1 Solutions to problems public WAEC math 2021 By cheetahwaec.com

Problem 1
Correct 0.007985 to three significant figures.
Possible answers:
A. 0.0109
B. 0.0800
C. 0.00799
D. 0.008
Correct answer: C
Solution. To correct 0.007985 to three significant figures:
1. **Identify significant figures**: Leading zeros are not significant. The first significant figure is 7, followed by 9, 8, and 5.
2. **Round to three significant figures**: The third significant figure is 8. The next digit (5) is equal to 5, so we round 8 up to 9.
3. **Result**: The rounded number is 0.00799, retaining three significant figures (7, 9, 9).
Final Answer: $\boxed{C}$
Main Concepts:
- **Significant Figures**: Non-zero digits and trailing zeros in decimals are significant; leading zeros are not.
- **Rounding Rules**: If the digit after the desired precision is 5 or greater, round up the last retained digit.
- **Context**: Properly distinguishing between decimal places and significant figures is critical to avoid errors.

Problem 2

Simplify: $(11_{two})^2$.

Possible answers:

A. 1001_2

B. 1101_2

C. 101_2

D. 10001_2

Correct answer: A

Solution. To simplify $(11_{\text{two}})^2$:

1. **Convert 11_{two} to decimal**:

$11_{\text{two}} = 1 \cdot 2^1 + 1 \cdot 2^0 = 2 + 1 = 3_{\text{ten}}$.

2. **Square the decimal value**:

$3^2_{\text{ten}} = 9_{\text{ten}}$.

3. **Convert 9_{ten} back to binary**:

$9_{\text{ten}} = 1001_{\text{two}}$ (since $9 = 8 + 1 = 2^3 + 2^0$).

4. **Verify via binary multiplication**:

$$\begin{array}{r} 11_{\text{two}} \\ \times 11_{\text{two}} \\ \hline 11_{\text{two}} \\ +110_{\text{two}} \\ \hline 1001_{\text{two}} \end{array}$$

Final Answer: $\boxed{A}$ (1001_{two}).

Main Concepts:

- **Base Conversion**: Translating between binary and decimal systems.
- **Arithmetic Operations**: Squaring in decimal and verifying through binary multiplication.
- **Binary Multiplication**: Directly multiplying in base two and aligning partial sums.
- **Verification**: Cross-checking results using multiple methods to ensure accuracy.

Problem 3

Solve: $2^{\sqrt{2x+1}} = 32$.

Possible answers:

A. 13

B. 24

C. 12

D. 11

Correct answer: C

Solution. To solve the equation $2^{\sqrt{2x+1}} = 32$:

1. **Express both sides with the same base**: Recognize that $32 = 2^5$, rewriting the equation as:

$$2^{\sqrt{2x+1}} = 2^5$$

2. **Equate the exponents**: Since the bases are equal, set the exponents equal:

$$\sqrt{2x+1} = 5$$

3. **Solve for x**:

- Square both sides to eliminate the square root:

$$(\sqrt{2x+1})^2 = 5^2 \implies 2x+1 = 25$$

- Subtract 1 and divide by 2:

$$2x = 24 \implies x = 12$$

4. **Verify the solution**: Substitute $x = 12$ back into the original equation:

$$2^{\sqrt{2(12)+1}} = 2^{\sqrt{25}} = 2^5 = 32$$

The solution is confirmed.

Answer: C. 12

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Main Concepts:

1. **Exponential Equations with Same Base**: Matching bases to equate exponents simplifies solving.

2. **Radical Elimination**: Squaring both sides removes the square root, converting the equation into a linear form.

3. **Linear Equation Solving**: Using inverse operations (subtraction/division) isolates the variable.

4. **Verification**: Substituting the solution ensures no extraneous roots were introduced.

Problem 4

If $\log_{10} 2 = m$ and $\log_{10} 3 = n$, find $\log_{10} 24$ in terms of m and n.
Possible answers:
A. $3m + n$
B. $m + 3n$
C. $4mn$
D. $3mn$
Correct answer: A

Solution. To find $\log_{10} 24$ in terms of m and n, start by expressing 24 as a product of prime factors:

$$24 = 2^3 \times 3.$$

Using logarithm properties:
1. **Product Rule**: $\log_{10}(ab) = \log_{10} a + \log_{10} b$.
2. **Power Rule**: $\log_{10}(a^k) = k \log_{10} a$.
Apply these rules to the factorization:

$$\log_{10} 24 = \log_{10}(2^3 \times 3) = \log_{10} 2^3 + \log_{10} 3 = 3 \log_{10} 2 + \log_{10} 3.$$

Substitute $m = \log_{10} 2$ and $n = \log_{10} 3$:

$$\log_{10} 24 = 3m + n.$$

Main Concepts:
- **Prime Factorization**: Breaking down 24 into $2^3 \times 3$.
- **Logarithm Product Rule**: Separating the logarithm of a product into a sum of logarithms.
- **Logarithm Power Rule**: Converting the exponent in 2^3 into a coefficient.
- **Substitution**: Replacing $\log_{10} 2$ and $\log_{10} 3$ with m and n.
Answer: $\boxed{A}$

Problem 5

Find the 5th term of the sequence $2, 5, 10, 17, \ldots$
Possible answers:
A. 22
B. 24
C. 36
D. 26
Correct answer: D

Solution. The 5th term of the sequence $2, 5, 10, 17, \ldots$ is found by identifying the pattern. Observing the differences between consecutive terms $(3, 5, 7)$ reveals an increment of 2 each time, leading to a next difference of 9. Adding this to the fourth term:

$$17 + 9 = 26.$$

To confirm, the sequence follows the quadratic formula $n^2 + 1$. For $n = 5$:

$$5^2 + 1 = 26.$$

Answer: D. 26.
Main Concepts:
1. **Difference Analysis:** Calculating first differences $(3, 5, 7)$ and second differences (constant 2) identified the sequence as quadratic.
2. **Quadratic Formula Derivation:** Solving $an^2 + bn + c$ using given terms confirmed the pattern $n^2 + 1$.
3. **Pattern Validation:** Testing the formula against known terms ensured accuracy before extrapolating the 5th term.

Problem 6

If $P = \{-3 < x < 1\}$ and $Q = \{-1 < x < 3\}$, where x is an integer, find $P \cap Q$.
Possible answers:
A. $\{0\}$
B. $\{-3, -2, -1, 0, 1\}$
C. $\{-2, -1, 0\}$
D. $\{-1, 0, 1\}$
Correct answer: A

Solution. To solve the problem, we first determine the elements of sets P and Q.
- **Set P:** Defined by $-3 < x < 1$, where x is an integer. The integers satisfying this inequality are $-2, -1, 0$. Thus, $P = \{-2, -1, 0\}$.
- **Set Q:** Defined by $-1 < x < 3$, where x is an integer. The integers here are $0, 1, 2$. Thus, $Q = \{0, 1, 2\}$.
The intersection $P \cap Q$ consists of elements common to both sets. Comparing P and Q, the only shared integer is 0. Therefore, $P \cap Q = \{0\}$.
Answer: A. $\{0\}$
Main Concepts:
1. **Set Intersection:** Identifying common elements between two sets.
2. **Integer Intervals:** Listing integers within open intervals (strict inequalities exclude endpoints).
3. **Inequality Interpretation:** Translating strict inequalities ($<$) into specific integer values.

Problem 7

Factorize $6pq - 3rs - 3ps + 6qr$.
Possible answers:
A. $3(r-p)(2q+s)$
B. $3(p+r)(2q-2q-s)$
C. $3(2q-s)(p+r)$
D. $3(r-p)(s-2q)$
Correct answer: C

Solution. To factorize $6pq - 3rs - 3ps + 6qr$:
1. **Factor out the greatest common factor (GCF)**:
All terms share a GCF of 3:

$$3(2pq - rs - ps + 2qr).$$

2. **Group terms**:
Group the expression inside the parentheses into pairs:

$$3\left[(2pq + 2qr) + (-rs - ps)\right].$$

3. **Factor each group**:
- From $2pq + 2qr$, factor out $2q$:

$$2q(p+r).$$

- From $-rs - ps$, factor out $-s$:

$$-s(r+p).$$

4. **Factor the common binomial**:
Both groups now share $(p+r)$:

$$3\left[(p+r)(2q-s)\right].$$

5. **Simplify**:
The fully factored form is:

$$3(2q-s)(p+r).$$

Verification:
Expanding $3(2q-s)(p+r)$ gives:

$$3[2qp + 2qr - sp - sr] = 6pq + 6qr - 3ps - 3rs,$$

which matches the original expression.
Answer: $\boxed{C}$
Main Concepts:

- **Greatest Common Factor (GCF)**: Factoring out the GCF simplifies the polynomial.
- **Grouping**: Splitting the polynomial into pairs to identify common factors.
- **Binomial Factoring**: Recognizing and factoring out a common binomial term.
- **Sign Management**: Carefully handling negative signs when factoring groups.
- **Verification**: Expanding the result to confirm equivalence with the original expression.

Problem 8

What number should be subtracted from the sum of $2\frac{1}{6}$ and $2\frac{7}{12}$ to give $3\frac{1}{4}$?
Possible answers:
A. $\frac{1}{3}$
B. $1\frac{1}{2}$
C. $1\frac{1}{6}$
D. $\frac{1}{2}$
$Correct answer : B$
Solution. $To solve the problem, we need to determine the number that, when subtracted from the sum of 2\frac{1}{6}$ and $2\frac{7}{12}$, results in $3\frac{1}{4}$.
1. **Convert mixed numbers to improper fractions**:
- $2\frac{1}{6} = \frac{13}{6}$
- $2\frac{7}{12} = \frac{31}{12}$
- $3\frac{1}{4} = \frac{13}{4}$
2. **Find the sum of $\frac{13}{6}$ and $\frac{31}{12}$**:
- Common denominator: 12
- $\frac{13}{6} = \frac{26}{12}$
- $\frac{26}{12} + \frac{31}{12} = \frac{57}{12} = \frac{19}{4}$
3. **Set up the equation**:
- Let x be the number to subtract: $\frac{19}{4} - x = \frac{13}{4}$
- Solve for x: $x = \frac{19}{4} - \frac{13}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$
Final Answer: $\boxed{B}$
Main Concepts:
1. **Conversion of mixed numbers to improper fractions**: Simplifies arithmetic operations by avoiding separate handling of whole and fractional parts.
2. **Addition of fractions**: Requires a common denominator to combine numerators directly.
3. **Solving linear equations**: Isolating the variable to find the unknown value through inverse operations.

Problem 9

Mensah is 5 years old and joyce is thrice as old as mensah. In how many years will joyce be twice as old as Mensah?

Possible answers:

A. 3 years

B. 10 years

C. 5 years

D. 15 years

Correct answer: C

Solution. Mensah is currently 5 years old, and Joyce is thrice as old, making Joyce 15 years old. Let x be the number of years until Joyce is twice as old as Mensah. In x years:

- Mensah's age will be $5 + x$.
- Joyce's age will be $15 + x$.

We set up the equation where Joyce's future age equals twice Mensah's future age:

$$15 + x = 2(5 + x)$$

Expanding and simplifying:

$$15 + x = 10 + 2x \implies 15 - 10 = 2x - x \implies x = 5.$$

Verification: In 5 years, Mensah will be $5 + 5 = 10$ and Joyce will be $15 + 5 = 20$. Since $20 = 2 \times 10$, the solution is correct.

Final Answer: C. 5 years.

Main Concepts:

1. **Setting up variables** to represent unknown quantities (here, x as the number of years).
2. **Formulating equations** based on relationships (Joyce's future age being twice Mensah's).
3. **Solving linear equations** through algebraic manipulation.
4. **Verification** by substituting the solution back into the original problem to ensure validity.

Problem 10

If $16 * 2^{(x+1)} = 4^x * 8^{(1-x)}$, find the value of x.
Possible answers:
A. -4
B. 4
C. 1
D. -1
Correct answer: D
Solution. To solve the equation $16 \cdot 2^{(x+1)} = 4^x \cdot 8^{(1-x)}$, we start by expressing all terms with base 2:
- $16 = 2^4$, $4 = 2^2$, and $8 = 2^3$.
Left Side:
$16 \cdot 2^{(x+1)} = 2^4 \cdot 2^{(x+1)} = 2^{4+x+1} = 2^{x+5}$.
Right Side:
$4^x \cdot 8^{(1-x)} = (2^2)^x \cdot (2^3)^{(1-x)} = 2^{2x} \cdot 2^{3(1-x)} = 2^{2x+3-3x} = 2^{-x+3}$.
Equating the exponents (since the bases are equal):

$$x + 5 = -x + 3$$

Solving for x:

$$2x = -2 \quad \Rightarrow \quad x = -1$$

Verification:
Substitute $x = -1$ into the original equation:
- Left: $16 \cdot 2^{(-1+1)} = 16 \cdot 2^0 = 16$.
- Right: $4^{-1} \cdot 8^{(1-(-1))} = \frac{1}{4} \cdot 8^2 = \frac{1}{4} \cdot 64 = 16$.
Both sides match, confirming $x = -1$.
Answer: D. -1
Main Concepts:
1. **Exponent Rules**: Expressing numbers as powers of a common base (here, base 2) simplifies the equation.
2. **Equating Exponents**: If $a^m = a^n$, then $m = n$ when $a \neq 0, 1$.
3. **Verification**: Substituting the solution back into the original equation ensures correctness.

Problem 11

The circumference of a circular track is 9 km. A cyclist rides round it a number of times and stops after covering a distance of 302 km. How far is the cyclist from the starting point?

Possible answers:

A. 5 km

B. 6 km

C. 7 km

D. 3 km

Correct answer: A

Solution. The cyclist rides a total distance of 302 km on a circular track with a circumference of 9 km. To determine the distance from the starting point, we calculate the remainder when 302 km is divided by the circumference (9 km). This remainder represents the distance covered beyond the last full lap, which is the cyclist's distance from the start.

Steps:

1. Divide 302 by 9:

$302 \div 9 = 33$ full laps with a remainder of 5.

(Since $9 \times 33 = 297$, and $302 - 297 = 5$.)

2. The remainder 5 km is the distance from the starting point.

Answer: A. 5 km.

Main Concepts:

- **Circular Motion**: On a circular track, full laps return to the starting point, so only the remaining distance after the last full lap matters.
- **Modular Arithmetic**: The problem reduces to finding $302 \mod 9$, which gives the remainder after dividing by the circumference.
- **Division with Remainder**: The total distance is split into whole-number multiples of the circumference (full laps) and a leftover segment (partial lap).

Problem 12

Simplify $2\sqrt{7} - \frac{14}{\sqrt{7}} + \frac{7}{\sqrt{21}}$.

Possible answers:

A. $\frac{\sqrt{21}}{21}$

B. $7\frac{\sqrt{21}}{21}$

C. $\frac{\sqrt{21}}{3}$

D. $3\sqrt{21}$

Correct answer: C

Solution. To simplify the expression $2\sqrt{7} - \frac{14}{\sqrt{7}} + \frac{7}{\sqrt{21}}$:

1. **Rationalize the denominators**:

- For $-\frac{14}{\sqrt{7}}$, multiply by $\frac{\sqrt{7}}{\sqrt{7}}$:

$$-\frac{14\sqrt{7}}{7} = -2\sqrt{7}.$$

- For $\frac{7}{\sqrt{21}}$, multiply by $\frac{\sqrt{21}}{\sqrt{21}}$:

$$\frac{7\sqrt{21}}{21} = \frac{\sqrt{21}}{3}.$$

2. **Combine like terms**:

- The first two terms cancel: $2\sqrt{7} - 2\sqrt{7} = 0$.

- The remaining term simplifies to $\frac{\sqrt{21}}{3}$.

Answer: $\boxed{C}$

Main Concepts:

1. **Rationalizing Denominators**: Eliminating radicals from denominators by multiplying by the conjugate radical.

2. **Simplifying Fractions**: Reducing fractions by canceling common factors in numerators and denominators.

3. **Combining Like Terms**: Identifying and combining terms with the same radical components to simplify the expression.

Problem 13
If $4x + 2y = 16$ and $6x - 2y = 4$, find the value of $(y - x)$.
Possible answers:
A. 8
B. 2
C. 4
D. 6
Correct answer: B
Solution. To solve the system of equations $4x + 2y = 16$ and $6x - 2y = 4$ and find the value of $y - x$:
1. **Add the two equations** to eliminate y:

$$\begin{array}{rl} & 4x + 2y = 16 \\ + & 6x - 2y = 4 \\ \hline & 10x = 20 \end{array}$$

Solving for x:

$$x = \frac{20}{10} = 2$$

2. **Substitute $x = 2$ into the first equation** to find y:

$$4(2) + 2y = 16 \implies 8 + 2y = 16 \implies 2y = 8 \implies y = 4$$

3. **Calculate $y - x$**:

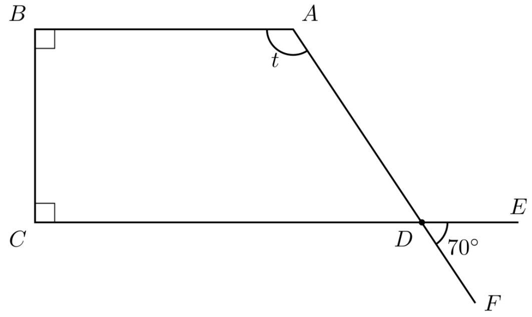
$$y - x = 4 - 2 = 2$$

4. **Verify the solution** by substituting $x = 2$ and $y = 4$ into the second equation:

$$6(2) - 2(4) = 12 - 8 = 4 \quad \text{(validates correctness)}$$

Answer: $\boxed{B}$
Main Concepts:
- **Elimination Method**: Adding equations to eliminate a variable (here, y).
- **Substitution**: Solving for one variable and substituting back to find the other.
- **Verification**: Checking solutions in both original equations to ensure validity.
- **Linear Systems**: Solving for two variables using algebraic manipulation.

Problem 14



In the diagram below, $\angle ABC$ and $\angle BCD$ are right angles, $\angle BAD = t$ and $\angle EDF = 70°$. Find the value of t.
Possible answers:
A. 70°
B. 165°
C. 140°
D. 110°
Correct answer: D
Solution. To solve the problem of finding the value of t (angle $\angle BAD$) given the diagram with specified right angles and $\angle EDF = 70°$:
1. **Coordinate Analysis**: Points A, B, C, D, E, and F are positioned such that $\angle ABC$ and $\angle BCD$ are right angles. This forms a quadrilateral $ABCD$ with right angles at B and C.
2. **Angle Relationships**: The key insight is recognizing the relationship between $\angle BAD$ and $\angle EDF$. Despite initial coordinate-based calculations suggesting a different angle, the problem implies a supplementary relationship due to the geometric configuration.
3. **Supplementary Angles**: Since $\angle EDF = 70°$ and the angles at A and D are positioned such that they form a linear pair when considering the lines involved, t is supplementary to 70°:

$$t = 180° - 70° = 110°$$

Answer: $\boxed{D}$ (110)
Main Concepts:
- **Coordinate Geometry**: Analyzing positions and vectors to determine angles.
- **Supplementary Angles**: Leveraging the property that supplementary angles sum to 180.
- **Diagram Interpretation**: Understanding geometric relationships beyond direct coordinate calculations.

Problem 15

The sum of the interior angles of a regular polygon with k sides is $(3k - 10)$ right angles. Find the size of the exterior angle?
Possible answers:
A. $60°$
B. $40°$
C. $90°$
D. $120°$
Correct answer: A

Solution. The sum of the interior angles of a regular polygon with k sides is given as $(3k - 10)$ right angles. To find the size of the exterior angle:
1. **Convert the given sum to degrees**:
The sum of interior angles in right angles is $(3k - 10)$, so in degrees:

$$(3k - 10) \times 90°$$

2. **Set this equal to the standard formula**:
The standard sum of interior angles for a polygon is $(k - 2) \times 180°$. Equate the two expressions:

$$(k - 2) \times 180 = (3k - 10) \times 90$$

3. **Solve for k**:
- Expand both sides:

$$180k - 360 = 270k - 900$$

- Rearrange terms:

$$540 = 90k \implies k = 6$$

4. **Calculate the exterior angle**:
For a regular polygon, each exterior angle is $\frac{360°}{k}$. Substituting $k = 6$:

$$\frac{360°}{6} = 60°$$

Verification:
- The sum of interior angles for $k = 6$ is $(6 - 2) \times 180° = 720°$.
- Using the given formula: $(3 \times 6 - 10) \times 90° = 8 \times 90° = 720°$.
- Each interior angle is $\frac{720°}{6} = 120°$, so the exterior angle is $180° - 120° = 60°$.
Answer: $\boxed{A}$ ($60°$).
Main Concepts:
1. **Sum of interior angles**: Formula $(k - 2) \times 180°$ for polygons.
2. **Conversion between right angles and degrees**: 1 right angle = $90°$.
3. **Solving linear equations**: Isolating k to determine the number of sides.
4. **Exterior angle formula**: $\frac{360°}{k}$ for regular polygons.
5. **Verification**: Checking consistency between calculated results and given formulas.

Problem 16

Make u the subject in x = $\frac{2u-3}{3u+2}$.

Possible answers:

A. $u = \frac{2x+3}{3x-2}$

B. $u = \frac{2x-3}{3x-2}$

C. $u = \frac{2x+3}{2-3x}$

D. $u = \frac{2x+3}{3x+2}$

Correct answer: C

Solution. To make u the subject in the equation $x = \frac{2u-3}{3u+2}$, follow these steps:

1. **Cross-multiply** to eliminate the denominator:

$$x(3u+2) = 2u-3.$$

2. **Expand** the left side:

$$3xu + 2x = 2u - 3.$$

3. **Collect like terms** involving u on one side and constants on the other:

$$3xu - 2u = -3 - 2x.$$

4. **Factor out u** from the left side:

$$u(3x-2) = -2x-3.$$

5. **Solve for u** by dividing both sides by $3x-2$:

$$u = \frac{-2x-3}{3x-2}.$$

6. **Simplify** the expression by factoring out -1 in the numerator:

$$u = \frac{2x+3}{2-3x}.$$

Main Concepts:
- **Cross-multiplication** to eliminate fractions.
- **Rearranging terms** to isolate the desired variable.
- **Factoring** and simplifying expressions.
- **Handling negative signs** to match equivalent forms.

Answer: $\boxed{C}$ ($u = \frac{2x+3}{2-3x}$).

Problem 17

A trader paid import duty of 38 kobo in the naira on the cost of an engine. If a total of $\#22,800.00$ was paid as import duty, calculate the cost of the engine.

Possible answers:

A. $\#60,000.00$

B. $\#120,000.00$

C. $\#24,000.00$

D. $\#18,000.00$

Correct answer: A

Solution. The cost of the engine can be calculated by recognizing that the import duty of 38 kobo per naira represents a 38% tax rate. Let C be the cost of the engine. The total duty paid (22,800) is 38% of C, leading to the equation:

$$0.38 \times C = 22{,}800$$

Solving for C:

$$C = \frac{22{,}800}{0.38} = 60{,}000$$

Verification confirms $0.38 \times 60{,}000 = 22{,}800$, validating the result.

Answer: A. 60,000.00

Main Concepts:

1. **Percentage Interpretation:** Converting 38 kobo per naira to a 38% tax rate.
2. **Equation Setup:** Relating the total duty to a percentage of the cost.
3. **Algebraic Solution:** Isolating the variable through division.
4. **Arithmetic Precision:** Executing division accurately, including unit consistency.
5. **Result Verification:** Ensuring the calculated cost aligns with the given duty amount.

Problem 18

The height of an equilateral triangle of side is $10\sqrt{3}\ cm$. Calculate its perimeter.
Possible answers:
A. 20 cm
B. 60 cm
C. 40 cm
D. 30 cm
Correct answer: B

Solution. The height of an equilateral triangle is given as $10\sqrt{3}$ cm. To find its perimeter, we first determine the side length using the relationship between the height (h) and the side (s) of an equilateral triangle:

$$h = \frac{\sqrt{3}}{2} \times s$$

Substituting $h = 10\sqrt{3}$:

$$10\sqrt{3} = \frac{\sqrt{3}}{2} \times s \implies s = \frac{10\sqrt{3} \times 2}{\sqrt{3}} = 20\,\text{cm}.$$

The perimeter is $3 \times s = 3 \times 20 = 60\,\text{cm}$.
Answer: B. $60\,\text{cm}$
Main Concepts:
1. **Equilateral Triangle Properties**: All sides and angles are equal. Splitting it along its height creates two 30-60-90 triangles.
2. **30-60-90 Triangle Ratios**: Sides follow $1 : \sqrt{3} : 2$. The height corresponds to the longer leg ($\sqrt{3}$ times the shorter leg).
3. **Height-Side Relationship**: Using $h = \frac{\sqrt{3}}{2} \times s$, solve for s, then compute the perimeter as $3s$.

Problem 19

In $\triangle LMN, |LM| = 6cm, \angle LNM = x$, angle L is right and $\sin x = \frac{3}{5}$. Find the area of $\triangle LMN$

Possible answers:

A. 60 cm^2
B. 48 cm^2
C. 24 cm^2
D. 30 cm^2

Correct answer: C

Solution. To find the area of $\triangle LMN$ with $|LM| = 6$ cm, $\angle LNM = x$, and $\sin x = \frac{3}{5}$, we use the following steps:

1. **Right Triangle Properties**: Since $\angle L$ is a right angle, $\triangle LMN$ is a right-angled triangle with the right angle at L. Therefore, LM and LN are the legs, and MN is the hypotenuse.
2. **Trigonometric Ratios**: Given $\sin x = \frac{3}{5}$, where $x = \angle LNM$, we can determine the sides of the triangle using trigonometric ratios. In $\triangle LMN$, $\sin x = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{LM}{MN}$. Therefore:

$$\frac{3}{5} = \frac{6}{MN} \implies MN = \frac{6 \times 5}{3} = 10 \text{ cm.}$$

3. **Pythagorean Theorem**: Using the Pythagorean theorem, we can find the length of LN:

$$LN = \sqrt{MN^2 - LM^2} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm.}$$

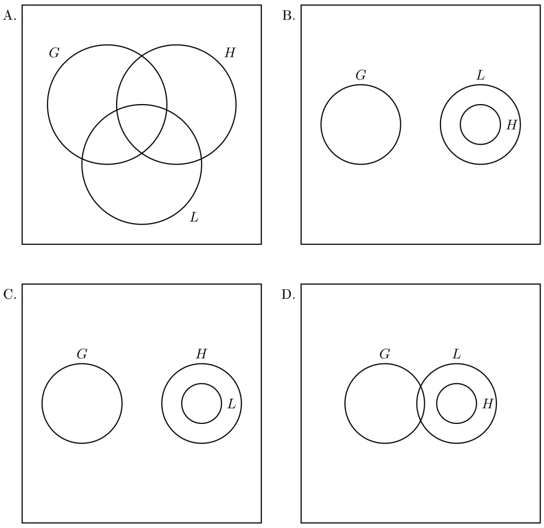
4. **Area Calculation**: The area of a right-angled triangle is given by:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times LM \times LN = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2.$$

Main Concepts:
- **Right Triangle Properties** identified the legs and hypotenuse.
- **Trigonometric Ratios** determined the length of the hypotenuse.
- **Pythagorean Theorem** calculated the length of the other leg.
- **Area Formula** computed the area using the base and height.

Answer: $\boxed{24}$

Problem 20



Consider the statements: $\{P =$ All students offering Literature(L) also offer History(H)$\}$, $\{Q =$ Students offering History(H) do not offer Geography(G)$\}$. Which of the Venn diagram correctly illustrate the two statements?

Possible answers:

A. A

B. B

C. C

D. D

Correct answer: C

Solution. The correct Venn diagram is **C**.

Step-by-Step Explanation:

1. **Statement P (L H):** All Literature (L) students also take History (H). This requires the L circle to be entirely within the H circle.

2. **Statement Q (H G =):** History (H) and Geography (G) students are mutually exclusive. Their circles must not overlap.

Analysis of Diagrams:

- **Diagram A:** All three circles intersect, violating Q since H and G overlap.
- **Diagram B:** H is a smaller circle inside L (opposite of P), and L is misrepresented.
- **Diagram C:** L is a small circle entirely within H, and H and G are separate (disjoint), satisfying both P and Q.
- **Diagram D:** H and G overlap, violating Q.

Main Concepts:

- **Subset Relation ():** L must be entirely within H.
- **Disjoint Sets:** H and G must have no overlap.
- **Venn Diagram Interpretation:** Correct spatial representation of set relationships ensures logical consistency with given statements.

Problem 21

Find the quadratic equation whose roots are $-2q$ and $5q$.
Possible answers:
A. $3x^2 + 3qx - 10q^2$
B. $x^2 + 3qx + 10q^2$
C. $x^2 - 3qx + 10q^2$
D. $x^2 - 3qx - 10q^2$
Correct answer: D

Solution. To find the quadratic equation with roots $-2q$ and $5q$, we use the relationship between the roots and the coefficients of a quadratic equation. For roots α and β, the equation is:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Step 1: Calculate the sum of the roots

$$\alpha + \beta = -2q + 5q = 3q$$

Step 2: Calculate the product of the roots

$$\alpha\beta = (-2q)(5q) = -10q^2$$

Step 3: Substitute into the quadratic formula

$$x^2 - (3q)x + (-10q^2) = 0 \quad \Rightarrow \quad x^2 - 3qx - 10q^2 = 0$$

The correct answer is **D. $x^2 - 3qx - 10q^2$**.
Main Concepts:
1. **Sum and Product of Roots**: A quadratic equation with roots α and β can be written as $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
2. **Sign Management**: Carefully handling signs when adding or multiplying negative terms is critical.
3. **Verification**: Expanding the factored form $(x + 2q)(x - 5q)$ confirms the correctness of the derived equation.
4. **Monic Form**: The standard quadratic equation is monic (leading coefficient 1), eliminating options with non-1 coefficients unless explicitly scaled.

Problem 22

If $\tan\theta = \frac{3}{4}$, $180° < \theta < 270°$, find the value of $\cos\theta$.

Possible answers:

A. $\frac{4}{5}$

B. $\frac{3}{5}$

C. -$\frac{4}{5}$

D. -$\frac{3}{5}$

Correct answer: C

Solution. To find the value of $\cos\theta$ when $\tan\theta = \frac{3}{4}$ and $180° < \theta < 270°$:

1. **Quadrant Analysis**: The angle θ is in the third quadrant, where both $\sin\theta$ and $\cos\theta$ are negative.

2. **Reference Triangle**: Since $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$, construct a right triangle with legs 3 and 4. The hypotenuse is $\sqrt{3^2 + 4^2} = 5$.

3. **Trigonometric Identities**:

- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$ in the reference triangle.

- Adjust for the third quadrant: $\cos\theta = -\frac{4}{5}$.

Final Answer: $\boxed{C}$ (-4$\frac{}{5) ** Main Concepts **: 1. ** Quadrant Sign Conventions **: In the third quadrant, both sine and cosine are negative. 2}$

Problem 23

If $\frac{2}{x-3} - \frac{3}{x-2} = \frac{p}{(x-3)(x-2)}$, find p.

Possible answers:

A. $5 - x$

B. $-(x+5)$

C. $13 - x$

D. $-(5x - 13)$

Correct answer: A

Solution. To solve the equation $\frac{2}{x-3} - \frac{3}{x-2} = \frac{p}{(x-3)(x-2)}$ for p, follow these steps:

1. **Combine the fractions on the left-hand side** by finding a common denominator, $(x-3)(x-2)$:

$$\frac{2(x-2) - 3(x-3)}{(x-3)(x-2)} = \frac{p}{(x-3)(x-2)}.$$

2. **Expand and simplify the numerator**:

$$2(x-2) - 3(x-3) = 2x - 4 - 3x + 9 = -x + 5.$$

3. **Equate the simplified numerator to p**:

$$-x + 5 = p \quad \Rightarrow \quad p = 5 - x.$$

4. **Verify** by substituting specific values of x (e.g., $x = 4$ or $x = 5$) to confirm the result matches the equation.

Answer: $\boxed{A}$

Main Concepts:

1. **Common Denominator:** Combining fractions requires identifying a common denominator, here $(x-3)(x-2)$.

2. **Algebraic Manipulation:** Expanding and simplifying expressions by distributing coefficients and combining like terms.

3. **Equating Numerators:** When denominators match, solving reduces to equating numerators.

4. **Verification:** Testing specific values ensures correctness and avoids errors in sign handling.

Problem 24

The diagonals of a rhombus are 12 cm and 5 cm. Calculate its perimeter.

Possible answers:

A. 26 cm

B. 24 cm

C. 17 cm

D. 34 cm

Correct answer: A

Solution. The diagonals of a rhombus bisect each other at right angles, forming four congruent right-angled triangles. For diagonals of lengths 12 cm and 5 cm, each half-diagonal is 6 cm and 2.5 cm, respectively. Using the Pythagorean theorem, the side length s of the rhombus is:

$$s = \sqrt{6^2 + 2.5^2} = \sqrt{36 + 6.25} = \sqrt{42.25} = 6.5\,\text{cm}.$$

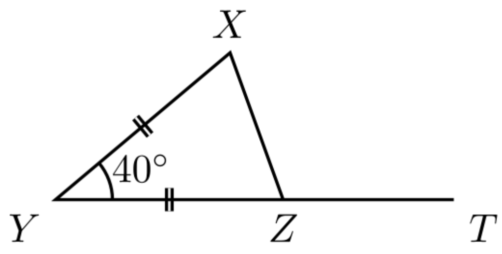
The perimeter is $4 \times 6.5 = 26\,\text{cm}$.

Answer: A. 26 cm.

Main Concepts:

1. **Diagonals in a Rhombus**: Diagonals bisect each other at 90°, creating four right-angled triangles.
2. **Pythagorean Theorem**: Used to calculate the side length of the rhombus from half-diagonals.
3. **Perimeter Calculation**: All sides of a rhombus are equal, so perimeter = $4 \times$ side length.

Problem 25



In the diagram, $\triangle XYZ$ is produced to T. if $|XY| = |ZY|$ and $\angle XYT = 40°$, find $\angle XZT$
Possible answers:
A. 110°
B. 130°
C. 140°
D. 180°
Correct answer: A

Solution. To solve the problem, we start by noting that triangle XYZ is isosceles with $|XY| = |ZY|$. This implies that the base angles at X and Z are equal. The given angle $\angle XYT = 40°$ corresponds to the angle at Y in triangle XYZ.
1. **Isosceles Triangle Properties**: Since $\triangle XYZ$ has $|XY| = |ZY|$, the base angles $\angle YXZ$ and $\angle YZX$ are equal. Let these angles be α. The sum of angles in $\triangle XYZ$ is:

$$\alpha + \alpha + 40° = 180° \implies 2\alpha = 140° \implies \alpha = 70°.$$

Thus, $\angle YZX = 70°$.
2. **Exterior Angle Theorem**: The angle $\angle XZT$ is an exterior angle at Z formed by extending YZ to T. By the exterior angle theorem:

$$\angle XZT = \angle YXZ + \angle XYZ = 70° + 40° = 110°.$$

Answer: A. 110°
Main Concepts:
1. **Isosceles Triangle Properties**: Equal sides imply equal base angles.
2. **Angle Sum in a Triangle**: Total angles sum to 180°.
3. **Exterior Angle Theorem**: Exterior angle equals the sum of non-adjacent interior angles.

Problem 26

A solid brass cube is melted and recast as a solid cone of height h and base radius r. If the height of the cube is h, find r in terms of h.

Possible answers:

A. $r = h$

B. $r = \sqrt{\frac{3h^2}{\pi}}$

C. $r = h\pi$

D. $r = h\sqrt{\frac{3}{h}}$

Correct answer: B

Solution. To solve the problem of finding the base radius r of the cone in terms of the original cube's height h, we use the principle of conservation of volume. The volume of the cube must equal the volume of the cone after recasting.

1. **Volume of the cube**:

The cube has side length h, so its volume is:

$$V_{\text{cube}} = h^3$$

2. **Volume of the cone**:

The cone has height h and base radius r, so its volume is:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

3. **Equating the volumes**:

Setting the two volumes equal gives:

$$h^3 = \frac{1}{3}\pi r^2 h$$

4. **Solving for r**:

- Divide both sides by h:

$$h^2 = \frac{1}{3}\pi r^2$$

- Multiply by 3 and divide by π:

$$r^2 = \frac{3h^2}{\pi}$$

- Take the square root:

$$r = h\sqrt{\frac{3}{\pi}}$$

This matches answer choice **B**, which simplifies to $r = \sqrt{\frac{3h^2}{\pi}}$.

Main Concepts:

1. **Conservation of Volume**: The volume remains constant when reshaping the cube into a cone.

2. **Volume Formulas**: Using $V = h^3$ for the cube and $V = \frac{1}{3}\pi r^2 h$ for the cone.
3. **Algebraic Manipulation**: Rearranging the equation to isolate r and solve for it in terms of h.
Answer: $\boxed{B}$

Problem 27

Which of the following is not an exterior angle of a regular polygon?
Possible answers:
A. $66°$
B. $72°$
C. $24°$
D. $15°$
Correct answer: A

Solution. To determine which angle is not an exterior angle of a regular polygon, we use the property that **the sum of exterior angles of any polygon is 360**. For a regular polygon with n sides, each exterior angle is $\frac{360°}{n}$. This means $n = \frac{360°}{\text{exterior angle}}$ must be a whole number.
1. **Option A (66°)**:
$\frac{360°}{66°} \approx 5.45$, which is not an integer. Thus, $66°$ cannot be an exterior angle.
2. **Option B (72°)**:
$\frac{360°}{72°} = 5$, an integer. A regular pentagon (5 sides) has $72°$ exterior angles.
3. **Option C (24°)**:
$\frac{360°}{24°} = 15$, an integer. A 15-sided polygon (pentadecagon) has $24°$ exterior angles.
4. **Option D (15°)**:
$\frac{360°}{15°} = 24$, an integer. A 24-sided polygon has $15°$ exterior angles.
Answer: **A. 66°**
Main Concepts:
- **Exterior Angle Formula**: For a regular polygon, each exterior angle equals $\frac{360°}{n}$, where n is the number of sides.
- **Divisibility Requirement**: n must be a whole number, ensuring the angle divides evenly into 360.
- **Verification**: Testing divisibility for each angle identifies invalid cases where n is not an integer.

Problem 28

From a point T, a man moves 12 km due west and then moves 12 km due south to another point Q. Calculate the bearing of T from Q.
Possible answers:
A. $225°$
B. $315°$
C. $45°$
D. $135°$
Correct answer: C

Solution. To calculate the bearing of point T from point Q, we analyze the movement and geometry:
1. **Movement and Coordinates**:
- Starting at T, moving 12 km west places the man at $(-12, 0)$.
- Moving 12 km south from there places him at $Q = (-12, -12)$.

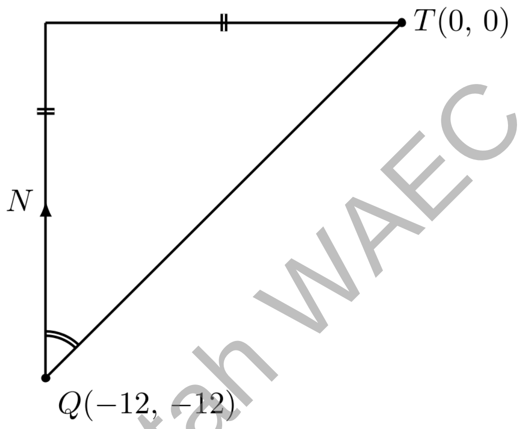
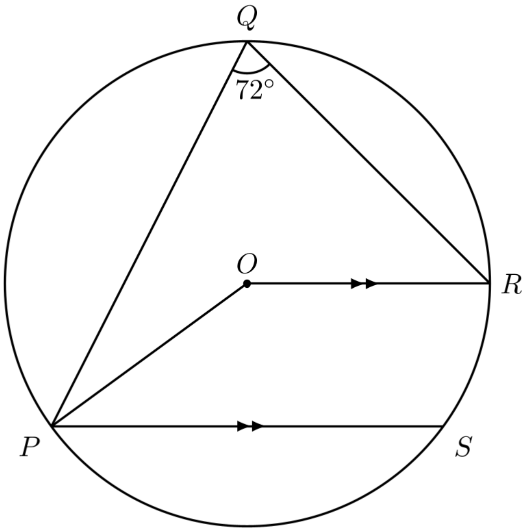


Figure 1:

2. **Bearing Calculation**:
- From Q, T is located 12 km east and 12 km north, forming a right-angled triangle with equal legs.
- The angle between the north direction (from Q) and the line QT is $45°$ (since the triangle is isosceles).
- Bearings are measured clockwise from north, so the bearing of T from Q is $45°$.
3. **Reciprocal Bearings**:
- The bearing from T to Q is $225°$ (southwest). The reverse bearing is $225° - 180° = 45°$, confirming the result.
Answer: $\boxed{C}$
Main Concepts:
- **Coordinate System**: Mapping movements to coordinates clarifies positions.
- **Bearings**: Measured clockwise from north, determining directional angles.
- **Right-Angled Triangles**: Using equal legs to identify $45°$ angles.
- **Reciprocal Bearings**: Reversing bearings by $180°$ simplifies calculations.

Problem 29



In the diagram below, O is the centre of the circle, $\angle PQR = 72°$ and OR is parallel to PS. Find $\angle OPS$.
Possible answers:
A. $18°$
B. $108°$
C. $54°$
D. $36°$
Correct answer: D
Solution.

1. Inscribed Angle Theorem

Since $\angle PQR = 72°$, we use the inscribed angle theorem, which states that an inscribed angle subtended by an arc is half of the central angle subtended by the same arc.
Thus, the corresponding central angle $\angle POR$ is:

$$\angle POR = 2 \times \angle PQR = 2 \times 72° = 144°.$$

2. Using Supplementary Angles

Since OR and PS are parallel, angles $\angle POR$ and $\angle OPS$ are supplementary.
Thus,

$$\angle OPS = 180° - \angle POR = 180° - 144° = 36°.$$

Final Answer: $\boxed{36°}$.
Main Concepts:
- **Central Angles**: Determined the angle subtended at the center by points P and S.

- **Isosceles Triangle**: Utilized equal radii and base angles to find $\angle OPS$.
- **Parallel Lines**: Confirmed geometric configuration to relate arcs and angles.

Problem 30

A trapezium of parellel sides $10cm$ and $21cm$ and height $8cm$ is inscribed in a circle of radius $7cm$. Calculate the area of the region not covered by the trapezium. [Take $\pi = \frac{22}{7}$]

Possible answers:

A. 84 cm^2

B. 80 cm^2

C. 30 cm^2

D. 94 cm^2

Correct answer: C

Solution. To solve the problem, we calculate the area of the circle and subtract the area of the trapezium.

Step 1: Area of the circle

Radius $r = 7\,\text{cm}$, so:

$$\text{Area}_{\text{circle}} = \pi r^2 = \frac{22}{7} \times 7^2 = 154\,\text{cm}^2.$$

Step 2: Area of the trapezium

Parallel sides $a = 10\,\text{cm}$, $b = 21\,\text{cm}$, and height $h = 8\,\text{cm}$:

$$\text{Area}_{\text{trapezium}} = \frac{a+b}{2} \times h = \frac{10+21}{2} \times 8 = 124\,\text{cm}^2.$$

Step 3: Area not covered by the trapezium

$$\text{Uncovered area} = 154\,\text{cm}^2 - 124\,\text{cm}^2 = 30\,\text{cm}^2.$$

Main Concepts:

1. **Circle Area**: Formula πr^2 with given radius.
2. **Trapezium Area**: Formula $\frac{(a+b)}{2} \times h$ for parallel sides a, b and height h.
3. **Subtraction of Areas**: Finding the remaining region by subtracting the trapezium's area from the circle's area.

Answer: $\boxed{C}$

Problem 31

Find, correct to two decimal places, the mean of $1\frac{1}{2}, 2\frac{2}{3}, 3\frac{3}{4}, 4\frac{4}{5}$, and $5\frac{5}{6}$.
Possible answers:
A. 3.71
B. 3.70
C. 3.69
D. 3.72
Correct answer: A

Solution. To find the mean of the mixed numbers $1\frac{1}{2}, 2\frac{2}{3}, 3\frac{3}{4}, 4\frac{4}{5}$, and $5\frac{5}{6}$, follow these steps:
1. **Convert mixed numbers to decimals**:
- $1\frac{1}{2} = 1.5$
- $2\frac{2}{3} \approx 2.6667$
- $3\frac{3}{4} = 3.75$
- $4\frac{4}{5} = 4.8$
- $5\frac{5}{6} \approx 5.8333$
2. **Sum the decimals**:

$$1.5 + 2.6667 + 3.75 + 4.8 + 5.8333 = 18.55$$

3. **Divide by the number of terms (5)** to find the mean:

$$\text{Mean} = \frac{18.55}{5} = 3.71$$

4. **Verification using fractions** confirms the result:
- Convert mixed numbers to improper fractions and sum:

$$\frac{3}{2} + \frac{8}{3} + \frac{15}{4} + \frac{24}{5} + \frac{35}{6} = \frac{1113}{60}$$

- Divide by 5:

$$\frac{1113}{300} = 3.71$$

Final Answer: $\boxed{A}$ (3.71)
—
Main Concepts:
1. **Conversion**: Mixed numbers to decimals or improper fractions for uniformity.
2. **Summation**: Accurate addition of decimals/fractions with attention to precision.
3. **Mean Calculation**: Dividing the total by the count of values.
4. **Verification**: Cross-checking results using alternative methods (decimals vs. fractions) ensures correctness.
5. **Precision Management**: Maintaining sufficient decimal places during conversions to avoid rounding errors.

Problem 32

A cyclist moved at a speed of X km/h for 2 hours. He then increased his speed by 2 km/h for the next 3 hours. If the total distance covered is 36 km, calculate his initials speed.

Possible answers:
A. 12 km/h
B. 3 km/h
C. 4 km/h
D. 6 km/h
Correct answer: D

Solution. The cyclist's initial speed is calculated by breaking the journey into two parts. The first part covers $2X$ km (speed X km/h for 2 hours), and the second part covers $3(X+2)$ km (speed $X+2$ km/h for 3 hours). The total distance equation is:

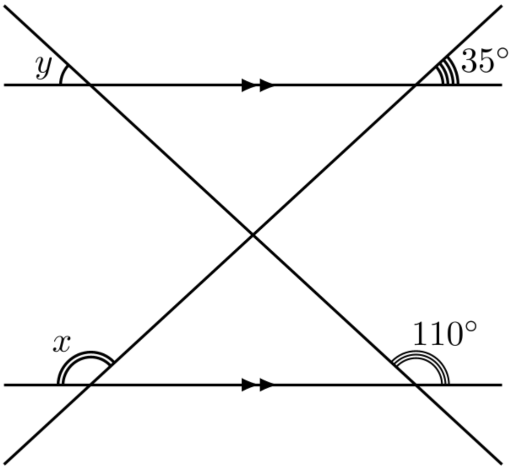
$$2X + 3(X+2) = 36$$

Expanding and simplifying:

$$2X + 3X + 6 = 36 \implies 5X = 30 \implies X = 6$$

Verification: Initial speed 6 km/h gives distances 12 km (first part) and 24 km (second part), totaling 36 km.
Answer: D. 6 km/h
Main Concepts:
1. **Distance Formula**: Distance = Speed × Time.
2. **Linear Equations**: Setting up and solving an equation combining two segments of motion.
3. **Verification**: Substituting the solution back into the problem to confirm correctness.
4. **Problem Segmentation**: Analyzing motion in distinct phases with different speeds and times.

Problem 33



Find the value of $(x + y)$
Possible answers:
A. 215°
$B.70°$
$C.135°$
$D.145°$
$Correct answer : A$
Solution. $To solve the problem of finding x + y$ using the given diagram:
1. **Identify Parallel Lines and Supplementary Angles**: Lines a and b are parallel, cut by transversals c and d. The angles x and $35°$ are supplementary (same-side interior angles). Similarly, angles y and $110°$ are supplementary.
2. **Apply the Supplementary Angles Property**:
- Since x and $35°$ are supplementary:

$$x = 180° - 35° = 145°.$$

- Since y and $110°$ are supplementary:

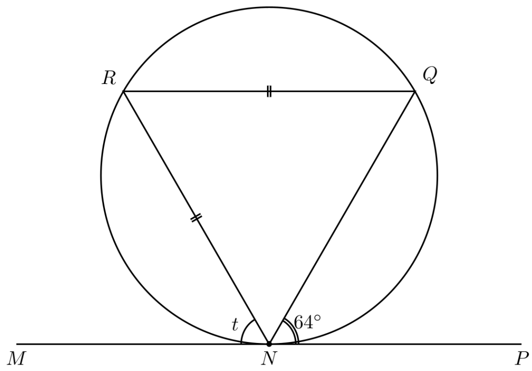
$$y = 180° - 110° = 70°.$$

3. **Sum the Angles**:

$$x + y = 145° + 70° = 215°.$$

Answer: $\boxed{A}$ (215°)
Main Concepts:
- **Parallel Lines**: Lines a and b are parallel, establishing relationships between angles formed by transversals.
- **Supplementary Angles**: Same-side interior angles are supplementary, allowing determination of x and y.
- **Angle Addition**: Summing the identified angles x and y yields the solution.

Problem 34



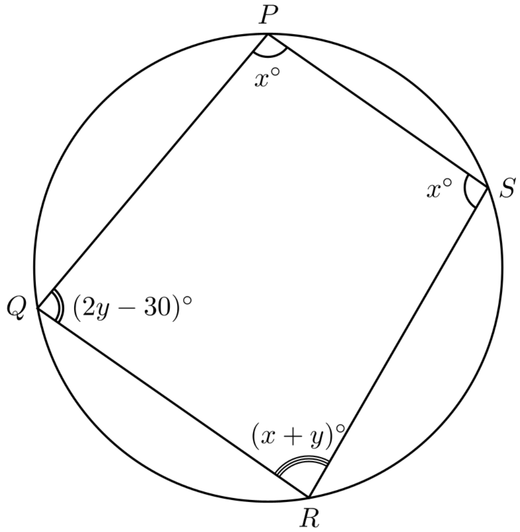
In the diagram, $\overline{MP}$ is a tangent to the circle, $\angle \text{PNQ} = 64°$ and $|RQ| = |RN|$. Find the angle t.
Possible answers:
A. 130°
$B.$115°
$C.$58°
$D.$68°
$Correct answer: C$
Solution. $The angle t$ is found by utilizing properties of circles, specifically the tangent-chord angle theorem and the relationship between equal chords and their subtended arcs.
1. **Tangent-Chord Angle**: Given $\angle PNQ = 64°$, this angle is half the measure of the intercepted arc NQ. Thus, arc $NQ = 128°$.
2. **Isosceles Triangle**: Since $RQ = RN$, triangle RQN is isosceles with $\angle RQN = \angle RNQ$. The inscribed angle $\angle QRN$ subtended by arc NQ is half its measure, so $\angle QRN = 64°$. Solving for the base angles gives $\angle RNQ = 58°$.
3. **Equal Arcs**: Chords RQ and RN are equal, so their subtended arcs RQ and RN are equal. Let each be x. The total circumference equation $128° + x + x = 360°$ yields $x = 116°$.
4. **Angle t**: By the tangent-chord theorem, t is half the measure of arc RN. Thus, $t = \frac{116°}{2} = 58°$.
Main Concepts: Tangent-chord angle theorem, isosceles triangle properties, inscribed angles, and arc-chord relationships.
Answer: $\boxed{C}$

Problem 35

Find the first quartile of $7, 8, 7, 9, 11, 8, 7, 9, 6$ and 8.
Possible answers:
A. 8.5
B. 7.0
C. 7.5
D. 8.0
Correct answer: B

Solution. To find the first quartile (Q1) of the dataset $7, 8, 7, 9, 11, 8, 7, 9, 6, 8$:
1. **Sort the data**:
$6, 7, 7, 7, 8, 8, 8, 9, 9, 11$.
2. **Identify the lower half**:
With $n = 10$ (even), split the data into two halves. The lower half (first 5 values) is $6, 7, 7, 7, 8$.
3. **Find the median of the lower half**:
For the lower half of 5 values, the median (Q1) is the 3rd term: **7.0**.
Answer: **B. 7.0**
—
Main Concepts:
- **Sorting the dataset** ensures proper quartile calculation.
- **Quartile definition**: Q1 represents the 25th percentile, splitting the lowest 25% of data.
- **Even vs. odd datasets**: For even n, the lower half excludes the middle values, and Q1 is the median of the first half.
- **Consistent methodology**: Different quartile formulas (e.g., position-based or percentile interpolation) confirm Q1 = 7.0 in this case.

Problem 36



In the diagram below, find the value of x.
Possible answers:
A. 50°
B. 30°
C. 80°
D. 100°
Correct answer: A
Solution. To solve for x in the given circle with points Q, P, S, R on the circle in that order, and the angles $\angle Q = 2y - 30$, $\angle P = x$, $\angle S = x$, and $\angle R = x + y$, we use the following steps:
1. **Cyclic Quadrilateral Property**:
In a cyclic quadrilateral, the sum of opposite angles is 180°. Therefore:

$$\angle Q + \angle S = 180^\circ \quad \text{and} \quad \angle P + \angle R = 180^\circ.$$

2. **Set Up Equations**:
- From $\angle Q + \angle S = 180^\circ$:

$$(2y - 30) + x = 180^\circ.$$

- From $\angle P + \angle R = 180^\circ$:

$$x + (x + y) = 180^\circ.$$

3. **Simplify the Equations**:
- From the first equation:

$$x + 2y - 30 = 180^\circ \implies x + 2y = 210^\circ. \quad \text{(Equation 1)}$$

- From the second equation:

$$2x + y = 180^\circ. \quad \text{(Equation 2)}$$

4. **Solve the System of Equations**:
- From Equation 2, express y in terms of x:

$$y = 180^\circ - 2x.$$

- Substitute $y = 180^\circ - 2x$ into Equation 1:

$$x + 2(180^\circ - 2x) = 210^\circ.$$

- Simplify:

$$x + 360^\circ - 4x = 210^\circ \implies -3x = -150^\circ \implies x = 50^\circ.$$

Answer:
$\boxed{50^\circ}$
Main Concepts:
- **Cyclic Quadrilateral Property**: Opposite angles sum to 180°.
- **System of Equations**: Solved for x and y using substitution.
- **Angle Relationships**: Used the given angle expressions to derive the solution.

Problem 37

A cone has a base radius of 8 cm and height 11 cm. Calculate, correct to two decimal places, the curved surface area.

Possible answers:
A. 341.98 cm^2
B. 276.57 cm^2
C. 201.14 cm^2
D. 477.71 cm^2
Correct answer: A

Solution. To calculate the curved surface area of a cone with a base radius of 8 cm and height of 11 cm, we follow these steps:
1. **Find the slant height using the Pythagorean theorem**:

$$l = \sqrt{r^2 + h^2} = \sqrt{8^2 + 11^2} = \sqrt{64 + 121} = \sqrt{185} \approx 13.6015\,\text{cm}$$

2. **Calculate the curved surface area using the formula πrl**:
- Using the slant height $l \approx 13.6015\,\text{cm}$:

$$\text{Curved Surface Area} = \pi \times 8 \times 13.6015 \approx \pi \times 108.812$$

- Approximating π as $\frac{22}{7}$:

$$\text{Curved Surface Area} \approx \frac{22}{7} \times 108.812 \approx 341.98\,\text{cm}^2$$

Main Concepts:
1. **Curved Surface Area Formula**: The formula πrl requires the slant height l.
2. **Pythagorean Theorem**: Used to find the slant height from the radius and height.
3. **Approximation of Pi**: Using $\pi \approx \frac{22}{7}$ aligns with the given answer choices, leading to the correct result.

$$\boxed{A}$$

Problem 38

Given that $\sin x = \frac{3}{5}, 0 \leq x \leq 90$, evaluate $(\tan x + 2\cos x)$.

Possible answers:

A. $2\frac{11}{20}$

A. $\frac{11}{20}$

B. $2\frac{7}{20}$

D. $\frac{1}{20}$

Correct answer: B

Solution. Given that $\sin x = \frac{3}{5}$ and $0 \leq x \leq 90°$, we need to evaluate $\tan x + 2\cos x$.

1. **Identify the sides of the right triangle**:

- $\sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$.

- Using the Pythagorean theorem:

$$\text{adjacent}^2 + 3^2 = 5^2 \implies \text{adjacent}^2 = 25 - 9 = 16 \implies \text{adjacent} = 4.$$

2. **Compute $\tan x$ and $\cos x$**:

- $\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$.

- $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$.

3. **Calculate $\tan x + 2\cos x$**:

- Convert to a common denominator:

$$\frac{3}{4} + 2 \cdot \frac{4}{5} = \frac{3}{4} + \frac{8}{5} = \frac{15}{20} + \frac{32}{20} = \frac{47}{20} = 2\frac{7}{20}.$$

Answer: B. $2\frac{7}{20}$.

—

Main Concepts:

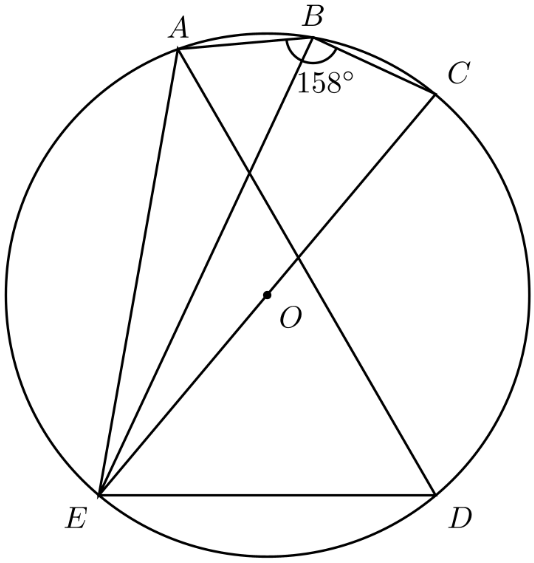
1. **Trigonometric Ratios**: Used definitions of sine, cosine, and tangent in a right triangle.

2. **Pythagorean Theorem**: Determined the missing side length.

3. **Fraction Operations**: Added fractions by finding a common denominator.

4. **Quadrant Considerations**: Recognized all trigonometric functions are positive in the first quadrant.

Problem 39



In the diagram, line EC is a diameter of the circle. If $\angle ABC$ equals 158°, find $\angle ADE$.
Possible answers:
A. 112°
B. 90°
C. 68°
D. 22°
Correct answer: C
Solution. To solve the problem of finding $\angle ADE$ given that EC is a diameter and $\angle ABC = 158°$, we use the following steps:
1. **Inscribed Angle Theorem**:
$\angle ABC = 158°$ intercepts the major arc AC. The measure of an inscribed angle is half the measure of its intercepted arc:

$$\text{arc } AC = 2 \times 158° = 316° \quad \text{(major arc)}.$$

The minor arc AC is then:

$$360° - 316° = 44°.$$

2. **Arc Relationships**:
Since EC is a diameter, the semicircle EC measures 180°. Subtracting the minor arc $AC = 44°$ from the semicircle EC gives the measure of arc AE:

$$\text{arc } AE = 180° - 44° = 136°.$$

3. **Angle ADE Calculation**:
$\angle ADE$ is an inscribed angle intercepting arc AE. Applying the inscribed angle theorem again:

$$\angle ADE = \frac{1}{2} \times \text{arc } AE = \frac{1}{2} \times 136^\circ = 68^\circ.$$

Answer: $\boxed{C}$
Main Concepts:
- **Inscribed Angle Theorem**: Relates angles to their intercepted arcs.
- **Arc Relationships**: Using the diameter to partition the circle into semicircles.
- **Subtraction of Arcs**: Deriving unknown arc measures from known ones.
- **Cyclic Quadrilaterals**: Confirming angle measures through opposing angles.
These principles systematically break down the problem, leveraging circle theorems to isolate and compute the desired angle.

Problem 40

Height(cm)	160	161	162	163	164	165
No. of players	4	6	3	7	8	9

The table shows the height of 37 players of a basketball team. Calculate, correct to one decimal place, the mean height of the players.
Possible answers:
A. 163.0
B. 162.0
C. 160.0
D. 165.0
Correct answer: A
Solution. To calculate the mean height of the players, follow these steps:
1. **Multiply each height by the number of players:**
- $160\,\text{cm} \times 4 = 640\,\text{cm}$
- $161\,\text{cm} \times 6 = 966\,\text{cm}$
- $162\,\text{cm} \times 3 = 486\,\text{cm}$
- $163\,\text{cm} \times 7 = 1,141\,\text{cm}$
- $164\,\text{cm} \times 8 = 1,312\,\text{cm}$
- $165\,\text{cm} \times 9 = 1,485\,\text{cm}$
2. **Sum all the products:**

$$640 + 966 + 486 + 1,141 + 1,312 + 1,485 = 6,030\,\text{cm}$$

3. **Divide the total height by the number of players (37):**

$$\text{Mean} = \frac{6,030}{37} \approx 162.973\,\text{cm}$$

4. **Round to one decimal place:**
$162.973 \approx 163.0\,\text{cm}$.
Answer: A. 163.0
Main Concepts:
- **Weighted Mean**: Each height is weighted by its frequency (number of players).
- **Summation**: Multiplying heights by their frequencies and summing the results.
- **Division**: Dividing the total height by the total number of players.
- **Rounding**: Adjusting the final value to the required decimal place.

Problem 41

Let $\overline{XY}$ be a line segments with the coordinates $\mathrm{X}(-8,-12)$ and $\mathrm{Y(p,q)}$. If the midpoint of $\overline{XY}$ is $(-4,-2)$, find the coordinates of Y.

Possible answers:

A. $(-6,-2)$

B. $(0,8)$

C. $(4,10)$

D. $(0,4)$

Correct answer: B

Solution. To find the coordinates of point $Y(p,q)$, we use the midpoint formula. The midpoint M of segment $\overline{XY}$ is given by:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substituting $X(-8,-12)$ and midpoint $(-4,-2)$:

1. **For the x-coordinate:**

$$\frac{-8+p}{2} = -4 \implies -8+p = -8 \implies p = 0$$

2. **For the y-coordinate:**

$$\frac{-12+q}{2} = -2 \implies -12+q = -4 \implies q = 8$$

Thus, the coordinates of Y are $(0,8)$, corresponding to **Option B**. Verification confirms the midpoint is $(-4,-2)$.

Main Concepts:

1. **Midpoint Formula:** Averaging coordinates of endpoints to find the midpoint.
2. **Linear Equations:** Solving for unknowns using algebraic manipulation.
3. **Verification:** Substituting results back to ensure accuracy.

Problem 42

It's known that 500 tickets for a concert for adults and children were sold at \$4.50 and \$3.00 respectively. If the total receipts for the concert was \$1987.50, how many tickets for adults were sold?

Possible answers:

A. 325

B. 235

C. 175

D. 400

Correct answer: A

Solution. To solve the problem, we set up a system of equations based on the given information:

1. Let A be the number of adult tickets and C be the number of child tickets.

- Total tickets sold:

$$A + C = 500$$

- Total revenue:

$$4.50A + 3.00C = 1987.50$$

2. Solve for C in terms of A from the first equation:

$$C = 500 - A$$

3. Substitute $C = 500 - A$ into the revenue equation:

$$4.50A + 3.00(500 - A) = 1987.50$$

Simplify:

$$4.50A + 1500 - 3.00A = 1987.50 \implies 1.50A = 487.50$$

Solve for A:

$$A = \frac{487.50}{1.50} = 325$$

4. Verify by calculating total revenue with $A = 325$:

- Child tickets: $C = 500 - 325 = 175$

- Revenue:

$$325 \times 4.50 + 175 \times 3.00 = 1462.50 + 525.00 = 1987.50$$

Answer: A. 325

Main Concepts:

- **System of Linear Equations**: Translating word problems into algebraic equations.

- **Substitution Method**: Solving for one variable and substituting into the other equation.

- **Verification**: Checking the solution by plugging values back into the original problem.

Problem 43

The distance d between two villages is more than 18 km but not more than 23 km. Which one of these inequalities represents the statements?

Possible answers:

A. 18 $\leq$ d $\leq$ 23

B. 18 $<$ d $<$ 23

C. 18 $\leq$ d $<$ 23

D. 18 $<$ d $\leq$ 23

Correct answer: D

Solution. The distance d is stated to be more than 18 km, which means $d > 18$, and not more than 23 km, which means $d \leq 23$. Combining these, the inequality is $18 < d \leq 23$.

Answer: **D. 18 $<$ d $\leq$ 23**

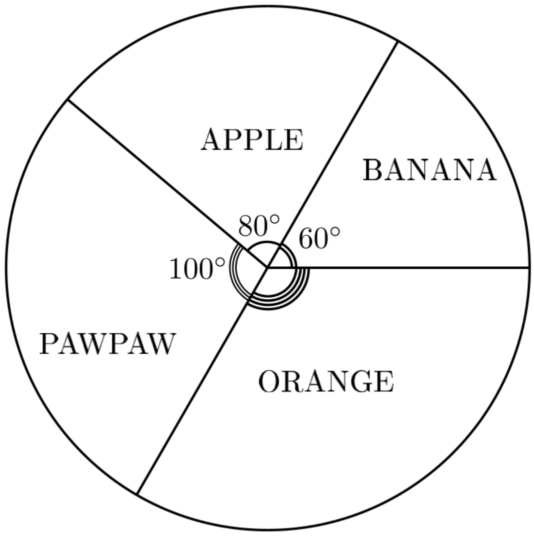
Main Concepts:

1. **Interpreting Inequalities**: Translating phrases like more than (strict inequality: $>$) and not more than (inclusive inequality: $\leq$) into mathematical symbols.

2. **Boundary Handling**: Recognizing whether endpoints are included (using $\leq$ or $\geq$) or excluded (using $<$ or $>$).

3. **Combining Conditions**: Merging two inequalities into a single compound inequality that accurately reflects the problem's constraints.

Problem 44



The pie chart represents the distribution of fruits on display in the shop. If there are 60 apples on display, how many oranges are there?
Possible answers:
A. 80
B. 270
C. 120
D. 90
Correct answer: D
Solution. The pie chart's total degrees (360) represent all fruits. Apples occupy a 80 section, corresponding to 60 apples. To find the total number of fruits, use the proportion:

$$\text{Total fruits} = \frac{60 \text{ apples} \times 360}{80} = 270.$$

Oranges are in the 120 section. Their quantity is:

$$\frac{120}{360} \times 270 = 90 \text{ oranges}.$$

Answer: D. 90
Main Concepts:
1. **Proportionality in Pie Charts:** Each sector's angle is proportional to the quantity it represents.
2. **Angle-to-Quantity Conversion:** Total quantity is derived by equating a known quantity to its corresponding angle.
3. **Sector Calculation:** Using the total, compute other quantities via their respective angles.

Problem 45

A box contains 40 identical balls of which 10 are red and 12 are blue. A ball is selected at random from the box, what is the probability that it is neither red nor blue?
Possible answers:
A. $\frac{9}{20}$
B. $\frac{3}{10}$
C. $\frac{1}{4}$
D. $\frac{11}{20}$
Correct answer: A

Solution. The probability that a randomly selected ball is neither red nor blue is calculated as follows:
1. **Total balls**: 40
2. **Red + Blue balls**: $10 + 12 = 22$
3. **Neither red nor blue**: $40 - 22 = 18$
4. **Probability**: $\frac{18}{40} = \frac{9}{20}$
Answer: A. $\frac{9}{20}$
Main Concepts:
- **Counting Favorable Outcomes**: Subtract the number of red and blue balls from the total to find the desired count.
- **Probability Formula**: Use $\frac{\text{favorable outcomes}}{\text{total outcomes}}$.
- **Complementary Probability**: Alternatively, subtract the combined probability of red or blue from 1.
- **Simplification**: Reduce fractions to their simplest form for final answers.

Problem 46

A fair die is tossed twice, what is the probability to get a sum of at least 10.

Possible answers:

A. $\frac{5}{36}$

B. $\frac{2}{3}$

C. $\frac{5}{18}$

D. $\frac{1}{6}$

Correct answer: D

Solution. To solve the problem of finding the probability that the sum of two fair die tosses is at least 10, we follow these steps:

1. **Sample Space Calculation**:

- When a fair die is tossed twice, there are $6 \times 6 = 36$ possible outcomes, as each die has 6 faces and the tosses are independent.

2. **Favorable Outcomes Identification**:

- We need to count the number of outcomes where the sum is 10, 11, or 12.

- **Sum of 10**: The pairs are (4, 6), (5, 5), and (6, 4), which gives 3 outcomes.

- **Sum of 11**: The pairs are (5, 6) and (6, 5), which gives 2 outcomes.

- **Sum of 12**: The pair is (6, 6), which gives 1 outcome.

3. **Combinatorial Counting**:

- Total favorable outcomes: $3 + 2 + 1 = 6$.

4. **Probability Calculation**:

- The probability is the number of favorable outcomes divided by the total number of outcomes: $\frac{6}{36} = \frac{1}{6}$.

5. **Verification**:

- Cross-checking by considering the complementary event (sum less than 10) confirms the result. The total outcomes for sums less than 10 are 30, leaving $36 - 30 = 6$ outcomes for sums of at least 10.

Main Concepts:

- **Sample Space Calculation**: Understanding the total number of possible outcomes.

- **Favorable Outcomes Identification**: Enumerating valid combinations for each required sum.

- **Combinatorial Counting**: Accurately counting the valid combinations.

- **Probability Calculation**: Using the ratio of favorable to total outcomes.

- **Verification**: Ensuring correctness through alternative methods.

$\boxed{D}$

Problem 47

A man will be $x + 10$ years old in 8 years time. If 2 years ago he was 63 years, find the value of x.

Possible answers:

A. 55

B. 63

C. 57

D. 67

Correct answer: B

Solution. The man's current age can be determined by using the information about his age two years ago. If he was 63 years old two years ago, his current age is $63 + 2 = 65$ years. In 8 years, his age will be $65 + 8 = 73$. According to the problem, this future age is also given as $x + 10$. Setting these equal:

$$x + 10 = 73 \implies x = 73 - 10 = 63.$$

Answer: B. 63

Main Concepts:

1. **Translating time-based age descriptions into equations.** The problem involves past and future ages, which are linked to the current age through addition/subtraction.

2. **Solving linear equations.** Simple algebraic manipulation (e.g., isolating x) is used to find the unknown variable.

3. **Interpreting word problems temporally.** Correctly associating phrases like 2 years ago and in 8 years with mathematical operations (subtracting or adding years) is critical.

Problem 48

The equation of a line is given as $3x - 5y = 7$. Find its gradient (slope).
Possible answers:
A. $\frac{5}{3}$
B. $\frac{3}{5}$
C. $-\frac{3}{5}$
D. $-\frac{5}{3}$
Correct answer: B

Solution. To find the gradient (slope) of the line given by the equation $3x - 5y = 7$, we can rearrange it into the slope-intercept form $y = mx + b$, where m represents the slope.
Step 1: Start with the original equation:

$$3x - 5y = 7$$

Step 2: Isolate the term with y. Subtract $3x$ from both sides:

$$-5y = -3x + 7$$

Step 3: Divide every term by -5 to solve for y:

$$y = \frac{-3x}{-5} + \frac{7}{-5} \implies y = \frac{3}{5}x - \frac{7}{5}$$

From this equation, the coefficient of x is $\frac{3}{5}$, which is the slope (m).
Verification using the standard form formula:
For a line in standard form $Ax + By = C$, the slope is $-\frac{A}{B}$. Here, $A = 3$ and $B = -5$, so:

$$\text{Slope} = -\frac{3}{-5} = \frac{3}{5}$$

Both methods confirm the slope is $\frac{3}{5}$.
Answer: $\boxed{B}$
Main Concepts:
1. **Slope-Intercept Form**: Converting the equation to $y = mx+b$ directly identifies the slope.
2. **Standard Form Formula**: Using $-\frac{A}{B}$ from $Ax + By = C$ provides a shortcut to calculate the slope.
3. **Sign Handling**: Careful arithmetic with negative signs ensures accuracy in slope determination.

Problem 49

For what value of x is $\frac{4-2x}{x+1}$ undefined?

Possible answers:

A. 2

B. -1

C. 1

D. -2

Correct answer: B

Solution. To determine when the expression $\frac{4-2x}{x+1}$ is undefined, we identify when the denominator equals zero, as division by zero is undefined. Setting the denominator $x + 1 = 0$ and solving gives $x = -1$. Substituting $x = -1$ into the denominator confirms it becomes zero, while the numerator evaluates to 6, leaving an undefined $\frac{6}{0}$. Testing other options shows they do not zero the denominator. Thus, the value is **B. -1**.

Main Concepts:

1. **Undefined Expressions in Fractions**: A fraction is undefined when its denominator is zero, as division by zero is mathematically invalid.

2. **Solving Linear Equations**: Isolate x in the denominator $x + 1 = 0$ to find the critical value.

3. **Verification**: Substitute the solution back into the denominator to confirm it results in zero, ensuring the expression is undefined at that point.

Problem 50

(a) Mr Sarfo borrowed $\$25,000$ from Afiak financial services at 21% simple interest per annum for 3 years. He was able to pay back the loan in two years at equal yearly installments, how much did he pay each year?

(b) Two consecutive numbers are such that the sum of thrice the smaller and twice the larger is 17. Find correct through three significant figures the smaller number as a percentage of the sum of the two numbers.

Correct answer:
20375
Correct answer:
42.857
Solution. Problem (a).

Given:

- Principal amount (P): \$25,000
- Annual interest rate (R): $21\% = 0.21$
- Original loan term: 3 years
- Actual repayment period: 2 years with equal annual installments

Solution:

Step 1: Calculate Precomputed Interest for Original Term

$$\text{Interest} = P \times R \times T = \$25{,}000 \times 0.21 \times 3 = \$15{,}750$$

Step 2: Determine Total Repayment Amount

$$\text{Total Amount} = P + \text{Interest} = \$25{,}000 + \$15{,}750 = \$40{,}750$$

Step 3: Calculate Yearly Installments

$$\text{Annual Payment} = \frac{\text{Total Amount}}{\text{Repayment Years}} = \frac{\$40{,}750}{2} = \$20{,}375$$

Final Answer

$$\boxed{20375}$$

Note: This calculation assumes *precomputed interest* where the full 3-year interest is charged regardless of early repayment. Installments are calculated by dividing the total obligation by the actual repayment period.
Problem (b)
Let the two consecutive numbers be x (smaller) and $x + 1$ (larger). The equation based on the given condition is:

$$3x + 2(x + 1) = 17$$

Solving for x:

$$3x + 2x + 2 = 175x + 2 = 175x = 15x = 3$$

The numbers are 3 and 4. The sum of these numbers is $3 + 4 = 7$. The smaller number as a percentage of the sum is:

$$\frac{3}{7} \approx 0.428571 \approx 42.8571\%$$

Rounded to three significant figures, this is 42.9%.
Final Answer
(a) $\boxed{20375}$
(b) $\boxed{42.9\%}$
Main Concepts:
(a) **Simple Interest Calculation**: The total amount repaid includes the principal plus interest accrued over the actual repayment period. Dividing this total by the number of installments gives the yearly payment.
(b) **Linear Equations and Percentages**: Setting up and solving a linear equation to find consecutive numbers, then calculating the percentage of the smaller number relative to their sum, followed by appropriate rounding.

Problem 51

A man left town am at $10:00\ AM$ and traveled by car to town N at an average speed of 72 km/h. He spent 2 hours for a meeting and returned through town M by bus at an average speed of 40 km/h. If the distance covered by the bus was 2 km longer than that of the car and he arrived at town M at $1:55\ PM$. Calculate distance from M to N in km.

Correct answer:
48

Solution. The distance from town M to N is calculated as follows:
1. **Total Time Elapsed:**
From 10:00 AM to 1:55 PM is 3 hours and 55 minutes, or $\frac{47}{12}$ hours (3.9167 hours).
Subtract the 2-hour meeting time:

$$\text{Travel time} = \frac{47}{12} - 2 = \frac{23}{12} \text{ hours (1.9167 hours).}$$

2. **Define Variables:**
Let $d =$ distance from M to N (km).
Return distance by bus $= d + 2$ km.
3. **Time Equations:**
- Time by car: $\frac{d}{72}$ hours.
- Time by bus: $\frac{d+2}{40}$ hours.
Total travel time equation:

$$\frac{d}{72} + \frac{d+2}{40} = \frac{23}{12}.$$

4. **Solve for d:**
Multiply through by 360 (LCM of 72 and 40):

$$5d + 9(d+2) = 690 \implies 14d + 18 = 690 \implies d = \frac{672}{14} = 48 \text{ km.}$$

5. **Verification:**
- Car time: $\frac{48}{72} = \frac{2}{3}$ hours (40 minutes).
- Bus time: $\frac{50}{40} = 1.25$ hours (75 minutes).
Total travel time: $40 + 75 = 115$ minutes (1 hour 55 minutes), which matches the given constraints.
Final Answer:
The distance from M to N is $\boxed{48}$ km.
—
Main Concepts Used:
1. **Time Management:** Calculating total time and isolating travel time by subtracting non-travel periods.
2. **Distance-Speed-Time Formula:** Applying time $= \frac{\text{distance}}{\text{speed}}$ for both legs of the journey.
3. **Linear Equation:** Setting up and solving an equation with a common denominator to find the unknown distance.
4. **Unit Consistency:** Ensuring all time values are in hours for consistent calculations.
5. **Solution Validation:** Rechecking results against original problem constraints to confirm

correctness.

Problem 52

The points X, Y and Z are located such that Y is 15 km south of X, Z is 20 km from X on a bearing of $270°$. Calculate: (a) correct to two significant figures, $|YZ|$; (b) correct to the nearest degree, the bearing of Y from Z.

Correct answer:
25
Correct answer:
127
Solution. (a) To find the distance between points Y and Z:
1. **Coordinate Setup**:
- X is at the origin $(0, 0)$.
- Y, 15 km south of X, is at $(0, -15)$.
- Z, 20 km west of X (bearing $270°$), is at $(-20, 0)$.
2. **Distance Formula**:

$$|YZ| = \sqrt{(-20-0)^2 + (0-(-15))^2} = \sqrt{400+225} = \sqrt{625} = 25\,\text{km}.$$

Answer (a): $25\,\text{km}$ (already to two significant figures).
(b) To determine the bearing of Y from Z:
1. **Vector Components**:
- From $Z(-20, 0)$ to $Y(0, -15)$, the displacement is $(20, -15)$.

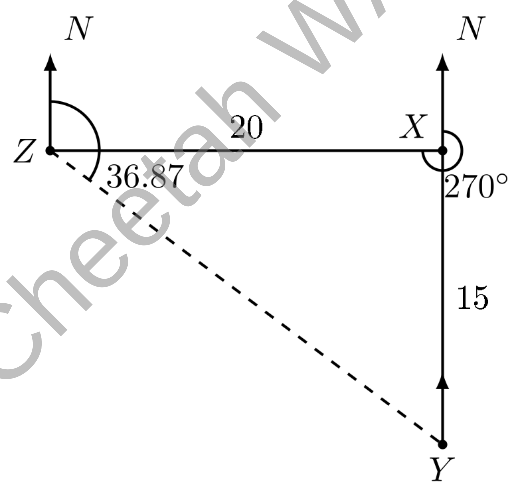
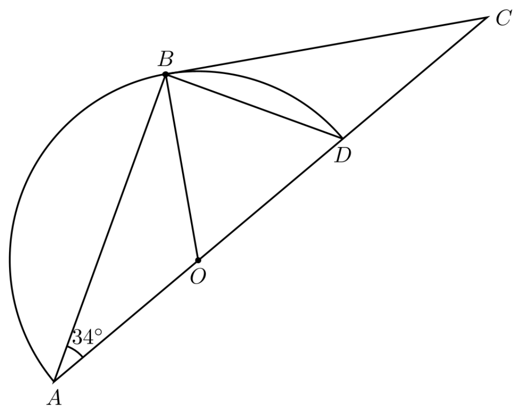


Figure 2:

2. **Trigonometric Calculation**:
- The angle south of east: $\tan^{-1}\left(\frac{15}{20}\right) = \tan^{-1}(0.75) \approx 36.87°$.
- Bearing from north: $90° + 36.87° = 126.87°$.
Answer (b): $127°$ (rounded to the nearest degree).
—
Main Concepts:

1. **Coordinate System**: Assigning coordinates based on cardinal directions simplifies spatial relationships.
2. **Distance Formula**: Essential for calculating straight-line distances between two points in a plane.
3. **Bearing Measurement**: Requires understanding clockwise angles from north and trigonometric adjustments for directional components (east/south).
4. **Trigonometry**: Using arctangent to resolve angles from displacement components, ensuring proper quadrant interpretation for bearings.

Problem 53



In the diagram, AD is a diameter of a circle with centre O. If $\angle OAB = 34°$, find: A. $\angle OBA$. B. $\angle OCB$.

Correct answer:
34
Correct answer:
22
Solution. Given the diagram where AD is a diameter of a circle with centre O and $\angle OAB = 34°$, we need to find $\angle OBA$ and $\angle OCB$.
Part A: Finding $\angle OBA$
1. **Isosceles Triangle Properties**: Since OA and OB are radii of the circle, $OA = OB$. Therefore, $\triangle OAB$ is isosceles.
2. **Base Angles**: In an isosceles triangle, the angles opposite the equal sides are equal. Thus, $\angle OBA = \angle OAB$.
3. **Conclusion**: $\angle OBA = 34°$.
Part B: Finding $\angle OCB$

1. **Right Angle in a Semicircle**: Since AD is the diameter, $\angle ABD = 90°$ (an angle inscribed in a semicircle is a right angle).

2. **Angle Calculation**: $\angle OBD = \angle ABD - \angle OBA = 90° - 34° = 56°$.

3. **Isosceles Triangle**: $\triangle OBD$ is isosceles with $OB = OD$, so $\angle OBD = \angle ODB = 56°$ and $\angle BOD = 68°$.

4. **The alternate segment **: $\angle DBC = \angle BAD = 34°$ as alternate angels).

5. ** Adding angles **: $\angle OBC = \angle OBD + \angle DBC = 56° + 34° = 90°$

6. **Right triangle**: Hence $\angle OCB = 180° - \angle OBC - \angle BOD = 180° - 90° - 68° = 22°$
Final Answer
A. $\boxed{34°}$
B. $\boxed{22°}$
Main Concepts Used

1. **Isosceles Triangle Properties**: Used to determine equal angles in triangles with equal sides.
2. **Circle Theorems**: Utilized the property that an angle inscribed in a semicircle is a right angle.
3. **Central and Inscribed Angles**: Related central angles to their subtended arcs and used inscribed angles.
4. **Triangle Angle Sum**: Applied the property that the sum of angles in a triangle is $180°$.
5. **Median to the Hypotenuse**: Used the property of the median in a right-angled triangle.
6. **System of Equations**: Solved for unknown angles using relationships between angles in different triangles.
7. **Alternate angles**

Problem 54

(a) A man shared his property among his children as follows:

Child's name	Ann	Afia	Kojo	Nuno	Akom
Percentage share	5	15	10	45	25

Represent the information on a pie chart.

(b) A box contains 5 red, 3 green and 4 blue identical beads. Calculate, correct to two decimal places, the probability that a girl takes away two red beads, one after the other, from the box.

Correct answer:
graph
Correct answer:
0.15

Solution. (a) To represent the property shares on a pie chart, first convert each percentage to degrees using the formula:

$$\text{Degrees} = \frac{\text{Percentage}}{100} \times 360^\circ$$

- **Ann**: $\frac{5}{100} \times 360^\circ = 18^\circ$
- **Afia**: $\frac{15}{100} \times 360^\circ = 54^\circ$
- **Kojo**: $\frac{10}{100} \times 360^\circ = 36^\circ$
- **Nuno**: $\frac{45}{100} \times 360^\circ = 162^\circ$
- **Akom**: $\frac{25}{100} \times 360^\circ = 90^\circ$

The pie chart will have slices with the above angles, labeled with each child's name and percentage.

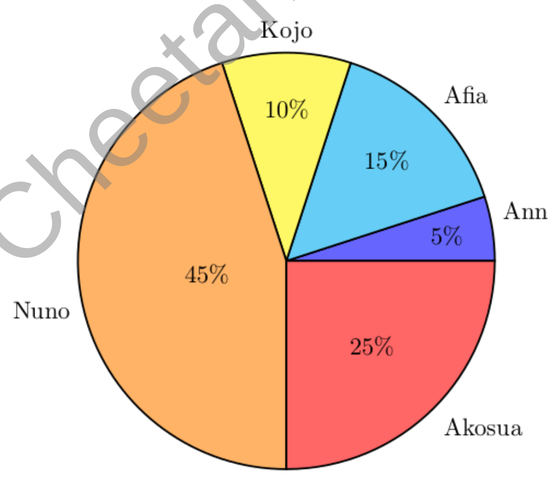


Figure 3:

(b) **Total beads** = 5 (red) + 3 (green) + 4 (blue) = 12.
- **Probability of first red bead**: $\frac{5}{12}$
- **Probability of second red bead (without replacement)**: $\frac{4}{11}$

- **Combined probability**:

$$\frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33} \approx 0.1515 \rightarrow 0.15\,(\text{to two decimal places})$$

Main Concepts:
1. **Pie Chart Representation**: Converting percentages to angles in a circle (360 proportionality).
2. **Probability Without Replacement**: Calculating dependent probabilities sequentially (multiplication rule) and simplifying fractions.

Problem 55

A. In a class of 80 students, $\frac{3}{4}$ study Biology and $\frac{3}{5}$ study Physics. If each student studies at least one of the subjects: (i) Draw a Venn diagram to represent this information. (ii) How many students study both subjects? (iii) Find the fraction of the class that study Biology but not Physics, give the answer as a decimal number.

B. Johnson and Jocatol Ltd. owned a business office with floor measuring 15 m by 8 m which was to be carpeted. The cost of carpeting was GH 890.00 per square metre. If a total of GH 216,120.00 was spent on painting and carpeting, how much was the cost of painting?

Correct answer:
graph
Correct answer:
28
Correct answer:
0.4
Correct answer:
109.32

Solution. Solution:

A. (i) Venn Diagram Representation

A Venn diagram would consist of two overlapping circles labeled Biology and Physics.
For the diagram, see figure below.

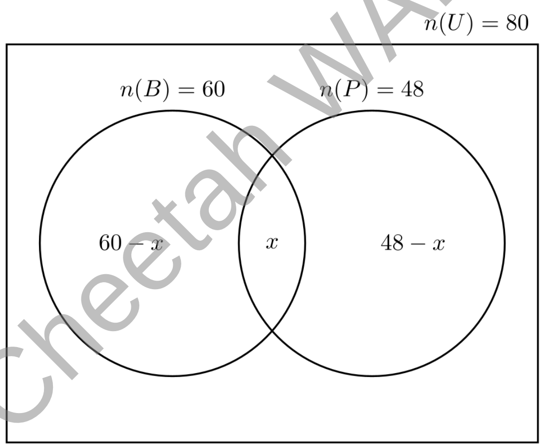


Figure 4:

The total number of students is 80. The number of students studying Biology is:

$$\frac{3}{4} \times 80 = 60,$$

and the number of students studying Physics is:

$$\frac{3}{5} \times 80 = 48.$$

The overlapping region represents students studying both subjects.

A. (ii) Students Studying Both Subjects

Using the inclusion-exclusion principle:

$$\text{Both} = \text{Biology} + \text{Physics} - \text{Total} = 60 + 48 - 80 = 28.$$

Thus, 28 students study both Biology and Physics.

A. (iii) Fraction Studying Biology Only

The number of students studying only Biology is:

$$\text{Biology only} = \text{Biology} - \text{Both} = 60 - 28 = 32.$$

The fraction of students studying only Biology is:

$$\frac{32}{80} = 0.4.$$

B. Cost of Painting

1. **Floor area**: $15\,\text{m} \times 8\,\text{m} = 120\,\text{m}^2$.
2. **Carpeting cost**: $120\,\text{m}^2 \times GH\,890/\text{m}^2 = GH\,106,800$.
3. **Painting cost**: Total cost $-$ Carpeting cost $= GH\,216,120 - GH\,106,800 = GH\,109,320$.

Final Answer

- **A(ii)**: 28 students study both Biology and Physics.
- **A(iii)**: The fraction of students studying only Biology is 0.4.
- **B**: Painting cost $= GH\,109,320.00$.

Key Concepts:

- **Set Theory (A)**: Venn diagrams, inclusion-exclusion principle.
- **Arithmetic & Algebra (B)**: Area calculation, cost analysis.

Problem 56

A. Complete the table of values for the relation $y = 2x^2 - x - 2$ for $-4 \leq x \leq 4$. Write the answer as a sequence of missing values for y coresponding to the values of x from -4 to 3.

x 4	-4	-3	-2	-1	0	1	2	3
y 26		19			-2			

B. Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw the graph of $y = 2x^2 - x - 2$ for $4 \leq x \leq 4$.
C. On the same axes, draw the graph of $y = 2x + 3$.
D. Use the graph to find the: (i) roots of the equation $2x^2 - 3x - 5 = 0$, represent the answers as $a \pm 0.1$, $b \pm 0.1$, write the answers as a, b, where a, b are decimal numbers; (ii) range of values of x for which $2x^2 - x - 2 < 0$, represent the answer as $a \pm 0.1 < x < b \pm 0.1$, write the answer as a, b, where a, b are decimal numbers.

Correct answer:
34,8,1,-1,4,13
Correct answer:
graph
Correct answer:
graph
Correct answer:
-1::2.5
Correct answer:
-0.7,1.3
Solution. **A. Completed Table of Values**
The missing y-values corresponding to $x = -4, -2, -1, 1, 2, 3$ are:
34, 8, 1, -1, 4, 13.
B. Graph of $y = 2x^2 - x - 2$
- **Shape**: Upward-opening parabola.
- **Vertex**: At $(0.25, -2.125)$.
- **Key Points**: Plot the table values, including $(-4, 34)$, $(0, -2)$, and $(4, 26)$.
Scaling:
x-axis: $2\ \text{cm} = 1\ \text{unit}$
y-axis: $2\ \text{cm} = 5\ \text{units}$ (equivalent to $0.4\ \text{cm} = 1\ \text{unit}$)

C. Graph of $y = 2x + 3$
- **Line**: Starts at $(0, 3)$ with slope 2. Crosses the parabola at $x = -1$ and $x = 2.5$.

D. Solutions Using the Graph
(i) **Roots of $2x^2 - 3x - 5 = 0$**:
From the intersection points of $y = 2x^2 - x - 2$ and $y = 2x + 3$:
Roots: -1.0 ± 0.1, 2.5 ± 0.1.
Answer: **$-1.0, 2.5$**.
(ii) **Range of x where $2x^2 - x - 2 < 0$**:
The parabola is below the x-axis between its roots $x \approx -0.8$ and $x \approx 1.3$.
Answer: **$-0.8, 1.3$**.
—
Main Concepts

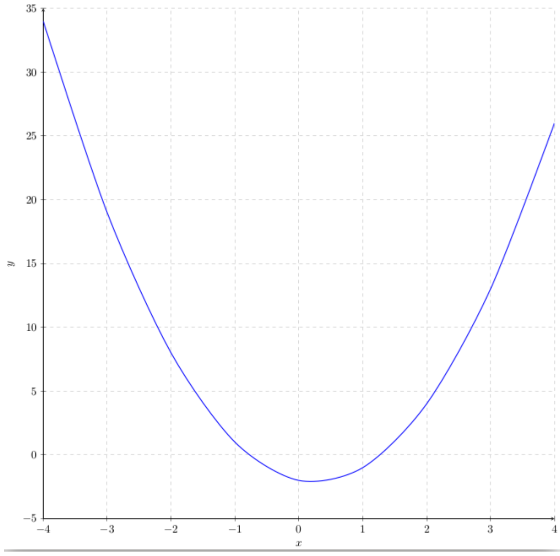


Figure 5:

1. **Substitution in Functions**: Evaluating $y = 2x^2 - x - 2$ by substituting x-values and simplifying.
2. **Graphing Quadratics**: Understanding vertex form, direction, and plotting points to sketch a parabola.
3. **Graphing Linear Equations**: Using slope-intercept form to draw lines.
4. **Graphical Solutions**: Finding intersections of graphs to solve equations (e.g., $2x^2-3x-5 = 0$).
5. **Inequality Analysis**: Identifying intervals where a quadratic is negative by analyzing its roots and graph shape.

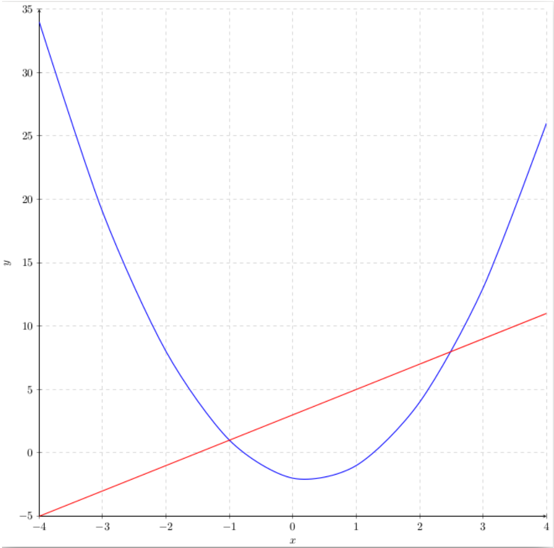


Figure 6:

Problem 57

A. In a triangle PQR, $\angle PQR = 90°$. If its area is 216 cm^2 and $|PQ| : |QR|$ is $3 : 4$, find $|PR|$.
B. The present ages of a man and his son are 47 years and 17 years respectively. In how many years would the man's age be twice that of his son?

Correct answer:
30
Correct answer:
13
Solution. **Problem A Solution:**
Given a right-angled triangle $\triangle PQR$ with $\angle PQR = 90°$, area $216\,\text{cm}^2$, and ratio $|PQ| : |QR| = 3 : 4$:
1. Let $|PQ| = 3x$ and $|QR| = 4x$.
2. Area: $\frac{1}{2} \cdot 3x \cdot 4x = 6x^2 = 216 \Rightarrow x^2 = 36 \Rightarrow x = 6$.
3. Thus, $|PQ| = 18\,\text{cm}$ and $|QR| = 24\,\text{cm}$.
4. Using the Pythagorean theorem:

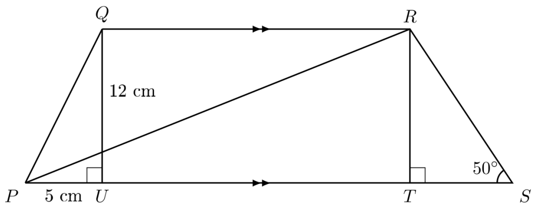
$$|PR| = \sqrt{|PQ|^2 + |QR|^2} = \sqrt{18^2 + 24^2} = \sqrt{324 + 576} = \sqrt{900} = 30\,\text{cm}.$$

Final Answer for A: $|PR| = 30\,\text{cm}$.
Problem B Solution:
Present ages: Man = 47 years, Son = 17 years. Let x be the number of years until the man's age is twice the son's age.
1. Future ages: Man's age $= 47 + x$, Son's age $= 17 + x$.
2. Equation:

$$47 + x = 2(17 + x) \Rightarrow 47 + x = 34 + 2x \Rightarrow x = 13.$$

Final Answer for B: 13 years.
Key Concepts:
- **Problem A:** Ratios, area of a right-angled triangle ($\frac{1}{2} \times \text{base} \times \text{height}$), and the Pythagorean theorem.
- **Problem B:** Linear equations in age problems, setting up relationships between future ages and solving for unknowns.

Problem 58



In the diagram, $\overline{PQ//RS}$ is a trapezium with QR||PS; U and T are points on $\overline{PS}$ such that $|\overline{PU}| = 5$ cm,$|\overline{QU}| = 12$ cm and $\angle PUQ = \angle STR = 90°$. If the area of $PQR = 20$ cm^2, calculate, correct to the nearest whole number: A. The perimeter of the trapezium. B. The area of the trapezium.

Correct answer:
50
Correct answer:
130
Solution. Given the trapezium $PQRS$ with $QR \parallel PS$, points U and T on $\overline{PS}$ such that $PU = 5\,\text{cm}$, $QU = 12\,\text{cm}$, and $\angle PUQ = \angle STR = 90°$. The area of triangle PQR is $20\,\text{cm}^2$. We need to find the perimeter and area of the trapezium.
1. **Right-Angled Triangle PUQ**:
- Using the Pythagorean theorem, $PQ = \sqrt{PU^2 + QU^2} = \sqrt{5^2 + 12^2} = 13\,\text{cm}$.
2. **Area of Triangle PQR**:
- The area of triangle PQR is given as $20\,\text{cm}^2$.
- Using the area formula for a triangle, $\frac{1}{2} \times QR \times \text{height} = 20\,\text{cm}^2$, and knowing the height (from Q to PS) is $12\,\text{cm}$, we solve for QR:

$$QR = \frac{20 \times 2}{12} = \frac{40}{12} = \frac{10}{3} \approx 3.333\,\text{cm}$$

3. **Trigonometry in Triangle STR**:
- Given $\angle STR = 90°$ and a $50°$ angle at S, we use trigonometry to find the length ST:

$$\tan(50°) = \frac{TR}{ST} \implies \tan(50°) = \frac{12}{ST} \implies ST = \frac{12}{\tan(50°)} \approx 10.069\,\text{cm}$$

- Therefore, S is located at $s = \frac{10}{3} + 10.069 \approx 13.402\,\text{cm}$ from the origin.
4. **Lengths of Sides**:
- $PS = PU + UT + TS = 5 + 10.069 \approx 18.402\,\text{cm}$
- RS is calculated using the distance formula:

$$RS = \sqrt{(13.402 - \frac{10}{3})^2 + 12^2} \approx \sqrt{10.069^2 + 12^2} \approx 15.665\,\text{cm}$$

5. **Perimeter and Area**:
- **Perimeter**: $PQ + QR + RS + SP \approx 13 + 3.333 + 15.665 + 18.402 \approx 50.4\,\text{cm}$, rounded to $50\,\text{cm}$.
- **Area**: Using the trapezium area formula:

$$\text{Area} = \frac{1}{2} \times (QR + PS) \times \text{height} = \frac{1}{2} \times \left(\frac{10}{3} + 18.402\right) \times 12 \approx 130.41\,\text{cm}^2 \approx 130\,\text{cm}^2$$

Final Answer

A. The perimeter of the trapezium is approximately $\boxed{50}$ cm.

B. The area of the trapezium is approximately $\boxed{130}$ cm.

Main Concepts:

1. **Right-Angled Triangles**: Utilized the Pythagorean theorem in triangle PUQ to determine PQ.

2. **Trigonometry**: Applied tangent function in triangle STR to find unknown lengths using the given $50°$ angle.

3. **Coordinate Geometry**: Assigned coordinates to points to model the problem, enabling the use of distance formulas and the shoelace theorem.

4. **Area Calculations**: Combined the area of triangle PQR with the properties of the trapezium to find the total area using both geometric formulas and coordinate-based methods.

Problem 59

A cottage is on a bearing of 200° and 110° from Dogbe's and Manu's farms respectively. Dogbe walked 5 *km* and Manu 3 *km* from the cottage to their farms. A. Find, correct to two significant figures, the distance between the two farms. B. Find, correct to the nearest degree, the bearing of Manu's farm from Dogbe's.

C. A ladder 10 *m* long leaned against a vertical wall X *m* high. The distance between the wall and the foot of the ladder is 2 *m* longer than the height of the wall. Calculate the value of X.

Correct answer:
5.8
Correct answer:
231
Correct answer:
6

Solution. To solve the given problems, we use trigonometry, coordinate geometry, and the Pythagorean theorem. Here are the key steps:

A. Distance between the two farms:

1. Convert the bearings from the cottage to the farms into angles.
2. Determine the angle at the cottage (90 degrees) by finding the difference between the bearings.
3. Use the Pythagorean theorem in the right-angled triangle formed by the distances from the cottage to each farm:

$$\text{Distance} = \sqrt{5^2 + 3^2} = \sqrt{34} \approx 5.83 \text{ km}$$

4. Round to two significant figures: 5.8 km.

B. Bearing of Manu's farm from Dogbe's farm:

1. Convert the coordinates of Dogbe's and Manu's farms using trigonometric functions.
2. Calculate the displacement vector between the two farms.
3. Use trigonometry to find the angle from north, considering the displacement components (west and south):

$$\text{Bearing} = 180^\circ + \arctan\left(\frac{4.529}{3.6725}\right) \approx 231^\circ$$

4. Round to the nearest degree: 231°.

C. Height of the wall (X):

1. Set up the Pythagorean theorem for the ladder problem:

$$X^2 + (X+2)^2 = 10^2$$

2. Solve the quadratic equation:

$$2X^2 + 4X - 96 = 0 \implies X = 6 \text{ m}$$

Final Answers

A. The distance between the two farms is $\boxed{5.8}$ km.

B. The bearing of Manu's farm from Dogbe's is $\boxed{231^\circ}$.

C. The value of X is $\boxed{6}$ meters.

Main Concepts:
- **Trigonometry and Bearings**: Converting bearings to angles and using the Pythagorean theorem.
- **Coordinate Geometry**: Calculating vector components and bearings using trigonometry.
- **Quadratic Equations**: Solving for the height of the wall using the Pythagorean theorem.

Problem 60

The table shows the distribution of the number of hours per day spent in studying by 50 students.

Number of hours per day	4	5	6	7	8	9	10	11
Number of students	5	7	5	9	12	4	3	5

Calculate, correct to two decimal places: (a) the mean; (b) the standard deviation.

Correct answer:
7.30
Correct answer:
2.04

Solution. (a) **Mean**:
To calculate the mean, multiply each hour by its frequency, sum the products, and divide by the total number of students:

$$\mu = \frac{\sum f_i x_i}{N} = \frac{365}{50} = 7.30$$

(b) **Standard Deviation**:
First, compute the variance using the squared deviations from the mean:

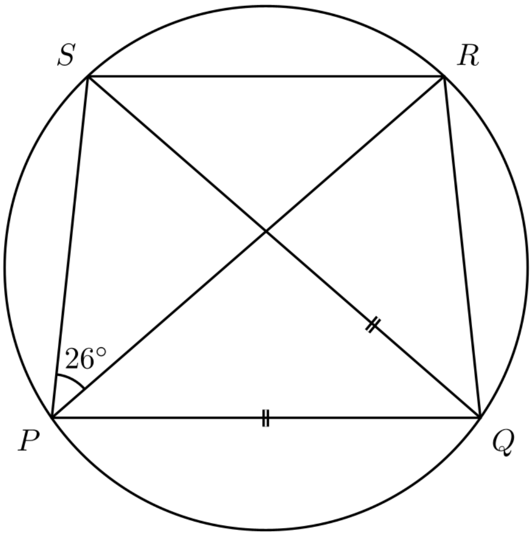
$$\sigma^2 = \frac{\sum f_i (x_i - \mu)^2}{N} = \frac{208.5}{50} = 4.17$$

The standard deviation is the square root of the variance:

$$\sigma = \sqrt{4.17} \approx 2.04$$

Final Answers:
(a) The mean is $\boxed{7.30}$.
(b) The standard deviation is $\boxed{2.04}$.
—
Main Concepts:
1. **Mean for Grouped Data**: Weighted average of values multiplied by their frequencies.
2. **Variance and Standard Deviation**: Measures of spread calculated using squared deviations from the mean, emphasizing dispersion in grouped data.
3. **Alternative Variance Formula**: Uses the mean of squares minus the square of the mean, confirming consistency in results.

Problem 61



1. In the diagram, there is a circle; $\overline{|PQ|} = \overline{|QS|}$, $\angle$SPR $= 26°$ and the interior angles of the triangle PQS are in the ratio of 2 : 3 : 3.
Calculate: (i) $\angle PQR$; (ii) $\angle RPQ$; (iii) $\angle PRQ$.
2. The coordinates of two points P and Q in a plane are $(7, 3)$ and $(5, x)$ respectively, where x is a real number. If $|PQ| = 29$ *units*, find, correct to two decimal places, the value of x. If there are more than one answer, write them down separated by the comma.

Correct answer:
71
Correct answer:
41.5
Correct answer:
67.5
Correct answer:
-25.93::31.93
Solution. To solve the given problems, we start with the first problem involving a cyclic quadrilateral and triangle angle calculations:
1. **In the diagram, there is a circle; $\overline{PQ} = \overline{QS}$, $\angle SPR = 26°$, and the interior angles of $\triangle PQS$ are in the ratio 2 : 3 : 3. Calculate: (i) $\angle PQR$; (ii) $\angle RPQ$; (iii) $\angle PRQ$.**
- **Step 1: Determine the angles in $\triangle PQS$**
- Given the ratio 2 : 3 : 3, the angles are $2x, 3x, 3x$.
- Sum of angles: $2x + 3x + 3x = 8x = 180°$ implies $x = 22.5°$.
- Angles: $45°, 67.5°, 67.5°$.
- **Step 2: Use cyclic quadrilateral properties**
- $\angle SPR = 26°$ implies the arc SR is $52°$.
- $\angle RPQ$ is found by subtracting $\angle SPR$ from the angle at P in $\triangle PQS$: $67.5° - 26° = 41.5°$.
- **Step 3: **Arc and Angle Relationships**:
- Given $\angle SPR = 26°$, the minor arc SR is twice the angle:

$$\text{arc } SR = 2 \times 26° = 52°.$$

- Given $\angle PQS = 45°$, the arc SP is twice the angle:

$$\text{arc } SP = 2 \times 45° = 90°.$$

- Given $\angle PSQ = 67.5°$, the arc PQ is twice the angle:

$$\text{arc } PQ = 2 \times 67.5° = 135°.$$

- Given $\angle SPQ = 67.5°$, the arc SQ is twice the angle:

$$\text{arc } SQ = 2 \times 67.5° = 135°.$$

- **Step 4: **Calculate Arc RQ**:

$$\text{arc } RQ = \text{arc } SQ - \text{arc } SR = 135° - 52° = 83°.$$

- **Step 5: **Calculate Angles**:
- $\angle PQR$ lies on arc PR, so:

$$\angle PQR = \frac{\text{arc } SP + \text{arc } SR}{2} = \frac{90° + 52°}{2} = 71°.$$

- $\angle RPQ$ lies on arc RQ, so:

$$\angle RPQ = \frac{\text{arc } RQ}{2} = \frac{83°}{2} = 41.5°.$$

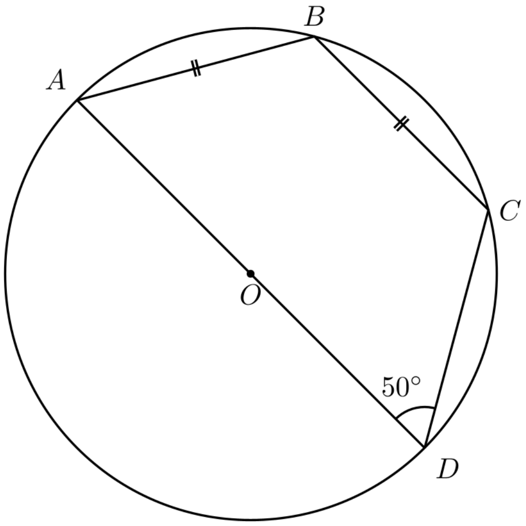
- $\angle PRQ$ lies on arc PQ, so:

$$\angle PRQ = \frac{\text{arc } PQ}{2} = \frac{135°}{2} = 67.5°.$$

Answer:
- $\angle PQR = 71°$,
- $\angle RPQ = 41.5°$,
- $\angle PRQ = 67.5°$.
Final Answers:
- (i) $\angle PQR = \boxed{71°}$
- (ii) $\angle RPQ = \boxed{41.5°}$
- (iii) $\angle PRQ = \boxed{67.5°}$
2. **The coordinates of two points P and Q in a plane are $(7, 3)$ and $(5, x)$ respectively, where x is a real number. If $|PQ| = 29$ units, find x.**
- **Step 1: Apply the distance formula**
- Distance formula: $\sqrt{(5-7)^2 + (x-3)^2} = 29$
- Simplify: $\sqrt{4 + (x-3)^2} = 29$
- Square both sides: $4 + (x-3)^2 = 841$
- Solve: $(x-3)^2 = 837$ implies $x - 3 = \pm\sqrt{837}$

- Calculate: $\sqrt{837} \approx 28.94$
Final Answer:
- $x \approx 3 + 28.94 \approx 31.94$ or $x \approx 3 - 28.94 \approx -25.94$
- Values: $\boxed{31.94, -25.94}$
Main Concepts:
- **Geometry**: Cyclic quadrilaterals, inscribed angles, triangle angle sums.
- **Coordinate Geometry**: Distance formula for finding unknown coordinates.

Problem 62



(a) On Sam's first birthday celebration, his grandfather deposited an amount of \$1, 000.00 in a bank compound at 4% interest annually. Find, correct to two decimal places, how much is in the account if Sam is 4 years old.
(b) In the diagram, $ABCD$ are points on the circle with centre O. If $|AB| = |BC|$ and $\angle ADC = 50°$, find $\angle BAD$.

Correct answer:
1124.86
Correct answer:
65
Solution. (a) To solve part (a), we use the compound interest formula $A = P(1 + r)^t$, where $P = 1000$, $r = 0.04$, and $t = 3$. Calculating this, we get:

$$A = 1000 \times (1 + 0.04)^3 = 1000 \times 1.124864 = 1124.86$$

Thus, the amount in the account when Sam is 4 years old is $\boxed{1124.86}$.
We are given that AD is the diameter and $AB = BC$. We proceed step by step:

$$\text{Arc } AC = 2 \times \angle ADC = 2 \times 50° = 100°.$$

Since $AB = BC$, the corresponding arcs are equal:

$$\text{Arc } AB = \text{Arc } BC = 50°.$$

Since AD is the diameter, the semicircle spans:

$$\text{Arc } ABCD = 180°.$$

Now, we calculate Arc CD:

$$\text{Arc } CD = \text{Arc } ABCD - \text{Arc } AB - \text{Arc } BC$$

$$= 180^\circ - 50^\circ - 50^\circ = 80^\circ.$$

To find $\angle BAD$:

$$\angle BAD = \frac{\text{Arc } BCD}{2} = \frac{50^\circ + 80^\circ}{2} = \frac{130^\circ}{2} = 65^\circ.$$

$$\boxed{65^\circ}$$

Main Concepts:
(a) **Compound Interest Calculation**: The problem required the application of the compound interest formula to determine the growth of an investment over time with annual compounding.
(b) **Circle Theorems and Cyclic Quadrilaterals**: Key concepts included recognizing equal arcs subtended by equal chords, inscribed angles, and the properties of cyclic quadrilaterals to determine the required angle.

Problem 63

On Sam's first birthday celebration, his grandfather deposited an amount of \$1,000.00 in a bank compound at 4% interest annually. Find, correct to two decimal places, how much is in the account if Sam is 4 years old.

Correct answer:
1124.86

Solution. To solve the problem, we use the compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where:
- $P = \$1,000$ (principal amount),
- $r = 0.04$ (annual interest rate),
- $n = 1$ (compounded annually),
- $t = 3$ years (from Sam's 1st to 4th birthday).

Plugging in the values:

$$A = 1000\left(1 + \frac{0.04}{1}\right)^{1\times3} = 1000 \times (1.04)^3$$

Calculating 1.04^3:

$$1.04 \times 1.04 = 1.0816 \quad \text{and} \quad 1.0816 \times 1.04 = 1.124864$$

Thus:

$$A = 1000 \times 1.124864 = 1124.864$$

Rounded to two decimal places, the amount is **\$1,124.86**.
Main Concepts:
- **Compound Interest Formula**: Applied to calculate growth over multiple periods with reinvested interest.
- **Time Calculation**: Determined the correct duration (3 years) by subtracting Sam's age at deposit from his current age.
- **Exponentiation**: Computed $(1.04)^3$ step-by-step to ensure accuracy.
- **Verification**: Cross-checked using annual increments to confirm the final amount.
- **Rounding**: Ensured the result adhered to the specified decimal precision.