \$3 more than Ben, so Ben has $x - 3$.
- **Jane's amount**: Mary has \$5 less than Jane, so Jane has $x + 5$.

Total for Jane and Ben:

$$(x - 3) + (x + 5) = 2x + 2$$

Verification with example:
If Mary has \$10:
- Ben: $10 - 3 = 7$
- Jane: $10 + 5 = 15$
- Total: $7 + 15 = 22$

Plug $x = 10$ into answer D: $2(10) + 2 = 22$, which matches.
Answer: D. \$$(2x + 2)$

Problem 15

Consider the statements: p: Stephen is intelligent q : Stephen is good at Mathematics If $p \Rightarrow q$, which of the following is a valid conclusion?
Possible answers:
A. If Stephen is good at Mathematics, then he is intelligent
B. If Stephen is not good at Mathematics, then he is not intelligent
C. If Stephen is not intelligent, then he is not good at Mathematics
D. If Stephen is not good at Mathematics, then he is intelligent
Correct answer: B

Solution. Given the implication $p \Rightarrow q$, where p is Stephen is intelligent and q is Stephen is good at Mathematics, we need to determine which of the given options is a valid conclusion.
1. **Understanding the contrapositive**: The contrapositive of $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. This is logically equivalent to the original implication and is always true if the original implication is true.
2. **Analyzing the options**:
- **Option A**: $q \Rightarrow p$ (converse) is not necessarily true.
- **Option B**: $\neg q \Rightarrow \neg p$ (contrapositive) is logically equivalent to $p \Rightarrow q$, hence valid.
- **Option C**: $\neg p \Rightarrow \neg q$ (inverse) is not necessarily true.
- **Option D**: $\neg q \Rightarrow p$ is not related to the original implication and is not valid.
By verifying through logical equivalences and examples, we confirm that the contrapositive $\neg q \Rightarrow \neg p$ (Option B) is the valid conclusion.

$$\boxed{B}$$

Problem 16

What value of p will make $(x^2 - 4x + p)$ a perfect square?
Possible answers:
A. -2
B. 16
C. 4
D. -8
Correct answer: C

Solution. To determine the value of p that makes the quadratic expression $x^2 - 4x + p$ a perfect square, we can use several methods:
1. **Comparing Coefficients**:
- A perfect square trinomial has the form $(x + a)^2$, which expands to $x^2 + 2ax + a^2$.
- Comparing $x^2 - 4x + p$ with $x^2 + 2ax + a^2$, we get $2a = -4$, leading to $a = -2$.
- The constant term a^2 is $(-2)^2 = 4$. Therefore, $p = 4$.
2. **Completing the Square**:
- For the quadratic $x^2 + bx + c$, the value that completes the square is $\left(\frac{b}{2}\right)^2$.
- Here, $b = -4$, so $\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$. Thus, $p = 4$.
3. **Using the Discriminant**:
- The discriminant of $x^2 - 4x + p$ must be zero for it to be a perfect square.
- The discriminant is $(-4)^2 - 4(1)(p) = 16 - 4p$.
- Setting the discriminant to zero: $16 - 4p = 0$ leads to $p = 4$.
All methods confirm that the value of p is 4. Therefore, the answer is $\boxed{C}$.

Problem 17
Find the value of x such that $\frac{1}{x} + \frac{4}{3x} - \frac{5}{6x} + 1 = 0$.
Possible answers:
A. $\frac{1}{6}$
B. $\frac{1}{4}$
C. -$\frac{3}{2}$
D. -$\frac{7}{6}$
Correct answer: C
Solution. To find the value of x such that $\frac{1}{x} + \frac{4}{3x} - \frac{5}{6x} + 1 = 0$, we start by combining the terms involving $\frac{1}{x}$.
First, we convert each term to have a common denominator of $6x$:

$$\frac{1}{x} = \frac{6}{6x}, \quad \frac{4}{3x} = \frac{8}{6x}, \quad -\frac{5}{6x} = -\frac{5}{6x}$$

Combining these terms:

$$\frac{6}{6x} + \frac{8}{6x} - \frac{5}{6x} = \frac{6+8-5}{6x} = \frac{9}{6x} = \frac{3}{2x}$$

This simplifies the equation to:

$$\frac{3}{2x} + 1 = 0$$

Subtracting 1 from both sides:

$$\frac{3}{2x} = -1$$

Multiplying both sides by $2x$:

$$3 = -2x$$

Solving for x:

$$x = \frac{3}{-2} = -\frac{3}{2}$$

Verification by substituting $x = -\frac{3}{2}$ back into the original equation confirms the solution is correct. Therefore, the answer is $\boxed{C}$.

Problem 18

Make t the subject of k $= m\sqrt{\frac{t-p}{r}}$.

Possible answers:

A. $\frac{k^2r+p}{m^2}$

B. $\frac{k^2r+pm^2}{m^2}$

C. $\frac{k^2r-p}{m^2}$

D. $\frac{k^2r+p^2}{m^2}$

Correct answer: B

Solution. To make t the subject of the equation k $= m\sqrt{\frac{t-p}{r}}$, follow these steps:

1. **Divide both sides by m**:

$$\frac{k}{m} = \sqrt{\frac{t-p}{r}}$$

2. **Square both sides** to eliminate the square root:

$$\left(\frac{k}{m}\right)^2 = \frac{t-p}{r} \quad \Rightarrow \quad \frac{k^2}{m^2} = \frac{t-p}{r}$$

3. **Multiply both sides by r**:

$$\frac{k^2r}{m^2} = t - p$$

4. **Add p to both sides** to isolate t:

$$t = \frac{k^2r}{m^2} + p$$

5. **Express p with the denominator m^2** to combine terms:

$$t = \frac{k^2r + pm^2}{m^2}$$

Answer: $\boxed{B}$

Problem 19

An exterior angle of a regular polygon is $22.5°$. Find the number of sides.

Possible answers:

A. 13

B. 14

C. 15

D. 16

Correct answer: D

Solution. To find the number of sides of a regular polygon given an exterior angle of $22.5°$, we use the fact that the sum of all exterior angles of any convex polygon is $360°$. For a regular polygon, each exterior angle is equal.

The number of sides n can be calculated by dividing the total sum of exterior angles by the measure of one exterior angle:

$$n = \frac{360°}{22.5°}$$

Performing the division:

$$n = \frac{360}{22.5} = 16$$

To verify, we can also consider the interior angle. The exterior angle is supplementary to the interior angle, so the interior angle is:

$$180° - 22.5° = 157.5°$$

Using the formula for the interior angle of a regular polygon:

$$\text{Interior angle} = \frac{(n-2) \times 180°}{n}$$

Setting up the equation:

$$157.5 = \frac{(n-2) \times 180}{n}$$

Solving for n:

$$157.5n = 180(n-2)$$

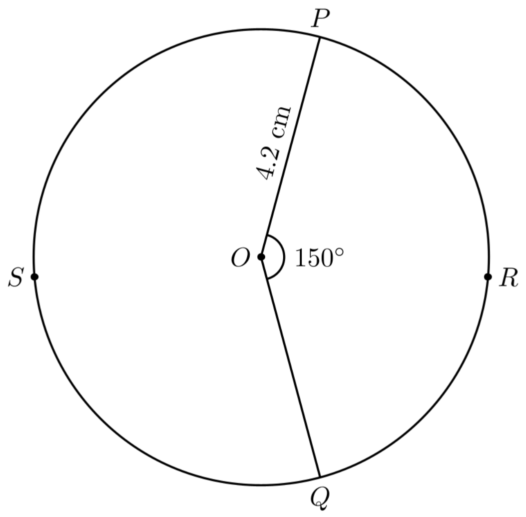
$$157.5n = 180n - 360$$

$$-22.5n = -360$$

$$n = \frac{-360}{-22.5} = 16$$

Thus, the number of sides is $\boxed{D}$.

Problem 20



In the diagram, $\angle POQ = 150°$ and the radius of the circle $PSQR$ is 4.2 cm. What is the length of the minor arc? [Take $\pi = \frac{22}{7}$]
Possible answers:
A. 11 cm
B. 15.4 cm
C. 17.64 cm
D. 23.10 cm
Correct answer: A
Solution. To find the length of the minor arc subtended by an angle of 150° in a circle with radius 4.2 cm, we use the formula for the arc length:

$$\text{Arc length} = \left(\frac{\theta}{360}\right) \times 2\pi r$$

where $\theta = 150°$, $r = 4.2\,\text{cm}$, and $\pi = \frac{22}{7}$.
1. Calculate the fraction of the circle:

$$\frac{150}{360} = \frac{5}{12}$$

2. Substitute the values into the formula:

$$\text{Arc length} = \left(\frac{5}{12}\right) \times 2 \times \frac{22}{7} \times 4.2$$

3. Simplify step by step:
- Calculate $2 \times 4.2 = 8.4$
- Multiply $8.4 \times \frac{22}{7}$:

$$8.4 \div 7 = 1.2 \quad \text{and} \quad 1.2 \times 22 = 26.4$$

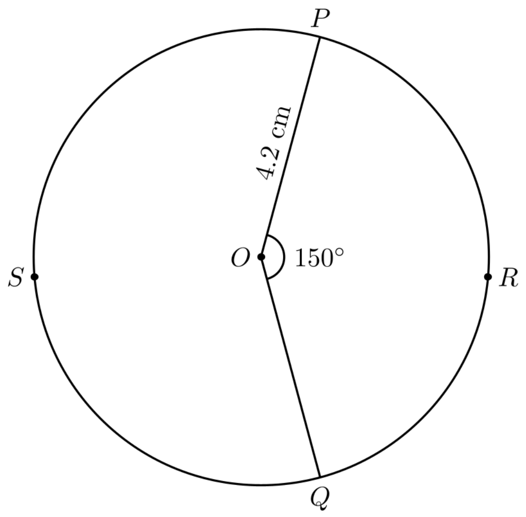
- Multiply $\frac{5}{12} \times 26.4$:

$$26.4 \div 12 = 2.2 \quad \text{and} \quad 2.2 \times 5 = 11$$

Thus, the length of the minor arc is 11 cm.

$$\boxed{A}$$

Problem 21



Find the area of the sector OPSQ.
Possible answers:
A. 15.40 cm^2
B. 17.64 cm^2
C. 23.10 cm^2
D. 32.34 cm^2
Correct answer: D

Solution. To find the area of sector OPSQ, we start by identifying the radius and the central angle. The diagram shows a circle with center O, and the radius OP is labeled as 4.2 cm. The central angle between OP and OQ is given as 150°, indicated by the arc in the diagram. However, sector OPSQ suggests the major sector passing through point S, which would correspond to a central angle of $360° - 150° = 210°$.
Calculations:
- **Radius (r):** 4.2 cm
- **Central Angle (θ):** 210° (major sector via S)
- **Sector Area Formula:**

$$\text{Area} = \left(\frac{\theta}{360}\right) \times \pi r^2$$

Substituting values:

$$\text{Area} = \left(\frac{210}{360}\right) \times \pi \times (4.2)^2 = \frac{7}{12} \times \pi \times 17.64 \approx 32.34\,\text{cm}^2$$

Answer: D. 32.34 cm^2

Problem 22

A ladder 6 m long leans against a vertical wall at an angle 53° to the horizontal. How high up the wall does the ladder reach?

Possible answers:

A. 3.611 m

B. 4.521 m

C. 4.792 m

D. 3.962 m

Correct answer: C

Solution. The problem involves finding the height up a wall that a 6-meter ladder reaches when it leans against the wall at an angle of 53° to the horizontal.

1. **Visualize the Right Triangle**: The ladder forms the hypotenuse of a right-angled triangle with the wall and the ground. The angle between the ladder and the horizontal (ground) is 53°.
2. **Trigonometric Ratio**: We use the sine function because we need the opposite side (height) relative to the given angle. The sine of an angle in a right triangle is the ratio of the length of the opposite side to the hypotenuse.

$$\sin(53^\circ) = \frac{\text{height}}{6}$$

3. **Calculate the Height**: Rearrange the formula to solve for the height:

$$\text{height} = 6 \times \sin(53^\circ)$$

4. **Approximate sin(53°)**: Using a calculator, $\sin(53^\circ) \approx 0.7986$.
5. **Compute the Result**:

$$\text{height} \approx 6 \times 0.7986 \approx 4.7916$$

Rounding to three decimal places gives 4.792 meters.

Thus, the correct answer is $\boxed{C}$.

Problem 23

A cylinder, opened at one end, has a radius of 3.5 cm and height 8 cm. Calculate the total surface area.

Possible answers:
A. 126.5 cm^2
B. 165.0 cm^2
C. 212.0 cm^2
D. 214.5 cm^2
Correct answer: D

Solution. The total surface area of an open cylinder (with one circular base) is the sum of the area of the base and the lateral surface area.
1. **Area of the base (one circle):**

$$\pi r^2 = \pi(3.5)^2 = \pi \times 12.25 \approx 38.48\,\text{cm}^2 \text{ (using } \pi \approx 3.1416)$$

2. **Lateral surface area:**

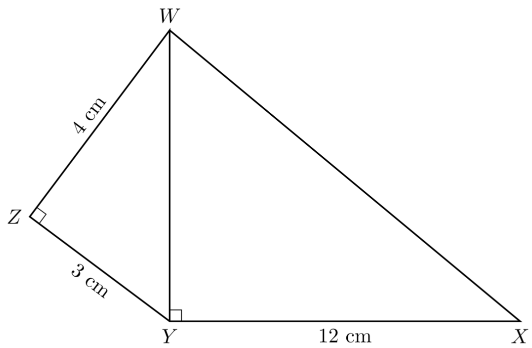
$$2\pi rh = 2\pi \times 3.5 \times 8 = 56\pi \approx 175.93\,\text{cm}^2 \text{ (using } \pi \approx 3.1416)$$

3. **Total surface area:**

$$38.48 + 175.93 \approx 214.41\,\text{cm}^2$$

Using $\pi = \frac{22}{7}$ for exact calculation:
- Base area: $\frac{22}{7} \times 12.25 = 38.5\,\text{cm}^2$
- Lateral surface area: $2 \times \frac{22}{7} \times 3.5 \times 8 = 176\,\text{cm}^2$
- Total: $38.5 + 176 = 214.5\,\text{cm}^2$
Answer: D. 214.5 cm^2

Problem 24



In the diagram below $\angle WZY$ and $\angle WYX$ are right angles. Find the perimeter of $WXYZ$.
Possible answers:
A. 30 cm
B. 32 cm
C. 35 cm
D. 37 cm
Correct answer: B
Solution. To find the perimeter of quadrilateral $WXYZ$ given that $\angle WZY$ and $\angle WYX$ are right angles, we start by analyzing the given lengths and using the Pythagorean theorem.
1. **Triangle WZY**:
- Right-angled at Z.
- Given $ZY = 3\,\text{cm}$ and $ZW = 4\,\text{cm}$.
- Using the Pythagorean theorem:

$$WY = \sqrt{ZY^2 + ZW^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5\,\text{cm}$$

2. **Triangle WYX**:
- Right-angled at Y.
- Given $YX = 12\,\text{cm}$ and $WY = 5\,\text{cm}$ (calculated above).
- Using the Pythagorean theorem:

$$WX = \sqrt{WY^2 + YX^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13\,\text{cm}$$

3. **Perimeter of $WXYZ$**:
- The sides of the quadrilateral are $WX = 13\,\text{cm}$, $XY = 12\,\text{cm}$, $YZ = 3\,\text{cm}$, and $ZW = 4\,\text{cm}$.
- Adding these lengths together:

$$\text{Perimeter} = WX + XY + YZ + ZW = 13 + 12 + 3 + 4 = 32\,\text{cm}$$

Thus, the perimeter of $WXYZ$ is $\boxed{B}$.

Problem 25

The length of a rectangle is 10 cm. If its perimeter is 28 cm, find the area.

Possible answers:

A. 30 cm^2

B. 40 cm^2

C. 60 cm^2

D. 80 cm^2

Correct answer: B

Solution. To find the area of the rectangle, start by using the perimeter formula for a rectangle:

$$\text{Perimeter} = 2 \times (\text{length} + \text{width})$$

Given the perimeter is 28 cm and the length is 10 cm, substitute these values:

$$28 = 2 \times (10 + \text{width})$$

Divide both sides by 2:

$$14 = 10 + \text{width} \implies \text{width} = 14 - 10 = 4\,\text{cm}$$

Now calculate the area using Area = length × width:

$$\text{Area} = 10 \times 4 = 40\,\text{cm}^2$$

Answer: B. 40 cm^2

Problem 26

A boy 1.4 m tall, stood 10 m away from a tree of height 12 m. Calculate, correct to the nearest degree, the angle of elevation of the top of the tree from the boy's eyes.

Possible answers:

A. 70°

B. 47°

C. 19°

D. 8°

Correct answer: B

Solution. To find the angle of elevation of the top of the tree from the boy's eyes, we need to consider the vertical distance between the boy's eyes and the top of the tree, and the horizontal distance between the boy and the tree.

1. The height of the tree is 12 meters, and the boy's height is 1.4 meters. Therefore, the vertical distance from the boy's eyes to the top of the tree is:

$$12\,\text{m} - 1.4\,\text{m} = 10.6\,\text{m}$$

2. The horizontal distance between the boy and the tree is 10 meters.

3. The angle of elevation can be found using the tangent function, which relates the opposite side (vertical distance) to the adjacent side (horizontal distance) in a right-angled triangle:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{10.6}{10} = 1.06$$

4. To find the angle θ, we take the arctangent of 1.06. Using a calculator or estimation, we find that:

$$\theta \approx \arctan(1.06) \approx 46.7^\circ$$

5. Rounding 46.7 to the nearest degree gives us 47.

Thus, the angle of elevation of the top of the tree from the boy's eyes is $\boxed{B}$.

Problem 27
Given that $\sin(5x-28)^\circ = \cos(3x-50)'', 0^\circ \leq x \leq 90^\circ$, find the value of x.
Possible answers:
A. 39
B. 32
C. 21
D. 14
Correct answer: C
Solution. Given the equation $\sin(5x-28)^\circ = \cos(3x-50)^\circ$ for $0^\circ \leq x \leq 90^\circ$, we need to find the value of x.
We use the co-function identity $\sin(\theta) = \cos(90^\circ - \theta)$. This allows us to rewrite the equation as:

$$\sin(5x - 28^\circ) = \sin(90^\circ - (3x - 50^\circ))$$

Simplifying the right-hand side:

$$\sin(5x - 28^\circ) = \sin(140^\circ - 3x)$$

For the sine values to be equal, the angles must be either coterminal or supplementary. We consider the coterminal case first:

$$5x - 28 = 140 - 3x + 360n$$

Solving for x:

$$8x = 168 + 360n \implies x = 21 + 45n$$

Considering $0^\circ \leq x \leq 90^\circ$, the valid solution is $x = 21$ (when $n = 0$).
Next, we consider the supplementary case:

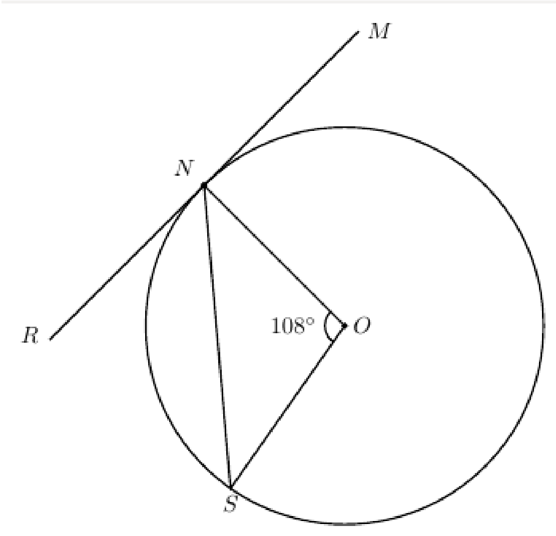
$$5x - 28 = 180 - (140 - 3x) + 360n$$

Simplifying:

$$5x - 28 = 40 + 3x + 360n \implies 2x = 68 + 360n \implies x = 34 + 180n$$

Considering $0^\circ \leq x \leq 90^\circ$, the valid solution is $x = 34$ (when $n = 0$), but this is not one of the given options.
Checking the given options, we verify $x = 21$:
- $\sin(5 \cdot 21 - 28) = \sin(77^\circ)$
- $\cos(3 \cdot 21 - 50) = \cos(13^\circ)$
Since $\sin(77^\circ) = \cos(13^\circ)$, the solution $x = 21$ is correct.
Thus, the final answer is $\boxed{C}$.

Problem 28



In the diagram, MNR is a tangent to the circle at N and $\angle NOS = 108°$. Find $\angle OSN$.
Possible answers:
A. 72°
B. 32°
C. 36°
D. 18°
Correct answer: C
Solution. Given the diagram where MN is a tangent to the circle at N and $\angle NOS = 108°$, we need to find $\angle OSN$.
1. **Identify the key elements**:
- O is the center of the circle.
- N and S are points on the circle, forming the central angle $\angle NOS = 108°$.
- MNR is a tangent to the circle at N, implying ON is perpendicular to MNR.
2. **Analyze triangle OSN**:
- Triangle OSN is isosceles with $ON = OS$ (both are radii of the circle).
- The central angle $\angle NOS = 108°$ is the vertex angle of the isosceles triangle OSN.
3. **Calculate the base angles**:
- The sum of the angles in triangle OSN is 180°.
- The base angles $\angle OSN$ and $\angle ONS$ are equal.
- Therefore, each base angle is $\frac{180° - 108°}{2} = \frac{72°}{2} = 36°$.
Thus, the measure of $\angle OSN$ is $\boxed{C}$.

Problem 30

Mrs Gabriel is pregnant. The probability that she will give birth to a girl is $\frac{1}{2}$ and with blue eyes is $\frac{1}{4}$. What is the probability that she will give birth to a girl with blue eyes?
Possible answers:
A. 1
B. $\frac{3}{4}$
C. $\frac{1}{8}$
D. $\frac{1}{4}$
Correct answer: C

Solution. To determine the probability that Mrs. Gabriel will give birth to a girl with blue eyes, we analyze the given probabilities:
1. **Probability of a girl**: $\frac{1}{2}$
2. **Probability of blue eyes (for any child)**: $\frac{1}{4}$
Assuming the gender and eye color are **independent** (as no dependence is specified), we multiply the two probabilities:

$$P(\text{girl and blue eyes}) = P(\text{girl}) \times P(\text{blue eyes}) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}.$$

Critical Analysis of Options:
- **A (1)**: Incorrect, as it exceeds both individual probabilities.
- **B ($\frac{3}{4}$)**: Too high; multiplying $\frac{1}{2}$ and $\frac{1}{4}$ cannot yield this.
- **C ($\frac{1}{8}$)**: Correct, derived from independent probabilities.
- **D ($\frac{1}{4}$)**: Incorrect; this matches the blue eyes probability alone, ignoring the gender condition.
Answer: $\boxed{C}$

Problem 31

The mean of a set of 10 numbers is 56. If the mean of the first nine numbers is 55, find the 10th number.

Possible answers:
A. 75
B. 65
C. 55
D. 45
Correct answer: B

Solution. The mean of a set of 10 numbers is 56, so the total sum of all 10 numbers is:

$$10 \times 56 = 560$$

The mean of the first nine numbers is 55, so the total sum of the first nine numbers is:

$$9 \times 55 = 495$$

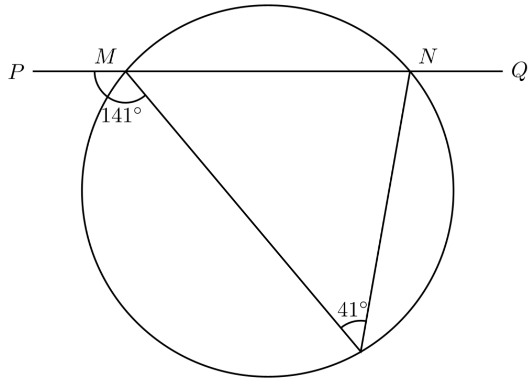
To find the 10th number, we subtract the sum of the first nine numbers from the total sum of all 10 numbers:

$$560 - 495 = 65$$

Thus, the 10th number is $\boxed{B}$.

Problem 32
Simplify $\frac{2-18m^2}{1+3m}$.
Possible answers:
A. $2[1+3m]$
B. $2[1-3\text{ m}]$
C. $2\left[1-3\text{ m}^2\right]$
D. $2\left[1+3\text{ m}^2\right]$
Correct answer: B
Solution. To simplify $\frac{2-18m^2}{1+3m}$:
1. **Factor the numerator**:
$2-18m^2=2(1-9m^2)$.
Recognize $1-9m^2$ as a difference of squares:
$1-9m^2=(1-3m)(1+3m)$.
Thus, the numerator becomes $2(1-3m)(1+3m)$.
2. **Cancel the common factor**:
The denominator $1+3m$ cancels with the $(1+3m)$ term in the numerator:
$\frac{2(1-3m)(1+3m)}{1+3m}=2(1-3m)$.
3. **Verification**:
Substituting test values for m confirms equivalence between the original expression and answer **B**. Other options fail to match when tested.
Answer: $\boxed{B}$

Problem 33



In the diagram, triangle MNR is inscribed in the circle, and line PQ is a straight line; $\angle MRN = 41°$ and $\angle RMP = 141°$, find $\angle QNR$.
Possible answers:
A. 39°
B. 80°
C. 110°
D. 141°
Correct answer: B
Solution. To solve the problem, we start by analyzing the given angles and the diagram. Triangle MNR is inscribed in the circle, making it a cyclic triangle. We are given $\angle MRN = 41°$ and $\angle RMP = 141°$. We need to find $\angle QNR$.
1. **Exterior Angle Theorem**: The exterior angle $\angle RMP = 141°$ at point M is equal to the sum of the remote interior angles in triangle MNR. Therefore, we have:

$$\angle RMP = \angle MRN + \angle MNR$$

Substituting the known values:

$$141° = 41° + \angle MNR$$

Solving for $\angle MNR$:

$$\angle MNR = 141° - 41° = 100°$$

2. **Supplementary Angles**: Since MNR and QNR are supplementary (they form a linear pair along the same straight line), we use:

$$\angle QNR = 180° - \angle MNR$$

Substituting the known value:

$$\angle QNR = 180° - 100° = 80°$$

Thus, the final answer is $\boxed{80^\circ}$.

Problem 34
Solve $\frac{y+2}{4} - \frac{y-1}{3} > 1$.
Possible answers:
A. $y < -10$
B. $y < -2$
C. $y < 2$
D. $y < 10$
Correct answer: B
Solution. To solve the inequality $\frac{y+2}{4} - \frac{y-1}{3} > 1$:
1. **Eliminate denominators** by multiplying both sides by 12 (LCM of 4 and 3):

$$12\left(\frac{y+2}{4}\right) - 12\left(\frac{y-1}{3}\right) > 12 \times 1$$

Simplifying each term:

$$3(y+2) - 4(y-1) > 12$$

2. **Expand and simplify**:

$$3y + 6 - 4y + 4 > 12 \implies -y + 10 > 12$$

3. **Solve for y**:

$$-y > 2 \implies y < -2 \quad \text{(inequality flips when multiplying by } -1)$$

4. **Verify** with test values confirms $y < -2$ is correct.
Answer: B. $y < -2$

Problem 35

The age in years of some members in a singing group are: $12, 47, 49, 15, 43, 41, 13, 39, 43, 41$ and 36. Find the lower quartile.

Possible answers:
A. 12
B. 13
C. 15
D. 20
Correct answer: C

Solution. To find the lower quartile (Q1) of the ages in the singing group, we first need to order the data from smallest to largest. The given ages are: 12, 47, 49, 15, 43, 41, 13, 39, 43, 41, and 36. Sorting these ages, we get:

$$12, 13, 15, 36, 39, 41, 41, 43, 43, 47, 49$$

There are 11 data points, which is an odd number. The median (Q2) is the 6th term, which is 41. The lower quartile (Q1) is the median of the lower half of the data, which excludes the overall median. The lower half of the data is the first five numbers:

$$12, 13, 15, 36, 39$$

Since there are 5 numbers in this lower half, the median of this subset is the 3rd term. The 3rd term in this ordered list is 15. Therefore, the lower quartile is 15.

$$\boxed{C}$$

Problem 36

The age in years of some members in a singing group are: $12, 47, 49, 15, 43, 41, 13, 39, 43, 41$ and 36. Find the mean.

Possible answers:

A. 33.35

B. 35.54

C. 34.45

D. 36.44

Correct answer: C

Solution. To find the mean age of the singing group members, follow these steps:

1. **Sum all the ages**:

$$12 + 47 + 49 + 15 + 43 + 41 + 13 + 39 + 43 + 41 + 36 = 379$$

2. **Count the number of members**:

There are 11 members.

3. **Calculate the mean**:

$$\text{Mean} = \frac{379}{11} \approx 34.4545\ldots$$

Rounded to two decimal places, this is **34.45**.

Answer: C. 34.45

Problem 37

Find, correct to two decimal places, the volume of a sphere whose radius is 3 cm. [Take $\pi = \frac{22}{7}$]
Possible answers:
A. 72.57 cm^3
B. 88.12 cm^3
C. 10529 cm^3
D. 113.14 cm^3
Correct answer: D

Solution. To find the volume of a sphere with a radius of 3 cm, we use the formula $V = \frac{4}{3}\pi r^3$. Given $\pi = \frac{22}{7}$, we proceed as follows:

1. Calculate the cube of the radius:

$$r^3 = 3^3 = 27$$

2. Multiply the result by π:

$$27 \times \frac{22}{7} = \frac{594}{7} \approx 84.8571$$

3. Multiply by $\frac{4}{3}$:

$$\frac{4}{3} \times 84.8571 \approx \frac{4}{3} \times 84.8571 = 113.1428$$

4. Round the result to two decimal places:

$$113.1428 \approx 113.14$$

Thus, the volume of the sphere, correct to two decimal places, is $113.14\,\text{cm}^3$.

$$\boxed{D}$$

Problem 38

The lengths of the parallel sides of a trapezium are 9 cm and 12 cm. If the area of the trapezium is 105 cm^2, find the perpendicular distance between the parallel sides.

Possible answers:

A. 5 cm

B. 7 cm

C. 10 cm

D. 15 cm

Correct answer: C

Solution. To find the perpendicular distance between the parallel sides of a trapezium with given lengths 9 cm and 12 cm, and area 105 cm, we use the formula for the area of a trapezium:

$$\text{Area} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

Plugging in the given values:

$$105 = \frac{1}{2} \times (9 + 12) \times h$$

First, compute the sum of the parallel sides:

$$9 + 12 = 21$$

Substitute this into the equation:

$$105 = \frac{1}{2} \times 21 \times h$$

Simplify the equation:

$$105 = 10.5 \times h$$

Solving for h:

$$h = \frac{105}{10.5} = 10$$

Thus, the perpendicular distance between the parallel sides is $\boxed{C}$.

Problem 39

Find the volume of a cone of radius 3.5 cm and vertical height 12 cm. [Take $pi = \frac{22}{7}$]
Possible answers:
A. 15.5 cm^3
B. 21.0 cm^3
C. 142 cm^3
D. 154 cm^3
Correct answer: D

Solution. To find the volume of a cone with radius 3.5 cm and vertical height 12 cm, we use the formula for the volume of a cone:

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Given:
- Radius $r = 3.5\,\text{cm}$
- Height $h = 12\,\text{cm}$
- $\pi = \frac{22}{7}$

First, calculate the base area of the cone:

$$r^2 = (3.5)^2 = 12.25$$

$$\text{Base Area} = \pi r^2 = \frac{22}{7} \times 12.25$$

Convert 12.25 to a fraction:

$$12.25 = \frac{49}{4}$$

$$\text{Base Area} = \frac{22}{7} \times \frac{49}{4} = \frac{22 \times 49}{7 \times 4} = \frac{1078}{28} = 38.5\,\text{cm}^2$$

Next, calculate the volume by multiplying the base area by the height and then dividing by 3:

$$\text{Volume} = \frac{1}{3} \times 38.5 \times 12$$

$$38.5 \times 12 = 462$$

$$\text{Volume} = \frac{462}{3} = 154\,\text{cm}^3$$

Thus, the volume of the cone is $\boxed{D}$.

Problem 40

A local community has two newspapers: the morning tomes and the evening dispatch. The morning times is read by 45% of the households. The Evening Dispatch is read by 60% of the households. Twenty percent of the households read both papers. What is the probability that a particular household reads at least one paper?

Possible answers:
A. 0.45
B. 0.65
C. 0.85
D. 0.95
Correct answer: C

Solution. To determine the probability that a household reads at least one of the newspapers, we use the **inclusion-exclusion principle**:

$$P(M \cup E) = P(M) + P(E) - P(M \cap E)$$

Given:
- $P(M) = 0.45$
- $P(E) = 0.60$
- $P(M \cap E) = 0.20$

Substitute into the formula:

$$P(M \cup E) = 0.45 + 0.60 - 0.20 = 0.85$$

Verification:
- Only Morning Times: $45\% - 20\% = 25\%$
- Only Evening Dispatch: $60\% - 20\% = 40\%$
- Both: 20%

Adding these: $25\% + 40\% + 20\% = 85\%$.
Answer: $\boxed{C}$

Problem 41

A rectangle with width $\frac{3}{4}$ cm and area $3\frac{3}{8}$ cm^2. Find the length of the rectangle.
Possible answers:
A. 6 cm
B. $4\frac{1}{2}$ cm
C. $2\frac{5}{8}$ cm
D. 12 cm
Correct answer: B

Solution. To find the length of the rectangle, use the formula for area:

$$\text{Length} = \frac{\text{Area}}{\text{Width}}$$

1. **Convert the mixed number to an improper fraction**:
The area $3\frac{3}{8}$ cm^2 becomes $\frac{27}{8}$ cm^2.
2. **Divide the area by the width**:

$$\text{Length} = \frac{27}{8} \div \frac{3}{4} = \frac{27}{8} \times \frac{4}{3} = \frac{27 \times 4}{8 \times 3} = \frac{108}{24} = \frac{9}{2} = 4\frac{1}{2} \text{ cm}$$

3. **Verification**:
Multiplying $\frac{3}{4}$ cm $\times\ 4\frac{1}{2}$ cm $= \frac{27}{8}$ cm^2, confirming the area matches.
Answer: B. $4\frac{1}{2}$ cm

Problem 42

The mean of two numbers x and y is 4. Find the mean of four numbers x, $2x$, y and $2y$.
Possible answers:
A. 2
B. 4
C. 6
D. 8
Correct answer: C

Solution. The mean of two numbers x and y is given as 4. This implies that:

$$\frac{x+y}{2} = 4$$

Multiplying both sides by 2, we get:

$$x + y = 8$$

Next, we need to find the mean of the four numbers x, $2x$, y, and $2y$. The mean is calculated by dividing the sum of these numbers by 4. First, we find the sum of the numbers:

$$x + 2x + y + 2y = 3x + 3y$$

Factoring out the common term, we get:

$$3(x + y)$$

Substituting $x + y = 8$ into the expression, we have:

$$3 \times 8 = 24$$

The mean of the four numbers is then:

$$\frac{24}{4} = 6$$

Thus, the mean of the four numbers x, $2x$, y, and $2y$ is $\boxed{C}$.

Problem 43

The straight line $y = mx - 4$ passes through the point $(-4, 16)$. Calculate the gradient of the line.

Possible answers:
A. -5
B. -3
C. 3
D. 5
Correct answer: A

Solution. To determine the gradient m of the line $y = mx - 4$ that passes through the point $(-4, 16)$:

1. Substitute $x = -4$ and $y = 16$ into the equation:

$$16 = m(-4) - 4$$

2. Solve for m:

$$16 = -4m - 4$$

$$16 + 4 = -4m$$

$$20 = -4m$$

$$m = \frac{20}{-4} = -5$$

3. Verification: Substitute $m = -5$ and $x = -4$ into the equation:

$$y = (-5)(-4) - 4 = 20 - 4 = 16$$

This matches the given point, confirming the solution.
Answer: A. -5

Problem 44

If the equations $x^2 - 5x + 6 = 0$ and $x^2 + px + 6 = 0$ have the same roots, find the value of p.
Possible answers:
A. 5
B. 6
C. -5
D. -6
Correct answer: C

Solution. To find the value of p such that the quadratic equations $x^2 - 5x + 6 = 0$ and $x^2 + px + 6 = 0$ have the same roots, we start by using Vieta's formulas.
For the first equation $x^2 - 5x + 6 = 0$:
- The sum of the roots is $-\frac{-5}{1} = 5$.
- The product of the roots is $\frac{6}{1} = 6$.
For the second equation $x^2 + px + 6 = 0$:
- The sum of the roots is $-\frac{p}{1} = -p$.
- The product of the roots is $\frac{6}{1} = 6$.
Since the products of the roots are already equal (both 6), we only need to equate the sums of the roots:

$$5 = -p$$

Solving for p, we get:

$$p = -5$$

To verify, we solve the first equation $x^2 - 5x + 6 = 0$ by factoring:

$$(x - 2)(x - 3) = 0$$

The roots are $x = 2$ and $x = 3$.
Substituting these roots into the second equation $x^2 + px + 6 = 0$:
- For $x = 2$: $2^2 + 2p + 6 = 0$ gives $4 + 2p + 6 = 0$ leading to $2p = -10$ and $p = -5$.
- For $x = 3$: $3^2 + 3p + 6 = 0$ gives $9 + 3p + 6 = 0$ leading to $3p = -15$ and $p = -5$.
Both substitutions confirm $p = -5$. Thus, the value of p is $\boxed{C}$.

Problem 45

A trader made a loss of 15% when an article was sold. Find the ratio (selling price):(cost price).
Possible answers:
A. 3 : 20
B. 3 : 17
C. 17 : 20
D. 20 : 23
Correct answer: C

Solution. The trader made a loss of 15% when selling an article. To find the ratio of selling price (SP) to cost price (CP), we start with the formula for loss percentage:

$$\text{Loss Percentage} = \left(\frac{\text{CP} - \text{SP}}{\text{CP}}\right) \times 100$$

Given a 15% loss, we substitute the values into the formula:

$$15 = \left(\frac{\text{CP} - \text{SP}}{\text{CP}}\right) \times 100$$

Dividing both sides by 100:

$$0.15 = \frac{\text{CP} - \text{SP}}{\text{CP}}$$

Multiplying both sides by CP:

$$0.15 \times \text{CP} = \text{CP} - \text{SP}$$

Rearranging to solve for SP:

$$\text{SP} = \text{CP} - 0.15 \times \text{CP}$$

Factoring out CP:

$$\text{SP} = \text{CP} \times (1 - 0.15) = 0.85 \times \text{CP}$$

Thus, the selling price is 85% of the cost price. Converting 0.85 to a fraction gives:

$$0.85 = \frac{85}{100} = \frac{17}{20}$$

Therefore, the ratio SP:CP is 17:20. Verifying with actual numbers, if CP is $200, the loss is 30$, making SP $170, which simplifies to the same ratio 17 : 20$.

$\boxed{C}$

Problem 46

Given that $\log_3 27 = 2x + 1$, find the value of x.
Possible answers:
A. 0
B. 1
C. 2
D. 3
Correct answer: B

Solution. To solve the equation $\log_3 27 = 2x + 1$, follow these steps:
1. **Convert the logarithmic equation to its exponential form**:

$$\log_3 27 = 2x + 1 \implies 3^{2x+1} = 27$$

2. **Express 27 as a power of 3**:

$$27 = 3^3$$

Substitute this into the equation:

$$3^{2x+1} = 3^3$$

3. **Set the exponents equal** (since the bases are the same):

$$2x + 1 = 3$$

4. **Solve for x**:

$$2x + 1 = 3 \implies 2x = 2 \implies x = 1$$

Verification: If $x = 1$, then $2x + 1 = 3$. Since $\log_3 27 = 3$, the equation holds true.
Answer: $\boxed{B}$

Problem 47

Solve $6x^2 = 5x - 1$.
Possible answers:
A. $x = 2, 3$
B. $x = 0, 3$
C. $x = \frac{1}{2}, \frac{1}{3}$
D. $x = \frac{1}{2}, \frac{-1}{3}$
Correct answer: C

Solution. To solve the equation $6x^2 = 5x - 1$, we first rearrange it into standard quadratic form:

$$6x^2 - 5x + 1 = 0$$

We need to factor the quadratic equation. We look for two numbers that multiply to $6 \times 1 = 6$ and add up to -5. These numbers are -2 and -3. Using these numbers to split the middle term, we get:

$$6x^2 - 2x - 3x + 1 = 0$$

Grouping the terms:

$$(6x^2 - 2x) + (-3x + 1) = 0$$

Factoring out the greatest common factor from each group:

$$2x(3x - 1) - 1(3x - 1) = 0$$

Factoring out the common term $(3x - 1)$:

$$(3x - 1)(2x - 1) = 0$$

Using the zero product property, we solve for x:

$$3x - 1 = 0 \quad \text{or} \quad 2x - 1 = 0$$

Solving these equations gives:

$$x = \frac{1}{3} \quad \text{or} \quad x = \frac{1}{2}$$

Verifying the solutions by substituting them back into the original equation confirms they are correct. The solutions are $x = \frac{1}{2}$ and $x = \frac{1}{3}$, which corresponds to option C.

$$\boxed{C}$$

Problem 48

(a) Given that $(7-2x), 9, (5x+17)$ are consecutive terms of a geometric progression with common ratio, $r > 0$, find the values of x. Represent the answers as $\frac{a}{b}$ and write the answers as a, b, where a, b are integers.
(b) Two positive numbers are in the ratio 3 : 4. The sum of thrice the first number and twice the second is 68. Find the smaller number.

Correct answer:
2,1::-19,10
Correct answer:
12

Solution. (a) Given that $(7-2x)$, 9, and $(5x+17)$ are consecutive terms of a geometric progression with common ratio $r > 0$, we need to find the values of x.
In a geometric progression, the ratio between consecutive terms is constant. Therefore, we have:

$$\frac{9}{7-2x} = \frac{5x+17}{9}$$

Cross-multiplying gives:

$$81 = (7-2x)(5x+17)$$

Expanding and simplifying the right-hand side:

$$81 = 35x + 119 - 10x^2 - 34x$$

$$81 = -10x^2 + x + 119$$

Rearranging terms:

$$10x^2 - x - 38 = 0$$

Solving this quadratic equation using the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$:

$$x = \frac{1 \pm \sqrt{1 + 4 \cdot 10 \cdot 38}}{20} = \frac{1 \pm 39}{20}$$

This gives the solutions:

$$x = \frac{40}{20} = 2 \quad \text{and} \quad x = \frac{-38}{20} = -\frac{19}{10}$$

Checking both solutions, we find that both result in positive terms and a positive common ratio. Therefore, the values of x are 2 and $-\frac{19}{10}$. Representing these as fractions $\frac{a}{b}$, we get $2 = \frac{2}{1}$ and $-\frac{19}{10}$.
(b) Two positive numbers are in the ratio 3 : 4. Let the numbers be $3k$ and $4k$. The sum of

thrice the first number and twice the second is 68:

$$3(3k) + 2(4k) = 68$$

$$9k + 8k = 68$$

$$17k = 68$$

$$k = 4$$

The smaller number is $3k = 3 \times 4 = 12$.
Final Answer
(a) $\boxed{2}$, $\boxed{1}$ and $\boxed{-19}$, $\boxed{10}$
(b) $\boxed{12}$

Problem 49

Given that y = $\left(\frac{pr}{m} - p^2r\right)^{\frac{-3}{2}}$:

(a) make r the subject; represent the answer as $r = \frac{am}{y^{\frac{2}{3}}[p+bmp^c]}$, write the answer as a, b, c, where a, b, c are integers.

(b) find the value of r when $y = -8, m = 1$ and $p = 3$. Represent the answer as $\frac{a}{b}$ and write the answer as a, b, where a is a negative integer, b is a positive integer.

Correct answer:
1,-1,2
Correct answer:
-1,24

Solution. Given the equation $y = \left(\frac{pr}{m} - p^2r\right)^{-\frac{3}{2}}$:

Part (a)

To make r the subject, we start by manipulating the given equation:

1. Raise both sides to the power of $-\frac{2}{3}$:

$$y^{-\frac{2}{3}} = \frac{pr}{m} - p^2r$$

2. Factor out r from the right-hand side:

$$y^{-\frac{2}{3}} = r\left(\frac{p}{m} - p^2\right)$$

3. Solve for r:

$$r = \frac{y^{-\frac{2}{3}}}{\frac{p}{m} - p^2}$$

4. Multiply numerator and denominator by m:

$$r = \frac{m}{y^{\frac{2}{3}}(p - mp^2)}$$

5. Factor the denominator:

$$r = \frac{m}{y^{\frac{2}{3}}p(1 - mp)}$$

6. Rewrite the denominator in the form $p + bmp^c$:

$$p - mp^2 = p + (-1)mp^2$$

Thus, $b = -1$ and $c = 2$.

Therefore, the values are $a = 1$, $b = -1$, and $c = 2$.

$$\boxed{1}, \boxed{-1}, \boxed{2}$$

Part (b)

To find the value of r when $y = -8$, $m = 1$, and $p = 3$:

1. Substitute $a = 1$, $b = -1$, $c = 2$, $m = 1$, $p = 3$, and $y = -8$ into the formula:

$$r = \frac{1 \cdot 1}{(-8)^{\frac{2}{3}}[3 + (-1) \cdot 1 \cdot 3^2]}$$

2. Calculate $(-8)^{\frac{2}{3}}$:

$$(-8)^{\frac{2}{3}} = \left((-8)^{\frac{1}{3}}\right)^2 = (-2)^2 = 4$$

3. Simplify the denominator:

$$3 + (-1) \cdot 1 \cdot 3^2 = 3 - 9 = -6$$

4. Combine the results:

$$r = \frac{1}{4 \cdot (-6)} = \frac{1}{-24} = -\frac{1}{24}$$

Thus, the value of r is $-\frac{1}{24}$, so $a = -1$ and $b = 24$.

$$\boxed{-1}, \boxed{24}$$

Problem 50

A chord subtends an angle of $72°$ at the centre of a circle of radius 24.5 m. Calculate, correct to one decimal place, the perimeter of the minor segment. [Take $\pi = \frac{22}{7}$]

Correct answer:
59.6

Solution. To calculate the perimeter of the minor segment formed by a chord subtending an angle of $72°$ at the centre of a circle with radius 24.5 meters, we need to find both the arc length and the chord length.

1. **Arc Length Calculation:**
The formula for the arc length is $\frac{\theta}{360°} \times 2\pi r$.

$$\text{Arc length} = \frac{72°}{360°} \times 2 \times \frac{22}{7} \times 24.5$$

Simplifying the angle ratio:

$$\frac{72}{360} = \frac{1}{5}$$

Calculating the arc length:

$$\text{Arc length} = \frac{1}{5} \times 2 \times \frac{22}{7} \times 24.5 = \frac{1}{5} \times 2 \times \frac{22}{7} \times \frac{49}{2} = \frac{1}{5} \times 154 = 30.8 \text{ meters}$$

2. **Chord Length Calculation:**
The formula for the chord length is $2r \sin\left(\frac{\theta}{2}\right)$.

$$\text{Chord length} = 2 \times 24.5 \times \sin\left(\frac{72°}{2}\right) = 49 \times \sin(36°)$$

Using $\sin(36°) \approx 0.5878$:

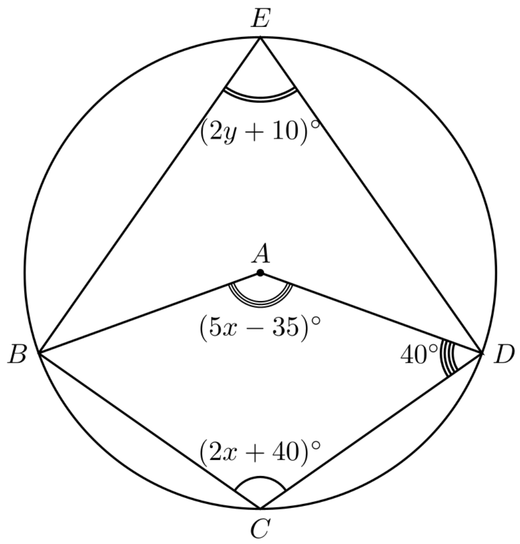
$$\text{Chord length} \approx 49 \times 0.5878 \approx 28.8 \text{ meters}$$

3. **Perimeter of the Minor Segment:**
The perimeter is the sum of the arc length and the chord length:

$$\text{Perimeter} = 30.8 + 28.8 = 59.6 \text{ meters}$$

Thus, the perimeter of the minor segment is $\boxed{59.6}$ meters.

Problem 51



In the diagram, $BCDE$ is a circle with centre A; $\angle BCD = (2x+40)^\circ, \angle BAD = (5x-35)^\circ, \angle BED = (2y+10)^\circ$ and $\angle ADC = 40^\circ$. Find the values of x and y.

Correct answer:
35
Correct answer:
30
Solution. Given the diagram with circle BCDE and center A, we need to find the values of x and y based on the given angles:
1. $\angle BCD = (2x+40)^\circ$
2. $\angle BAD = (5x-35)^\circ$
3. $\angle BED = (2y+10)^\circ$
4. $\angle ADC = 40^\circ$
Key Steps:
1. **Cyclic Quadrilateral BCDE**:
- Opposite angles in a cyclic quadrilateral sum to 180°.
- $\angle BCD + \angle BED = 180^\circ$
- Substituting the given angles:

$$(2x+40)+(2y+10)=180$$

- Simplifying:

$$2x+2y+50=180 \implies 2x+2y=130 \implies x+y=65$$

2. **Central and Inscribed Angles**:
- $\angle BAD$ is a central angle subtended by arc BD, so it is twice the inscribed angle subtended by the same arc.
- However, since $\angle BCD$ is an inscribed angle subtended by the major arc BD, we use the rela-

tionship:

$$\angle BCD = \frac{1}{2} \times \text{measure of major arc BD}$$

- The measure of major arc BD is $360^\circ - \angle BAD$:

$$2x + 40 = \frac{1}{2}(360 - (5x - 35))$$

- Solving this equation:

$$4x + 80 = 360 - 5x + 35 \implies 9x = 315 \implies x = 35$$

3. **Finding y**:
- Using the equation $x + y = 65$:

$$35 + y = 65 \implies y = 30$$

4. **Verification**:
- $\angle BAD = 5x - 35 = 5(35) - 35 = 140^\circ$
- $\angle BCD = 2x + 40 = 2(35) + 40 = 110^\circ$
- $\angle BED = 2y + 10 = 2(30) + 10 = 70^\circ$
- Check cyclic quadrilateral: $110^\circ + 70^\circ = 180^\circ$
- $\angle ADC = 40^\circ$ confirms the isosceles triangle properties.

Final Answer
The values of x and y are $\boxed{35}$ and $\boxed{30}$ respectively.

Problem 52
(a) Given that m = $\tan 30°$ and n = $\tan 45°$, simplify, without using calculator, $\frac{m-n}{mn}$, leaving the answer in the form p $- \sqrt{q}$. Write the answer as p, q, where p, q are integers.
(b) There are 20 women in a bus; 15 of them wear glasses and 10 wear wristwatches. If a woman is chosen at random from the bus, find the probability that she wears both glasses and wristwatches. Write down the answer as a decimal number.

Correct answer:
1,3
Correct answer:
0.25
Solution. Given that m = $\tan 30°$ and n = $\tan 45°$, we need to simplify $\frac{\mathrm{m-n}}{\mathrm{mn}}$.
First, we recall the values of the tangents:
- $\tan 30° = \frac{\sqrt{3}}{3}$
- $\tan 45° = 1$

Substituting these values into the expression:

$$\frac{\mathrm{m-n}}{\mathrm{mn}} = \frac{\frac{\sqrt{3}}{3} - 1}{\left(\frac{\sqrt{3}}{3}\right) \cdot 1}$$

Simplifying the denominator:

$$\left(\frac{\sqrt{3}}{3}\right) \cdot 1 = \frac{\sqrt{3}}{3}$$

This converts the expression into:

$$\frac{\frac{\sqrt{3}}{3} - 1}{\frac{\sqrt{3}}{3}} = \left(\frac{\sqrt{3}}{3} - 1\right) \cdot \frac{3}{\sqrt{3}}$$

Distributing the multiplication:

$$\left(\frac{\sqrt{3}}{3} \cdot \frac{3}{\sqrt{3}}\right) - \left(1 \cdot \frac{3}{\sqrt{3}}\right) = 1 - \sqrt{3}$$

Thus, the simplified form is $1 - \sqrt{3}$, which gives us $p = 1$ and $q = 3$.
For part (b), we need to find the probability that a randomly chosen woman wears both glasses and wristwatches. Using the principle of inclusion-exclusion:
- Number of women wearing glasses, $|G| = 15$
- Number of women wearing wristwatches, $|W| = 10$
- Total number of women, $|U| = 20$

The number of women wearing both is given by:

$$|G \cap W| = |G| + |W| - |U| = 15 + 10 - 20 = 5$$

The probability is then:

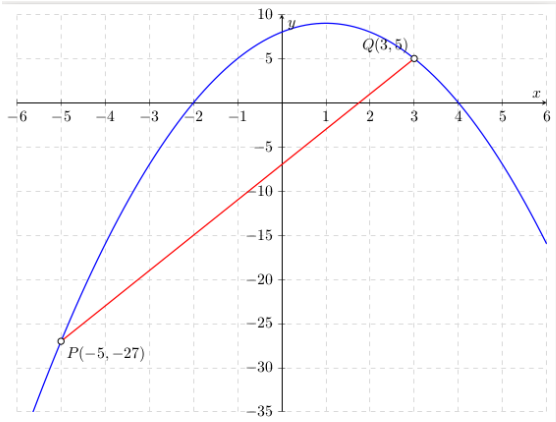
$$\frac{5}{20} = 0.25$$

Final Answer

(a) $\boxed{1, 3}$

(b) $\boxed{0.25}$

Problem 53



The graph shows the relation of the form $y = mx^2 + nx + r$, where m, n and r are constants. Using the graph:
A. State the scale used on both axes. Represent the answers as (1 cm):(x unit), (1 cm):(y unit). Write the answer as x, y.
B. Find the values of m, n and r.
C. Find the gradient of the line through P and Q.
D. State the range of values of x for which $y > 0$. Represent the answer as an interval (a, b), where a, b are integers, write the answer as a, b

Correct answer:
1,5
Correct answer:
-1
Correct answer:
2
Correct answer:
8
Correct answer:
4
Correct answer:
-2,4
Solution. (a) Each division measures 2 cm (must be measured on paper exam)

x-axis: 2 cm to 2 units or 1 cm to 1 unit

y-axis: 2 cm to 10 units or 1 cm to 5 units
$\boxed{1, 5}$
(b) Zeroes of the function are x = -2, x = 4 (from the graph)

According to fundamental theorem of algebra:

$$y = a(x + 2)(x - 4) = 0$$

Expand:

$$y = a(x^2 - 4x + 2x - 8) = 0$$

Simplify:

$$y = a(x^2 - 2x - 8) = 0$$

To find a plug in any point on the graph. (0,8) belongs to the graph so

$$8 = -(0^2 - 2 * 0 - 8) = 0$$

$$a = -1$$

$$y = -(x^2 - 2x - 8) = 0$$

$$y = -x^2 + 2x + 8 = 0$$

$\boxed{-1, 2, 8}$
(c) P(-5, -27), Q(3, 5) (from the graph)

$$\text{Gradient} = \frac{5 + 27}{3 + 5}$$

$$\text{Gradient} = \frac{32}{8} = 4$$

$\boxed{4}$
(d) We need to find the values of x for which the graph of our parabola y is above X axis. From the graph we can see that in happens when
Range: x: $-2 < x < 4$
$\boxed{-2, 4}$

Problem 54

(a) A man purchased 180 copies of a book at N250.00 each. He sold y copies at N300.00 each and the rest at a discount of 5 kobo in the Naira of the cost price. If he made a profit of N$7,125.00$, find the value of y.

(b) A trader bought x bags of rice at a cost C = 24x+103 and sold them at a price, S = $33x - \frac{x^2}{20}$.

(i) Find the expression for the profit; represent the answer as $ax^2 + bx + c$, where a, b, c are decimal numbers, writhe down the answer as a, b, c (ii) If 20 bags of rice were sold, calculate the percentage profit.

Correct answer:
150
Correct answer:
-0.05,9,-103
Correct answer:
9.78

Solution. (a) A man purchased 180 copies of a book at N250.00 each. The total cost price is calculated as follows:

$$180 \times 250 = 45,000$$

He sold y copies at N300.00 each and the rest at a discount of 5 kobo in the Naira, which is 95% of the cost price (N237.50 per copy). The total revenue from selling y copies at N300.00 each is $300y$. The revenue from selling the remaining $180 - y$ copies at N237.50 each is $(180 - y) \times 237.50$. The total revenue is:

$$300y + (180 - y) \times 237.50$$

The profit is given as N7,125.00. Setting up the equation for profit:

$$300y + (180 - y) \times 237.50 - 45,000 = 7,125$$

Expanding and simplifying:

$$300y + 42,750 - 237.5y - 45,000 = 7,125$$

$$62.5y - 2,250 = 7,125$$

Solving for y:

$$62.5y = 9,375 \implies y = \frac{9,375}{62.5} = 150$$

Thus, the value of y is $\boxed{150}$.

(b) (i) A trader bought x bags of rice at a cost $C = 24x + 103$ and sold them at a price

$S = 33x - \frac{x^2}{20}$. The expression for the profit is:

$$\text{Profit} = S - C = \left(33x - \frac{x^2}{20}\right) - (24x + 103) = 9x - \frac{x^2}{20} - 103$$

Therefore, the answer is $\boxed{-0.05, 9, -103}$.
(ii) The percentage profit is calculated as follows. The cost price for 20 bags is:

$$C = 24 \times 20 + 103 = 583$$

The selling price for 20 bags is:

$$S = 33 \times 20 - \frac{20^2}{20} = 640$$

The percentage profit (loss) is:

$$\frac{(S - C)}{C} \times 100 \approx 9.78\%$$

Thus, the percentage profit is $\boxed{9.78\%}$.

Problem 55

Item	food & drinks	fuel	rent	building project	education	savings
Percentage%	35	7.5	10	15	17.5	x

The table shows the monthly expenditure (in percentages) of Mr. Okafor's salary.
A. Calculate the percentage of Mr. Okafor's salary that was put into savings.
B. Illustrate the information on a pie chart.
C. If Mr. Okafor's annual gross salary is $\$28,800.00$ and he pays tax of 12%, calculate; (i) his monthly tax; (ii) the amount saved each month.

Correct answer:
15
Correct answer:
graph
Correct answer:
288
Correct answer:
316.8
Solution. To solve the given problem, we follow these steps:
Part A: Calculate the percentage of Mr. Okafor's salary that was put into savings.
The total percentage of all expenditures must sum to 100%. Adding the given percentages:

$$35\% + 7.5\% + 10\% + 15\% + 17.5\% = 85\%$$

The remaining percentage allocated to savings is:

$$100\% - 85\% = 15\%$$

Final Answer for Part A: The percentage put into savings is $\boxed{15\%}$.
Part B: Illustrate the information on a pie chart.
To create a pie chart, each percentage is converted to degrees by multiplying by 3.6 (since $360°/100\% = 3.6$ per percentage):
- Food drinks: $35\% \times 3.6 = 126°$
- Fuel: $7.5\% \times 3.6 = 27°$
- Rent: $10\% \times 3.6 = 36°$
- Building project: $15\% \times 3.6 = 54°$
- Education: $17.5\% \times 3.6 = 63°$
- Savings: $15\% \times 3.6 = 54°$
Final Answer for Part B: The pie chart is shown in the Figure below.

Part C: Calculate tax and monthly savings.
Given the annual gross salary is $28,800.00:
1. **Monthly gross salary:**

$$\frac{28,800}{12} = 2,400 \text{ dollars}$$

2. **Monthly tax (12% of gross salary):**

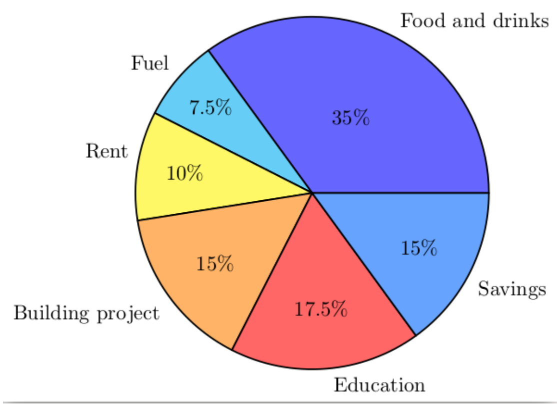


Figure 1:

$$0.12 \times 2,400 = 288 \text{ dollars}$$

3. **Net monthly salary (after tax):**

$$2,400 - 288 = 2,112 \text{ dollars}$$

4. **Monthly savings (24% of net salary):**

$$0.15 \times 2,112 = 316.8 \text{ dollars}$$

Final Answers for Part C:
(i) Monthly tax is $\boxed{\$288.00}$; (ii) Monthly savings amount is $\boxed{\$316.8}$.

Problem 56

A. Copy and complete the table of values for $y = 3\,\mathrm{Sin\,x} + 7\,\mathrm{Cos\,x}$, write the answer as a sequence of missing values for y coresponding to the values of x from 20 to 180.

x°	0	20	40	60	80	100	120	140	160	180
y	7.0				4.2		-0.9			

B. Using a scale of 2 cm to 20° on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of $y = 3\,\mathrm{Sin}\,x + 7\,\mathrm{Cos}\,x$ for all the values of x from the table.

C. Using the graph, find the: (i) value of y when $\mathrm{x} = 150°$, represent the answer as $a \pm 0.2$, where a as a decimal number and write down the answer as a; (ii) range of values of x for which $\mathrm{y} > 0$, represent the answer as $a° \leq x \leq b°$ and where a, b are integers, and write down the answer as a, b.

Correct answer:
7,7.6,7.3,6.1,4.2,1.7,-0.9,-3.4,-5.6,7.0
Correct answer:
graph
Correct answer:
-4.6
Correct answer:
0,113

Solution. A. To complete the table of values for $y = 3\sin x + 7\cos x$, we calculate the missing y values for x from 20 to 180:

- For $x = 20$:

$$y = 3\sin 20 + 7\cos 20 \approx 3 \times 0.3420 + 7 \times 0.9397 \approx 1.026 + 6.5779 \approx 7.6$$

- For $x = 40$:

$$y = 3\sin 40 + 7\cos 40 \approx 3 \times 0.6428 + 7 \times 0.7660 \approx 1.9284 + 5.3622 \approx 7.3$$

- For $x = 60$:

$$y = 3\sin 60 + 7\cos 60 \approx 3 \times 0.8660 + 7 \times 0.5 \approx 2.598 + 3.5 \approx 6.1$$

- For $x = 100$:

$$y = 3\sin 100 + 7\cos 100 \approx 3 \times 0.9848 + 7 \times (-0.1736) \approx 2.9544 - 1.2152 \approx 1.7$$

- For $x = 140$:

$$y = 3\sin 140 + 7\cos 140 \approx 3 \times 0.6428 + 7 \times (-0.7660) \approx 1.9284 - 5.362 \approx -3.4$$

- For $x = 160$:

$$y = 3\sin 160 + 7\cos 160 \approx 3 \times 0.3420 + 7 \times (-0.9397) \approx 1.026 - 6.5779 \approx -5.6$$

- For $x = 180$:

$$y = 3\sin 180 + 7\cos 180 = 0 + 7 \times (-1) = -7.0$$

The completed table values for y are: 7.6, 7.3, 6.1, 1.7, -3.4, -5.6, -7.0.
B. We draw the graph using values from part A.
Scaling:
x-axis: $2\,\text{cm} = 20^\circ$ (equivalent to $0.1\ \ \text{cm}/^\circ$).
$y - axis: 2\ \ \text{cm} = 2\ \ \text{units}(equivalentto 1\,\text{cm/unit})$.
$For the graph see Figure below.$

$For part C(i), using the graph, the value of y$ when $x = 150$ is approximately -4.6 ± 0.2, so the answer is -4.6.
For part C(ii), the range of x for which $y > 0$ is found by solving $3\sin x + 7\cos x = 0$. The solution is approximately 113.2, so the integer range is $0 \leq x \leq 113$.
Final Answer
A. The missing values are $\boxed{7.6}$, $\boxed{7.3}$, $\boxed{6.1}$, $\boxed{1.7}$, $\boxed{-3.4}$, $\boxed{-5.6}$, and $\boxed{-7.0}$.
C. (i) $\boxed{-4.6}$
(ii) $\boxed{0}$, $\boxed{113}$

Problem 57

Age	3	4	5	6	7	8	9	10
No. of children	2	6	5	x	6	9	8	5

The table below shows the distribution of ages of a number of children in a school. If the mean of the distribution is 7, find: A. value of x. B. standard deviation of their ages.

Correct answer:
4
Correct answer:
2.087
Solution. To find the value of x and the standard deviation of the ages, we start with the given distribution table and the mean of 7.
Part A: Finding the value of x
1. **Calculate the total number of children:**

$$2+6+5+x+6+9+8+5=41+x$$

2. **Calculate the sum of (Age Number of children):**

$$3\times 2+4\times 6+5\times 5+6\times x+7\times 6+8\times 9+9\times 8+10\times 5=291+6x$$

3. **Set up the equation for the mean:**

$$\frac{291+6x}{41+x}=7$$

4. **Solve for x:**

$$291+6x=7(41+x)291+6x=287+7x4=x$$

Thus, the value of x is $\boxed{4}$.
Part B: Calculating the standard deviation
1. **Calculate the total number of children with $x=4$:**

$$41+4=45$$

2. **Compute the squared differences from the mean (7) for each age and multiply by the number of children:**

$$(3-7)^2 \times 2 = 16 \times 2 = 32,$$

$$(4-7)^2 \times 6 = 9 \times 6 = 54,$$

$$(5-7)^2 \times 5 = 4 \times 5 = 20,$$

$$(6-7)^2 \times 4 = 1 \times 4 = 4,$$

$$(7-7)^2 \times 6 = 0 \times 6 = 0,$$

$$(8-7)^2 \times 9 = 1 \times 9 = 9,$$

$$(9-7)^2 \times 8 = 4 \times 8 = 32,$$

$$(10-7)^2 \times 5 = 9 \times 5 = 45.$$

3. **Sum these values:**

$$32 + 54 + 20 + 4 + 0 + 9 + 32 + 45 = 196$$

4. **Calculate the variance:**

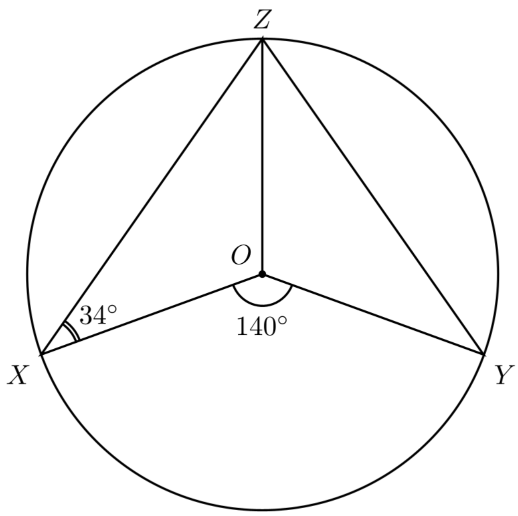
$$\frac{196}{45}$$

5. **Calculate the standard deviation:**

$$\sqrt{\frac{196}{45}} = \frac{14}{\sqrt{45}} = \frac{14\sqrt{5}}{15} = 2.087$$

Thus, the standard deviation is $\boxed{2.087}$.

Problem 58



(a) In the diagram, O is the centre of the circle, $\angle ZXO = 34°$ and $\angle XOY = 146°$. Find $\angle OYZ$.
(b) The exterior angles of a polygon are $42°, 38°, 57°, x°, (x + y)°, (2x - 15)°, (3x - y)°$. If x is 7 less than y, find the values of x and y.

Correct answer:
39
Correct answer:
34
Correct answer:
41

Solution. (a) In the diagram, O is the centre of the circle, $\angle ZXO = 34°$, and $\angle XOY = 146°$. We need to find $\angle OYZ$.
1. **Triangle ZXO**: Since OX and OZ are radii, triangle ZXO is isosceles with $\angle ZXO = \angle OZX = 34°$. The central angle $\angle ZOX$ is calculated as:

$$\angle ZOX = 180° - 34° - 34° = 112°$$

2. **Central Angles**: The given central angle $\angle XOY = 146°$. The total central angles around point O sum to $360°$. Therefore, the central angle $\angle YOZ$ is:

$$\angle YOZ = 360° - 112° - 146° = 102°$$

3. **Triangle OYZ**: Since OY and OZ are radii, triangle OYZ is isosceles. The base angles $\angle OYZ$ and $\angle OZY$ are equal. Thus:

$$\angle OYZ = \frac{180° - 102°}{2} = 39°$$

(b) The exterior angles of a polygon are given as $42°, 38°, 57°, x°, (x + y)°, (2x - 15)°, (3x - y)°$. We know x is 7 less than y (i.e., $x = y - 7$). We need to find x and y.
1. **Sum of Exterior Angles**: The sum of the exterior angles of any polygon is $360°$. Setting

up the equation:

$$42 + 38 + 57 + x + (x + y) + (2x - 15) + (3x - y) = 360$$

2. **Simplifying**: Combine constants and variables separately:

$$42 + 38 + 57 - 15 = 122$$

$$x + (x + y) + 2x + 3x - y = 7x$$

Thus, the equation becomes:

$$122 + 7x = 360$$

3. **Solving for x and y**:

$$7x = 238 \implies x = 34$$

$$y = x + 7 = 34 + 7 = 41$$

Final Answer
(a) $\boxed{39^\circ}$
(b) $\boxed{x = 34}$ and $\boxed{y = 41}$

Problem 59

(a) The probability that an athlete will not win any of three races is $\frac{1}{4}$. If the athlete runs in all the races, what is the probability that the athlete will win:

(i) only the second race, correct the answer to three decimal places; (ii) all the three races, correct the answer to three decimal places; (ii) only two of the races, correct the answer to two decimal places?

(b) A cone with perpendicular height 24 cm has a volume of 1200 cm^3. Find the volume of a cone with same base radius and height 84 cm. [Take $\pi = \frac{22}{7}$]

Correct answer:
0.047
Correct answer:
0.422
Correct answer:
0.422
Correct answer:
4200
Solution.

Solution

(a) Probability Calculations

Given that the probability that an athlete will not win a particular race is $\frac{1}{4}$, we define:

$$p = \frac{1}{4} = 0.25$$

Thus, the probability of winning a race is:

$$q = 1 - p = 1 - 0.25 = 0.75$$

Now, we compute the required probabilities.

(i) Probability of winning only the second race

For the athlete to win only the second race, they must lose the first race, win the second, and lose the third:

$$P(\text{only second race}) = p \cdot q \cdot p$$

$$= (0.25) \times (0.75) \times (0.25)$$

$$= 0.046875$$

$$\approx 0.047 \quad \text{(rounded to three decimal places)}$$

(ii) Probability of winning all three races

The probability that the athlete wins all three races is:

$$P(\text{all three races}) = q^3$$

$$= (0.75)^3$$

$$= 0.421875$$

$$\approx 0.422 \quad \text{(rounded to three decimal places)}$$

(iii) Probability of winning exactly two races

The athlete must win exactly two races and lose one. This can occur in three different ways:
1. Win the first and second, lose the third.
2. Win the first and third, lose the second.
3. Win the second and third, lose the first.

Each of these cases has probability:

$$P(\text{winning two races}) = q^2 \cdot p$$

$$= (0.75)^2 \times (0.25)$$

$$= 0.140625$$

Since there are three ways this can happen, the total probability is:

$$3 \times 0.140625 = 0.421875$$

$$\approx 0.422 \quad \text{(rounded to three decimal places)}$$

(b) Volume of a Cone

The volume of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h$$

For the first cone:
- **Height:** $h_1 = 24$ cm
- **Volume:** $V_1 = 1200$ cm^3
For the second cone:
- **Height:** $h_2 = 84$ cm
- **Same base radius** r
Since the radius remains the same, the volume scales in direct proportion to height:

$$\frac{V_2}{V_1} = \frac{h_2}{h_1}$$

$$V_2 = V_1 \times \frac{h_2}{h_1}$$

$$= 1200 \times \frac{84}{24}$$

$$= 1200 \times 3.5$$

$$= 4200 \text{ cm}^3$$

Final Answers

- (a) (i) $\boxed{0.047}$
- (ii) $\boxed{0.422}$
- (iii) $\boxed{0.422}$
- (b) $\boxed{4200}$

Problem 60

(a) The diameter of a cylinder closed at both ends is 7 cm. If the total surface area is 209 cm^2, calculate the height. [Take $\pi = \frac{22}{7}$]

(b) The points X and Y, 19 m apart are on the same side of a tree. The angles of elevation of the top, T, of the tree from X and Y on the horizontal ground with the foot of the tree are $43°$ and $38°$ respectively.

(i) Illustrate the information in a diagram. (ii) Find, correct to one decimal place, the height of the tree.

Correct answer:
6
Correct answer:
graph
Correct answer:
91.6

Solution. (a) The diameter of the cylinder is 7 cm, so the radius r is 3.5 cm. The total surface area (TSA) of a closed cylinder is given by the formula:

$$\text{TSA} = 2\pi r^2 + 2\pi rh$$

Given the TSA is 209 cm and using $\pi = \frac{22}{7}$:

$$209 = 2\left(\frac{22}{7}\right)(3.5)^2 + 2\left(\frac{22}{7}\right)(3.5)h$$

Calculating the terms:

$$2\left(\frac{22}{7}\right)(3.5)^2 = 2\left(\frac{22}{7}\right)(12.25) = 77 \text{ cm}^2$$

$$2\left(\frac{22}{7}\right)(3.5)h = 22h$$

Thus:

$$209 = 77 + 22h \implies 132 = 22h \implies h = 6 \text{ cm}$$

(b) (i)
See the figure below.

(b) (ii) The height of the tree can be found using trigonometry. Let h be the height of the tree, x be the distance from the foot of the tree to point X, and $x + 19$ be the distance to point Y. Using the angles of elevation:

$$\tan(43°) = \frac{h}{x} \quad \text{and} \quad \tan(38°) = \frac{h}{x+19}$$

Equating the expressions for h:

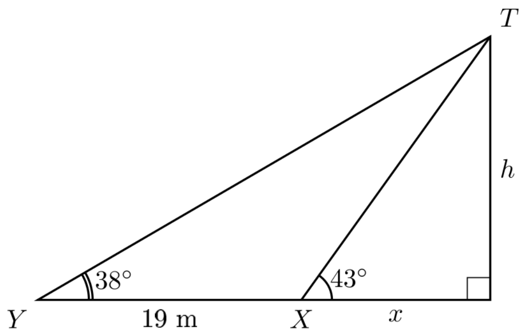


Figure 3:

$$x \tan(43^\circ) = (x + 19) \tan(38^\circ)$$

Solving for x:

$$x(\tan(43^\circ) - \tan(38^\circ)) = 19 \tan(38^\circ)$$

$$x = \frac{19 \tan(38^\circ)}{\tan(43^\circ) - \tan(38^\circ)}$$

Using $\tan(43^\circ) \approx 0.9325$ and $\tan(38^\circ) \approx 0.7813$:

$$x \approx \frac{19 \times 0.7813}{0.9325 - 0.7813} \approx \frac{14.8447}{0.1512} \approx 98.1792 \text{ m}$$

Calculating h:

$$h = x \tan(43^\circ) \approx 98.16 \times 0.9325 \approx 91.6 \text{ m}$$

Final answers:
(a) $\boxed{6 \text{ cm}}$
(b)(ii) $\boxed{91.6 \text{ m}}$