1 Solutions to problems public WAEC math 2020 By cheetahwaec.com

Problem 1
Evaluate and correct to two decimal places: $75.0785 - 34.624 + 9.83$.
Possible answers:
A. 30.60
B. 50.29
C. 50.28
D. 30.63
Correct answer: C
Solution. To evaluate and correct to two decimal places:
$75.0785 - 34.624 + 9.83$
Step 1: Subtract 34.624 from 75.0785

$$75.0785 - 34.624 = 40.4545$$

Step 2: Add 9.83 to the result

$$40.4545 + 9.83 = 50.2845$$

Step 3: Round 50.2845 to two decimal places
The third decimal digit is 4 (less than 5), so the result remains **50.28**.
Final Answer
$\boxed{C}$
Main Concepts:
1. **Order of Operations**: Performing subtraction and addition from left to right.
2. **Decimal Arithmetic**: Aligning decimals for accurate subtraction and addition.
3. **Rounding**: Truncating to two decimal places after calculating the final value.

Problem 2

If $X = \{x : x < 7\}$ and $Y = \{y : y \text{ is a factor of } 24\}$ are subsets of $\mu = \{1, 2, 3 \ldots 10\}$, find $X \cap Y$.
Possible answers:
A. $\{2, 3, 4, 6\}$
B. $\{1, 2, 3, 4, 6\}$
C. $\{2, 3, 4, 6, 8\}$
D. $\{1, 2, 3, 4, 6, 8\}$
Correct answer: B

Solution. To solve the problem, we first determine the elements of sets X and Y:
- **Set X:** Defined as $\{x : x < 7\}$. Since the universal set $\mu = \{1, 2, 3, \ldots, 10\}$, $X = \{1, 2, 3, 4, 5, 6\}$.
- **Set Y:** Defined as factors of 24 within μ. The factors of 24 are $1, 2, 3, 4, 6, 8, 12, 24$, but only $\{1, 2, 3, 4, 6, 8\}$ belong to μ.
The intersection $X \cap Y$ includes elements common to both sets:

$$X \cap Y = \{1, 2, 3, 4, 6\}.$$

Answer: B. $\{1, 2, 3, 4, 6\}$.
Main Concepts:
1. **Set Definition:** Understanding how sets X (elements less than 7) and Y (factors of 24) are constructed within the universal set μ.
2. **Factors:** Identifying factors of 24 and restricting them to μ.
3. **Intersection:** Determining common elements between X and Y.

Problem 3

Simplify $[(\frac{16}{9})^{-\frac{3}{2}} \times 16^{-\frac{3}{4}}]^{\frac{1}{3}}$.

Possible answers:

A. $\frac{3}{4}$

B. $\frac{9}{16}$

C. $\frac{3}{8}$

D. $\frac{1}{4}$

Correct answer: C

Solution. To simplify the expression $\left[\left(\frac{16}{9}\right)^{-\frac{3}{2}} \times 16^{-\frac{3}{4}}\right]^{\frac{1}{3}}$, we proceed as follows:

1. **Simplify $\left(\frac{16}{9}\right)^{-\frac{3}{2}}$:**

- Negative exponent: $\left(\frac{9}{16}\right)^{\frac{3}{2}}$.

- Fractional exponent: $\left(\sqrt{\frac{9}{16}}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$.

2. **Simplify $16^{-\frac{3}{4}}$:**

- Negative exponent: $\frac{1}{16^{\frac{3}{4}}}$.

- Fractional exponent: $\sqrt[4]{16^3} = \sqrt[4]{2^{12}} = 2^3 = 8$, so $16^{-\frac{3}{4}} = \frac{1}{8}$.

3. **Multiply the results:**

$\frac{27}{64} \times \frac{1}{8} = \frac{27}{512}$.

4. **Apply the outer exponent $\frac{1}{3}$:**

$\left(\frac{27}{512}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}$.

Final Answer: $\boxed{C}$

Main Concepts:

- **Negative Exponents:** Converted to reciprocals (e.g., $a^{-n} = \frac{1}{a^n}$).
- **Fractional Exponents:** Interpreted as roots and powers (e.g., $a^{\frac{m}{n}} = \sqrt[n]{a^m}$).
- **Exponent Rules:** Combined using $a^m \times a^n = a^{m+n}$ and $(a^m)^n = a^{m \cdot n}$.
- **Prime Factorization:** Simplified bases to primes (e.g., $16 = 2^4$) for clarity.
- **Simplification:** Applied stepwise reduction to avoid errors.

Problem 4

Express $1 + 2log_{10}3$ in the form of $log_{10}x$.
Possible answers:
A. $log_{10}90$
B. $log_{10}19$
C. $log_{10}9$
D. $log_{10}6$
Correct answer: A

Solution. To express $1 + 2\log_{10} 3$ in the form $\log_{10} x$, follow these steps:
1. **Apply the Power Rule**: Use the logarithm property $a \log_b c = \log_b(c^a)$ to rewrite $2\log_{10} 3$:

$$2\log_{10} 3 = \log_{10}(3^2) = \log_{10} 9.$$

The expression becomes $1 + \log_{10} 9$.
2. **Express 1 as a Logarithm**: Recognize that $1 = \log_{10} 10$ (since $10^1 = 10$). Substitute this into the expression:

$$1 + \log_{10} 9 = \log_{10} 10 + \log_{10} 9.$$

3. **Apply the Product Rule**: Combine the logarithms using $\log_b a + \log_b c = \log_b(a \cdot c)$:

$$\log_{10} 10 + \log_{10} 9 = \log_{10}(10 \cdot 9) = \log_{10} 90.$$

Final Answer: $\boxed{A}$ ($\log_{10} 90$).
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Main Concepts Used:
- **Power Rule**: Converted the coefficient 2 into an exponent for the logarithmic term.
- **Logarithm of Base**: Recognized $1 = \log_{10} 10$ to rewrite the constant term as a logarithm.
- **Product Rule**: Combined two logarithms into a single logarithm by multiplying their arguments.
These steps systematically transformed the original expression into the required logarithmic form.

Problem 5

If $101_{\text{two}} + 21_{\text{y}} = 33_{\text{five}}$, find the value of y.

Possible answers:

A. 8

B. 7

C. 6

D. 5

Correct answer: C

Solution. 1. Convert all numbers to base **10**:

- $101_{\text{two}} = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 5_{\text{ten}}$.
- $21_{\text{y}} = 2 \cdot y^1 + 1 \cdot y^0 = (2y+1)_{\text{ten}}$.
- $33_{\text{five}} = 3 \cdot 5^1 + 3 \cdot 5^0 = 15 + 3 = 18_{\text{ten}}$.

2. Set up the equation:

$$5 + (2y+1) = 18 \Longrightarrow y = 6$$

3. Find answer choices:

The calculated value $y = 6$ matches C. 6.

Answer: $\boxed{A}$

Main Concepts:

- **Base Conversion**: Translating numbers from bases 2 and 5 to base 10.
- **Equation Solving**: Isolating the variable y after unifying terms in decimal.

Problem 6

An amount of N 550, 000.00 was realized when a principal, x was saved at 2% simple interest for 5 years. Find the value of x.

Possible answers:

A. N470, 000.00

B. N480, 000.00

C. N490, 000.00

D. N500, 000.00

Correct answer: D

Solution. The value of the principal x is calculated using the simple interest formula.

Step-by-Step Solution:

1. **Formula for Total Amount with Simple Interest:**

$$A = P(1 + rt)$$

Where A is the total amount, P is the principal, r is the annual interest rate (in decimal), and t is the time (in years).

2. **Substitute Known Values:**

$$550{,}000 = P(1 + 0.02 \times 5)$$

$$550{,}000 = P(1.10)$$

3. **Solve for P:**

$$P = \frac{550{,}000}{1.10} = 500{,}000$$

Verification:

Interest over 5 years:

$0.02 \times 500{,}000 \times 5 = 50{,}000$.

Total amount: $500{,}000 + 50{,}000 = 550{,}000$, which matches the given amount.

Answer: D. N 500,000.00

Main Concepts:

- **Simple Interest Formula:** Understanding how total amount (A) relates to principal (P), rate (r), and time (t).

- **Algebraic Manipulation:** Rearranging the formula to solve for P.

- **Unit Conversion:** Converting percentage rate (2%) to decimal (0.02).

- **Verification:** Checking the solution by recalculating interest and total amount.

Problem 7
Given that $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}} = x + y\sqrt{15}$, find the value of $(x + y)$.
Possible answers:
A. $1\frac{3}{5}$
B. $1\frac{2}{5}$
C. $1\frac{1}{5}$
D. $\frac{1}{5}$
Correct answer: C
Solution. To solve the equation $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}} = x + y\sqrt{15}$, follow these steps:
1. **Split the numerator**:

$$\frac{\sqrt{3}}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{5}}.$$

Simplify the second term:

$$\frac{\sqrt{5}}{\sqrt{5}} = 1.$$

2. **Rationalize $\frac{\sqrt{3}}{\sqrt{5}}$**:
Multiply numerator and denominator by $\sqrt{5}$:

$$\frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{5}.$$

3. **Combine terms**:
The expression becomes:

$$1 + \frac{\sqrt{15}}{5}.$$

This matches the form $x + y\sqrt{15}$, so $x = 1$ and $y = \frac{1}{5}$.
4. **Calculate $x + y$**:

$$x + y = 1 + \frac{1}{5} = \frac{6}{5} = 1\frac{1}{5}.$$

Final Answer: $\boxed{C}$
Main Concepts:
- **Rationalizing the denominator** to eliminate radicals from the denominator.
- **Splitting fractions** into simpler components for easier manipulation.
- **Equating coefficients** of like terms ($\sqrt{15}$ and constants) to solve for unknowns x and y.

Problem 8

If $x = 3$ and $y = -1$, evaluate $2(x^2 - y^3)$.

Possible answers:

A. 24

B. 22

C. 20

D. 16

Correct answer: C

Solution. To evaluate $2(x^2 - y^3)$ when $x = 3$ and $y = -1$:

1. Substitute $x = 3$ and $y = -1$ into the expression:

$$2\left(3^2 - (-1)^3\right)$$

2. Calculate $3^2 = 9$ and $(-1)^3 = -1$.

3. Simplify inside the parentheses:

$$9 - (-1) = 9 + 1 = 10$$

4. Multiply by 2:

$$2 \times 10 = 20$$

Final Answer: $\boxed{C}$

Main Concepts Used:

- **Substitution**: Replacing variables with given numerical values.
- **Exponents**: Correctly computing 3^2 and $(-1)^3$, noting that odd powers of negative numbers remain negative.
- **Order of Operations**: Following PEMDAS/BODMAS rules to resolve parentheses and exponents before multiplication.
- **Integer Operations**: Handling subtraction of a negative value $(9 - (-1))$ as addition.

Problem 9

Solve $3x - 2y = 10$ and $x + 3y = 7$ simultaneously.
Possible answers:
A. $x = -4$ and $y = 1$
B. $x = -1$ and $y = -4$
C. $x = 1$ and $y = 4$
D. $x = 4$ and $y = 1$
Correct answer: D

Solution. To solve the system of equations $3x - 2y = 10$ and $x + 3y = 7$, we use the **substitution method**:
1. **Solve the second equation for x**:

$$x = 7 - 3y$$

2. **Substitute $x = 7 - 3y$ into the first equation**:

$$3(7 - 3y) - 2y = 10$$

Simplify:

$$21 - 9y - 2y = 10 \quad \Rightarrow \quad 21 - 11y = 10$$

Solve for y:

$$-11y = -11 \quad \Rightarrow \quad y = 1$$

3. **Substitute $y = 1$ back into $x = 7 - 3y$**:

$$x = 7 - 3(1) = 4$$

Verification:
- First equation: $3(4) - 2(1) = 12 - 2 = 10$
- Second equation: $4 + 3(1) = 4 + 3 = 7$

Answer: $\boxed{D}$ ($x = 4$ and $y = 1$).
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Main Concepts:
1. **Substitution Method**: Solving one equation for a variable and substituting it into the other equation to reduce the system to a single variable.
2. **Linear Equations**: Manipulating equations by distributing, combining like terms, and isolating variables.
3. **Verification**: Checking the solution in both original equations to ensure consistency and correctness.

Problem 10

The implication x $\to$ y is equivalent to?

Possible answers:

A. $\sim y \to \sim x$

B. $y \to \sim x$

C. $\sim x \to \sim y$

D. $y \to x$

Correct answer: A

Solution. The implication $x \to y$ is logically equivalent to its contrapositive, $\sim y \to \sim x$. This equivalence holds because both statements share the same truth values under all possible scenarios for x and y. The contrapositive maintains the logical relationship by negating both components and reversing their order, ensuring equivalence. Other options like the converse ($y \to x$) or the inverse ($\sim x \to \sim y$) do not preserve this equivalence, as their truth tables diverge from the original implication.

Main Concepts:

1. **Contrapositive**: The contrapositive of $x \to y$ is $\sim y \to \sim x$, which is logically equivalent to the original implication.

2. **Truth Table Verification**: By constructing truth tables for $x \to y$ and $\sim y \to \sim x$, their equivalence is confirmed through identical truth values.

3. **Distinguishing Equivalents**: Recognizing that the converse (swapping x and y) and inverse (negating both x and y) are not equivalent to the original implication.

Answer: A. $\sim y \to \sim x$.

Problem 11

The first term of a geometric progression is 3 and the 5th term is 48. Find the common ratio.
Possible answers:
A. 2
B. 4
C. 8
D. 16
Correct answer: A

Solution. To find the common ratio r of the geometric progression where the first term $a_1 = 3$ and the fifth term $a_5 = 48$, we use the formula for the n-th term of a geometric sequence:

$$a_n = a_1 \cdot r^{n-1}$$

For the fifth term ($n = 5$):

$$a_5 = 3 \cdot r^4 = 48$$

Solving for r:

$$r^4 = \frac{48}{3} = 16 \implies r = \sqrt[4]{16}$$

The fourth root of 16 is 2 (since $2^4 = 16$). Although $r = -2$ is also mathematically valid, the answer choices include only positive numbers. Substituting $r = 2$ confirms the sequence $3, 6, 12, 24, 48$, matching the given fifth term.
Answer: A. 2
—
Main Concepts:
1. **Geometric Progression Formula**: The n-th term is $a_n = a_1 \cdot r^{n-1}$, where a_1 is the first term and r is the common ratio.
2. **Solving Exponential Equations**: Isolate r by dividing and taking roots. Here, $r^4 = 16$ leads to $r = \sqrt[4]{16}$.
3. **Contextual Interpretation**: Recognizing that negative ratios are mathematically possible but excluded by the provided answer choices.
4. **Verification**: Substituting $r = 2$ back into the sequence confirms the fifth term matches the problem statement.

Problem 12

Solve $\frac{1}{3}(5-3x) < \frac{2}{5}(3-7x)$.

Possible answers:

A. $x > \frac{7}{22}$

B. $x < \frac{7}{22}$

C. $x > -\frac{7}{27}$

D. $x < -\frac{7}{27}$

Correct answer: D

Solution. To solve the inequality $\frac{1}{3}(5-3x) < \frac{2}{5}(3-7x)$, follow these steps:

1. **Eliminate fractions** by multiplying both sides by 15 (the least common multiple of 3 and 5):

$$15 \cdot \frac{1}{3}(5-3x) < 15 \cdot \frac{2}{5}(3-7x) \implies 5(5-3x) < 6(3-7x).$$

2. **Expand both sides**:

$$25 - 15x < 18 - 42x.$$

3. **Move variable terms to one side** by adding $42x$ to both sides:

$$25 + 27x < 18.$$

4. **Isolate the variable term** by subtracting 25 from both sides:

$$27x < -7.$$

5. **Solve for x** by dividing by 27:

$$x < -\frac{7}{27}.$$

Verification: Testing $x = -1$ (which satisfies $x < -\frac{7}{27}$) in the original inequality confirms the solution is valid. Testing $x = 0$ (which does not satisfy the inequality) shows it fails, further validating the result.

Answer: $\boxed{D}$

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Main Concepts:

1. **Eliminating denominators** by multiplying both sides by the least common multiple.
2. **Distributive property** to expand expressions.
3. **Rearranging terms** to isolate the variable.
4. **Inequality direction**: No reversal needed since division/multiplication by a positive number preserves the inequality direction.
5. **Verification via substitution** to confirm the solution.

Problem 13

Make m the subject of the relation $k = \sqrt{\frac{m-y}{m+1}}$.

Possible answers:

A. $m = \frac{y+k^2}{k^2+1}$

B. $m = \frac{y+k^2}{1-k^2}$

C. $m = \frac{y-k^2}{k^2+1}$

D. $m = \frac{y-k^2}{1-k^2}$

Correct answer: B

Solution. To make m the subject of the relation $k = \sqrt{\frac{m-y}{m+1}}$, follow these steps:

1. **Square both sides** to eliminate the square root:

$$k^2 = \frac{m-y}{m+1}.$$

2. **Cross-multiply** to isolate terms involving m:

$$k^2(m+1) = m - y.$$

3. **Expand** the left side:

$$k^2 m + k^2 = m - y.$$

4. **Rearrange terms** to group m-terms on one side:

$$k^2 m - m = -y - k^2.$$

5. **Factor out m** on the left side:

$$m(k^2 - 1) = -y - k^2.$$

6. **Solve for m** by dividing both sides by $k^2 - 1$:

$$m = \frac{-y-k^2}{k^2-1}.$$

7. **Simplify** by factoring out -1 in the numerator and denominator:

$$m = \frac{y+k^2}{1-k^2}.$$

Final Answer:

$$\boxed{B.\ m = \frac{y+k^2}{1-k^2}}$$

—

Main Concepts:
1. **Squaring both sides** to eliminate the square root.
2. **Cross-multiplication** to simplify the equation.
3. **Rearranging terms** to isolate m.
4. **Factoring** and **simplifying** algebraic expressions.
5. **Verification** through substitution to ensure correctness.

Problem 14

Find the quadratic equation whose roots are $\frac{1}{2}$ and $-\frac{1}{3}$.
Possible answers:
A. $3x^2 + x + 1 = 0$
B. $6x^2 + x - 1 = 0$
C. $3x^2 + x - 1 = 0$
D. $6x^2 - x - 1 = 0$
Correct answer: D

Solution. To find the quadratic equation with roots $\frac{1}{2}$ and $-\frac{1}{3}$, start by expressing the equation in factored form:

$$\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right) = 0.$$

Expanding this product:

$$x^2 + \frac{1}{3}x - \frac{1}{2}x - \frac{1}{6} = x^2 - \frac{1}{6}x - \frac{1}{6} = 0.$$

To clear the fractions, multiply the entire equation by 6:

$$6x^2 - x - 1 = 0.$$

This matches **Option D**.
Key Concepts:
1. **Roots to Factored Form**: Use the roots to write the equation as $(x - \text{root}_1)(x - \text{root}_2) = 0$.
2. **Polynomial Expansion**: Multiply the binomials and simplify.
3. **Eliminating Fractions**: Scale the equation by the least common denominator to obtain integer coefficients.
4. **Verification via Sum/Product**: Confirm using Vieta's formulas (sum of roots $= -\frac{b}{a}$, product $= \frac{c}{a}$).
Final Answer: $\boxed{D}$

Problem 15

Given that x is directly proportional to y and inversely proportional to z, $x = 15$ when $y = 10$ and $z = 4$, find the equation connecting x, y and z.
Possible answers:
A. $x = \frac{6y}{z}$
B. $x = \frac{12y}{z}$
C. $x = \frac{3y}{z}$
D. $x = \frac{3y}{2z}$
Correct answer: A

Solution. To solve the problem, we start by recognizing the relationships given: x is directly proportional to y and inversely proportional to z. This means the relationship can be expressed as:

$$x = k \cdot \frac{y}{z}$$

where k is the constant of proportionality.
Step 1: Substitute the given values to solve for k
Given $x = 15$, $y = 10$, and $z = 4$:

$$15 = k \cdot \frac{10}{4}$$

Simplify $\frac{10}{4} = \frac{5}{2}$:

$$15 = k \cdot \frac{5}{2}$$

Solve for k:

$$k = 15 \cdot \frac{2}{5} = 6$$

Step 2: Write the final equation
Substitute $k = 6$ back into the proportionality equation:

$$x = \frac{6y}{z}$$

Verification
Using $y = 10$, $z = 4$:

$$x = \frac{6 \cdot 10}{4} = \frac{60}{4} = 15$$

This matches the given value, confirming correctness.
Answer
$\boxed{A}$
Main Concepts
1. **Direct and Inverse Proportionality**: Understanding that $x \propto y$ implies $x = ky$, and $x \propto \frac{1}{z}$ implies $x = \frac{k}{z}$. Combining these gives $x = k \cdot \frac{y}{z}$.

2. **Solving for Constants**: Substituting known values into the proportionality equation to find the constant k.
3. **Verification**: Checking the derived equation by plugging in original values to ensure consistency.
4. **Equation Construction**: Using the constant k to finalize the relationship between x, y, and z.

Problem 16

Two buses start from the same station at 9.00 *am* and travel in opposite directions along the same straight road. The first bus travel at a speed of 72 km/h and the second at 48 km/h. At what time will they be 240 *km* apart?

Possible answers:

A. $1:00$ *pm*

B. $12:00$ *noon*

C. $11:00$ *am*

D. $10:00$ *am*

Correct answer: C

Solution. The two buses are traveling in opposite directions, so their relative speed is the sum of their individual speeds:

$$72\,\text{km/h} + 48\,\text{km/h} = 120\,\text{km/h}.$$

To find the time t when they are 240 km apart, use the formula:

$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{240}{120} = 2\,\text{hours}.$$

Starting at $9:00\,\text{am}$, adding $2\,\text{hours}$ gives $11:00\,\text{am}$.

Answer: C. $11:00\,\text{am}$.

Main Concepts:

1. **Relative Speed in Opposite Directions:** When objects move in opposite directions, their relative speed is the sum of their individual speeds.
2. **Time-Distance Relationship:** Time is calculated by dividing the total distance by the relative speed.
3. **Unit Conversion:** Consistent use of units (hours for time, km/h for speed) ensures accurate calculations.

Problem 17

A solid cuboid has a length of 7 cm, a width of 5 cm, and a height of 4 cm. Calculate its total surface area.

Possible answers:

A. 280 cm^2

B. 166 cm^2

C. 140 cm^2

D. 83 cm^2

Correct answer: B

Solution. To calculate the total surface area of a solid cuboid with dimensions $7\,\text{cm}$ (length), $5\,\text{cm}$ (width), and $4\,\text{cm}$ (height), we use the formula:

$$\text{TSA} = 2(lw + lh + wh)$$

Substituting the given values:

$$lw = 7 \times 5 = 35\,\text{cm}^2,$$

$$lh = 7 \times 4 = 28\,\text{cm}^2,$$

$$wh = 5 \times 4 = 20\,\text{cm}^2.$$

Adding these:

$$35 + 28 + 20 = 83\,\text{cm}^2.$$

Finally, multiplying by 2:

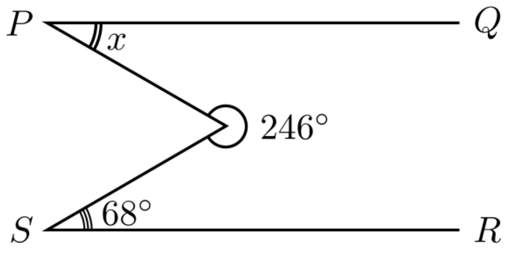
$$\text{TSA} = 2 \times 83 = 166\,\text{cm}^2.$$

Answer: **B. 166 cm^2**

Main Concepts:

1. **Cuboid Geometry**: A cuboid has six rectangular faces, with opposite faces being equal in area.
2. **Surface Area Formula**: The total surface area formula $2(lw + lh + wh)$ accounts for all six faces by summing the areas of three distinct face pairs and doubling the result.
3. **Dimensional Analysis**: Correct substitution of length, width, and height into the formula ensures accurate calculation.
4. **Common Errors**: Avoiding confusion with volume (lwh) or partial sums (e.g., omitting multiplication by 2).

Problem 18



In the diagram, $PQ||SR$. Find the value of x.
Possible answers:
A. 34
B. 46
C. 57
D. 68
Correct answer: B

Solution. 1. **Parallel Lines and Transversals:** Since $PQ \parallel SR$, the angles formed by the transversals OP and OS with these lines are related.
2. **Angle Between Transversals:** The reflex angle at O is given as $246°$, so the smaller angle between OP and OS is $360° - 246° = 114°$.
3. **Angle Relationships:** The angles x (at P) and $68°$ (at S) are exterior angles relative to the transversals. Since these angles lie on opposite sides of the transversals, their sum equals the angle between OP and OS: $x + 68° = 114°$.
4. **Solving for x:** By rearranging the equation, $x = 114° - 68° = 46°$.
Thus, the value of x is determined using properties of parallel lines and angle relationships formed by transversals.
The value of x is $\boxed{B}$.
Main Concepts Used:
1. **Parallel Lines and Transversals**
2. **Angle Between Transversals**
3. **Angle Relationships**

Problem 19

Find the equation of the line parallel to $2y = 3(x-2)$ and passes through the point $(2,3)$.
Possible answers:
A. $y = \frac{2}{3}x - 3$
B. $y = \frac{2}{3}x - 2$
C. $y = \frac{3}{2}x$
D. $y = -\frac{2}{3}x$
Correct answer: C

Solution. To find the equation of the line parallel to $2y = 3(x-2)$ and passing through $(2,3)$:
1. **Determine the slope of the given line**:
Rewrite $2y = 3(x-2)$ in slope-intercept form ($y = mx + b$):

$$2y = 3x - 6 \implies y = \frac{3}{2}x - 3$$

The slope (m) is $\frac{3}{2}$.
2. **Use the slope for the parallel line**:
Parallel lines share the same slope, so the new line also has $m = \frac{3}{2}$.
3. **Apply the point-slope formula**:
Using the point $(2,3)$ and slope $\frac{3}{2}$:

$$y - 3 = \frac{3}{2}(x-2)$$

Simplify to slope-intercept form:

$$y - 3 = \frac{3}{2}x - 3 \implies y = \frac{3}{2}x$$

Final Answer: $\boxed{C}$
Key Concepts:
1. **Slope of a line**: Derived by rewriting the equation in slope-intercept form ($y = mx + b$).
2. **Parallel lines**: Share identical slopes.
3. **Point-slope form**: Used to construct the equation of a line given a point and a slope.
4. **Simplification**: Converting the equation to slope-intercept form for clarity and comparison to answer choices.

Problem 20

The expression $\frac{5x+3}{6x(x+1)}$ will be undefined when x equals:.
Possible answers:
A. $\{0, 1\}$
B. $\{0, -1\}$
C. $\{-3, -11\}$
D. $\{-3, 0\}$
Correct answer: B

Solution. The expression $\frac{5x+3}{6x(x+1)}$ is undefined when its denominator $6x(x+1)$ equals zero. To solve for x:
1. Set the denominator equal to zero: $6x(x+1) = 0$.
2. Apply the zero product property: $x = 0$ or $x + 1 = 0$.
3. Solve each equation:
- $x = 0$
- $x = -1$.
Thus, the expression is undefined at $x = 0$ and $x = -1$.
Answer: $\boxed{B}$
Main Concepts:
- A rational expression is **undefined** when its **denominator equals zero**.
- **Factoring** the denominator and applying the **zero product property** to find critical values.
- Solving simple linear equations ($x = 0$ and $x + 1 = 0$) to determine where the denominator is zero.

Problem 21

A man is five times as old as his son. In four years' time, the product of their ages would be 340. If the son's age is y, express the product of their ages in terms of y.

Possible answers:

A. $5y^2 - 16y - 380 = 0$

B. $5y^2 - 24y - 380 = 0$

C. $5y^2 - 16y - 330 = 0$

D. $5y^2 + 24y - 324 = 0$

Correct answer: D

Solution. To solve the problem, let's denote the son's current age as y. The man's current age is then $5y$. In four years, their ages will be $y + 4$ (son) and $5y + 4$ (man). The product of their future ages is given as 340, leading to the equation:

$$(y + 4)(5y + 4) = 340$$

Expanding the left-hand side:

$$5y^2 + 4y + 20y + 16 = 5y^2 + 24y + 16$$

Setting this equal to 340:

$$5y^2 + 24y + 16 = 340$$

Subtracting 340 from both sides to form a quadratic equation:

$$5y^2 + 24y - 324 = 0$$

Main Concepts:

1. **Variable Definition:** Assigning y as the son's current age and expressing the man's age as $5y$.

2. **Future Ages:** Adjusting both ages by adding 4 years for the future time frame.

3. **Product Equation:** Setting up the equation for the product of their future ages equal to 340.

4. **Quadratic Expansion:** Expanding the product and simplifying to standard quadratic form.

5. **Equation Rearrangement:** Moving all terms to one side to match the structure of the answer choices.

Answer: D. $5y^2 + 24y - 324 = 0$

Problem 22

Simplify $\frac{a}{b} - \frac{b}{a} - \frac{c}{b}$.

Possible answers:

A. $\frac{a-b+c}{ab}$

B. $\frac{ab-bc-ac}{ab}$

C. $\frac{a2-b2+ac}{ab}$

D. $\frac{a2-b2-ac}{ab}$

Correct answer: D

Solution. To simplify the expression $\frac{a}{b} - \frac{b}{a} - \frac{c}{b}$, follow these steps:

1. **Identify the common denominator**: The denominators are b, a, and b. The least common multiple (LCM) of a and b is ab.

2. **Rewrite each term with the common denominator ab**:

- $\frac{a}{b} = \frac{a \cdot a}{b \cdot a} = \frac{a^2}{ab}$,
- $\frac{-b}{a} = \frac{-b \cdot b}{a \cdot b} = \frac{-b^2}{ab}$,
- $\frac{-c}{b} = \frac{-c \cdot a}{b \cdot a} = \frac{-ac}{ab}$.

3. **Combine the numerators**:

$$\frac{a^2}{ab} - \frac{b^2}{ab} - \frac{ac}{ab} = \frac{a^2 - b^2 - ac}{ab}.$$

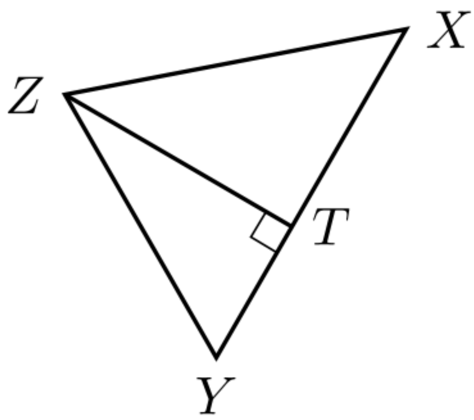
The simplified expression is $\frac{a^2-b^2-ac}{ab}$, which corresponds to **Option D**.

Main Concepts:

- **Common Denominator**: Identifying ab as the LCM of a and b.
- **Fraction Manipulation**: Adjusting each term to the common denominator by multiplying numerator and denominator appropriately.
- **Sign Management**: Carefully preserving negative signs during term conversion and combination.
- **Simplification**: Combining numerators while ensuring algebraic accuracy.

Answer: $\boxed{D}$

Problem 23



In the diagram, XYZ is an equilateral triangle of side 6 cm and Y is the midpoint of XY. Find $\tan(\angle XZT)$.
Possible answers:
A. $\frac{1}{\sqrt{3}}$
B. $\frac{\sqrt{3}}{2}$
C. $\sqrt{3}$
D. $\frac{1}{2}$
Correct answer: A
Solution. To solve the problem, we start by noting that triangle XYZ is equilateral with each side measuring $6\,\text{cm}$. Point T is the foot of the perpendicular from Z to XY, making T the midpoint of XY due to the symmetry of the equilateral triangle. This implies $XT = 3\,\text{cm}$ and $ZT = 3\sqrt{3}\,\text{cm}$ (the altitude of the equilateral triangle).
In right triangle XZT, we focus on $\angle XZT$. Using the tangent ratio:

$$\tan(\angle XZT) = \frac{\text{opposite}}{\text{adjacent}} = \frac{XT}{ZT} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

Answer: $\boxed{A}$
Main Concepts:
1. **Equilateral Triangle Properties**: All sides and angles are equal; altitudes bisect sides and form right angles.
2. **Trigonometric Ratios**: Using tangent in a right triangle to relate opposite and adjacent sides.
3. **Coordinate Geometry**: Simplifying angle and distance calculations by positioning points in a coordinate system.
4. **Vector Analysis**: Cross-verifying results using vector dot and cross products to ensure accuracy.

Problem 24

A fence 2.4 m tall, is 10 m away from a tree of height 16 m. Calculate the angle of elevation of the top of the tree from the top of the fence.

Possible answers:

A. $76.11°$

B. $53.67°$

C. $52.40°$

D. $51.32°$

Correct answer: B

Solution. To calculate the angle of elevation of the top of the tree from the top of the fence, we follow these steps:

1. **Determine the vertical difference**:

The tree is $16\,\text{m}$ tall, and the fence is $2.4\,\text{m}$ tall. The vertical distance between their tops is:

$$16\,\text{m} - 2.4\,\text{m} = 13.6\,\text{m}.$$

2. **Identify the horizontal distance**:

The fence is $10\,\text{m}$ away from the tree, which serves as the horizontal side of the right triangle.

3. **Apply the tangent function**:

The angle of elevation θ satisfies:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{13.6}{10} = 1.36.$$

4. **Calculate the angle using arctangent**:

Using a calculator or approximation, $\theta = \arctan(1.36) \approx 53.67°$.

Answer: $\boxed{B}$

—

Main Concepts:

1. **Angle of Elevation**: The angle formed between the horizontal line and the line of sight upward.

2. **Right Triangle Trigonometry**: The problem forms a right triangle with vertical and horizontal sides.

3. **Tangent Function**: Used to relate the opposite ($13.6\,\text{m}$) and adjacent ($10\,\text{m}$) sides.

4. **Inverse Trigonometric Function**: Arctangent converts the tangent ratio to the angle.

5. **Height Difference**: Correctly computing $16\,\text{m} - 2.4\,\text{m}$ ensures accurate vertical distance.

Problem 25

Fati buys milk at $\$x$ per tin sells each at a profit of $\$y$. If she sells 10 tins of milk, how much does she receives from the sales?
Possible answers:
A.$\$(xy + 10)$
B. $\$(x + 10y)$
C. $\$(10x + y)$
D. $\$10(x + y)$
Correct answer: D

Solution. Fati buys each tin of milk for $\$x$ and sells it with a profit of $\$y$ per tin. The selling price per tin is therefore $\$(x + y)$. For 10 tins, the total revenue is calculated by multiplying the selling price per tin by the number of tins sold:

$$10 \times (x + y) = \$(10(x + y)).$$

Answer: D. $\$10(x + y)$
Main Concepts:
1. **Profit Calculation:** Profit per unit is added to the cost price to determine the selling price.
2. **Total Revenue:** Total earnings from sales are found by multiplying the selling price per unit by the quantity sold.
3. **Algebraic Expression:** Combining terms and scaling by the quantity (10 tins) to derive the final expression.

Problem 26

If $\tan y$ is positive and $\sin y$ is negative, in which quadrant would y lie?
Possible answers:
A. First and third only
B. First and second only
C. Third only
D. Second only
Correct answer: C

Solution. To determine the quadrant where y lies given that $\tan y$ is positive and $\sin y$ is negative, we analyze the signs of trigonometric functions in each quadrant:
1. **Quadrant I**: All trigonometric functions (sin, cos, tan) are positive. This is inconsistent with $\sin y$ being negative.
2. **Quadrant II**: sin is positive, cos and tan are negative. This contradicts $\tan y$ being positive.
3. **Quadrant III**: tan is positive (since both sin and cos are negative, and their ratio is positive), and sin is negative. This satisfies both conditions.
4. **Quadrant IV**: cos is positive, sin and tan are negative. This conflicts with $\tan y$ being positive.
Main Concepts:
- **Signs of Trigonometric Functions**: Knowing which functions are positive/negative in each quadrant (e.g., All Students Take Calculus mnemonic).
- **Quadrant Analysis**: Identifying overlaps between conditions (positive tan implies Quadrants I or III; negative sin implies Quadrants III or IV).
- **Intersection of Conditions**: Quadrant III is the only overlap where both $\tan y > 0$ and $\sin y < 0$.
Answer: C. Third only.

Problem 27

The dimension of a rectangular base of a right pyramid is 9 cm by 5 cm. If the volume of the pyramid is 105 cm^3, how high is the pyramid?

Possible answers:

A. 10 cm

B. 6 cm

C. 8 cm

D. 7 cm

Correct answer: D

Solution. To determine the height of the pyramid, we use the formula for the volume of a pyramid:

$$\text{Volume} = \frac{1}{3} \times \text{Base Area} \times \text{Height}.$$

First, calculate the **base area** of the rectangular base:

$$\text{Base Area} = 9\,\text{cm} \times 5\,\text{cm} = 45\,\text{cm}^2.$$

Substitute the given volume ($105\,\text{cm}^3$) and the base area into the formula:

$$105 = \frac{1}{3} \times 45 \times \text{Height}.$$

Simplify the equation:

$$105 = 15 \times \text{Height}.$$

Solve for the height by dividing both sides by 15:

$$\text{Height} = \frac{105}{15} = 7\,\text{cm}.$$

Main Concepts:

1. **Volume Formula for Pyramids**: The volume of a pyramid is one-third the product of its base area and height.

2. **Rectangular Base Area**: The area of a rectangle is calculated as length multiplied by width.

3. **Algebraic Rearrangement**: Isolating the variable (height) by manipulating the volume equation.

Answer: D. 7 cm.

Problem 28

Each interior angle of a regular polygon is $168°$. Find the number of sides of the polygon.
Possible answers:
A. 30
B. 36
C. 24
D. 18
Correct answer: A

Solution. To determine the number of sides of a regular polygon with each interior angle measuring $168°$, we use the relationship between interior angles and the number of sides.
1. **Interior Angle Formula**:
For a regular polygon with n sides, each interior angle is given by:

$$\frac{(n-2)\times 180°}{n} = 168°$$

Solving for n:

$$\frac{(n-2)}{n} = \frac{168}{180} \implies \frac{n-2}{n} = \frac{14}{15}$$

Cross-multiplying:

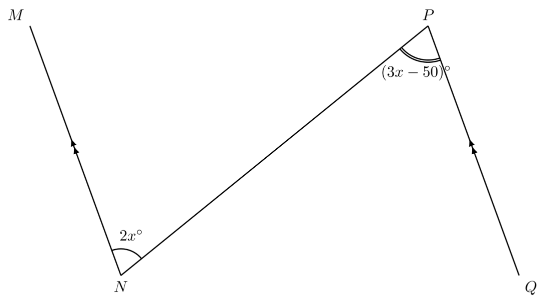
$$15(n-2) = 14n \implies 15n - 30 = 14n \implies n = 30$$

2. **Exterior Angle Verification**:
Each exterior angle is $180° - 168° = 12°$. Since the sum of exterior angles of any polygon is $360°$:

$$n = \frac{360°}{12°} = 30$$

Both methods confirm that the polygon has **30 sides**.
Answer: A. 30
—
Main Concepts:
1. **Interior Angle Formula**: The formula $\frac{(n-2)\times 180°}{n}$ relates the number of sides (n) to the measure of each interior angle in a regular polygon.
2. **Exterior Angle Property**: Each exterior angle of a polygon is supplementary to its interior angle ($180° -$ interior angle), and the sum of all exterior angles is always $360°$.
3. **Equation Solving**: Cross-multiplication and simplification are key algebraic steps to isolate n and solve for the number of sides.
4. **Verification**: Using multiple methods (interior and exterior angles) ensures consistency and accuracy in the solution.

Problem 29



In the diagram, $\overline{MN}||\overline{PQ}$, ∠MNP = 2x, and ∠NPQ = (3x − 50)°. Find the value of ∠NPQ.
Possible answers:
A. 200°
B. 100°
C. 120°
D. 90°
Correct answer: B

Solution. To solve the problem, we use the properties of parallel lines and transversals. Since $\overline{MN} \parallel \overline{PQ}$ and $\overline{NP}$ is a transversal, the alternate interior angles $\angle MNP$ and $\angle NPQ$ are equal. Setting their expressions equal gives:

$$2x = 3x - 50$$

Solving for x:

$$2x - 3x = -50 \implies -x = -50 \implies x = 50$$

Substitute $x = 50$ into $\angle NPQ$:

$$\angle NPQ = 3x - 50 = 3(50) - 50 = 100°$$

Answer: $\boxed{B}$
Main Concepts:
1. **Parallel Lines and Transversals**: Identified $\overline{NP}$ as a transversal intersecting parallel lines $\overline{MN}$ and $\overline{PQ}$.
2. **Alternate Interior Angles**: Applied the property that alternate interior angles are congruent.
3. **Equation Setup**: Equated the expressions for the alternate interior angles.
4. **Linear Equation Solving**: Solved for x and substituted back to find $\angle NPQ$.

Problem 30

The length of an arc of a circle of radius 3.5 cm is $1\frac{19}{36}$ cm. Calculate, correct to the nearest degree, the angle substended by the centre of the circle. [Take $\pi = \frac{22}{7}$]

Possible answers:
A. 55°
B. 36°
C. 25°
D. 22°
Correct answer: C

Solution. To calculate the angle subtended at the center of the circle, we use the formula for arc length in degrees:

$$s = \frac{\theta}{360} \times 2\pi r$$

Given:
- Arc length $s = 1\frac{19}{36}\,\text{cm} = \frac{55}{36}\,\text{cm}$,
- Radius $r = 3.5\,\text{cm} = \frac{7}{2}\,\text{cm}$,
- $\pi = \frac{22}{7}$.

Substitute into the formula:

$$\frac{55}{36} = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times \frac{7}{2}$$

Simplify $2 \times \frac{22}{7} \times \frac{7}{2} = 22$:

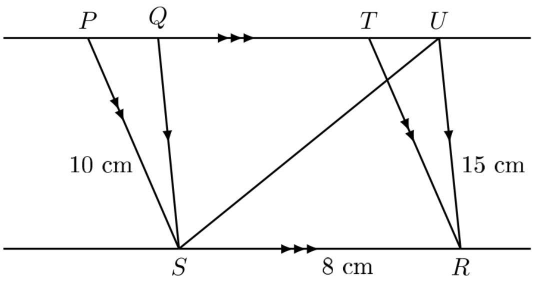
$$\frac{55}{36} = \frac{22\theta}{360}$$

Solve for θ:

$$\theta = \frac{55}{36} \times \frac{360}{22} = \frac{55 \times 10}{22} = 25°$$

Answer: C. 25°
Main Concepts:
1. **Arc Length Formula:** The relationship $s = \frac{\theta}{360} \times 2\pi r$ connects arc length, radius, and central angle in degrees.
2. **Unit Conversion:** Mixed numbers (e.g., $1\frac{19}{36}$) were converted to improper fractions for calculation.
3. **Simplification:** Canceling common factors (e.g., $\frac{7}{2}$ with $\frac{22}{7}$) streamlined computations.
4. **Verification:** Re-substituting the angle into the formula confirmed the result matched the given arc length.

Problem 31



In the diagram, $PU||SR$, $PS||TR$, $QS||UR$, $|UR| = 15\ cm$, $|SR| = 8\ cm$, $|PS| = 10\ cm$ and the area of the triangle SUR is $24\ cm^2$. Calculate the area of $PTRS$.
Possible answers:
A. 40 cm^2
B. 48 cm^2
C. 80 cm^2
D. 120 cm^2
Correct answer: B
Solution.

Solution

We notice that $PTRS$ is a parallelogram, so to calculate the area of $PTRS$ in two steps, we proceed as follows:
1. **Determine the height of the parallelogram:**
Given the area of triangle SUR as $24\,\text{cm}^2$ and the base $SR = 8\,\text{cm}$, we use the formula for the area of a triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

Substituting the known values:

$$24 = \frac{1}{2} \times 8 \times \text{height}$$

Solving for the height:

$$\text{height} = \frac{24 \times 2}{8} = 6\,\text{cm}$$

2. **Calculate the area of parallelogram $PTRS$:**
The area of a parallelogram is given by:

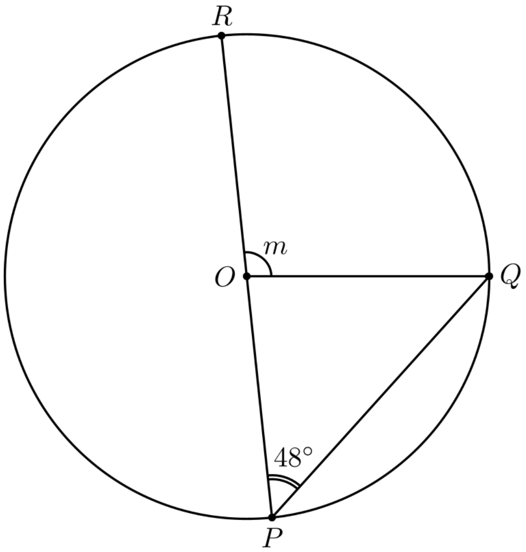
$$\text{Area} = \text{base} \times \text{height}$$

Using the base $SR = 8\,\text{cm}$ and the height $6\,\text{cm}$:

$$\text{Area} = 8 \times 6 = 48\,\text{cm}^2$$

Thus, the area of quadrilateral $PTRS$ is $48\,\text{cm}^2$.
Answer: $\boxed{B}$
Main Concepts:
- **Parallelogram Properties**: Opposite sides are parallel and equal, and the area is base multiplied by height.
- **Coordinate Geometry**: Assigning coordinates to points and using vectors to verify parallelism and calculate distances.
- **Area Relationships**: Using the given area of triangle SUR to determine the height, which applies to the parallelogram $PTRS$.

Problem 32



In the diagram, there is a circle with center O. If $\angle OPQ = 48°$, find the value of m.
Possible answers:
A. $96°$
B. $90°$
C. $68°$
D. $42°$
Correct answer: A
Solution. To solve the problem, we start by analyzing triangle OPQ. Since OP and OQ are radii of the circle, triangle OPQ is isosceles with $OP = OQ$. Given $\angle OPQ = 48°$, the base angles at P and Q are equal. Thus, $\angle OQP = 48°$. The remaining angle at O is calculated as:

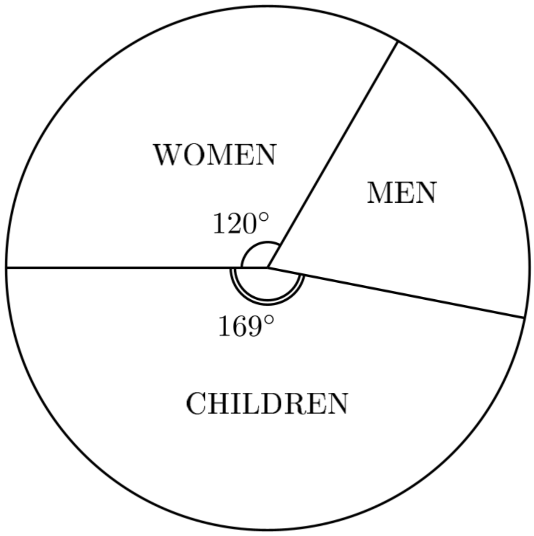
$$\angle POQ = 180° - 48° - 48° = 84°$$

This angle $\angle POQ$ measures $84°$.
Next:

$$m° + \angle POQ = 180°$$

Hence, angle m is $96°$.
Final Answer:
$\boxed{A}$
Main Concepts:
1. **Isosceles Triangle Properties**: Identified equal sides $OP = OQ$ to determine base angles.
2. **Straight Line Angle:** The sum of angles around point O on the straight line is $180°$.

Problem 33



The pie chart shows the population of men, women, and children in a city. If the population of the city is $1,800,000$, how many men are in the city?
Possible answers:
A. $845,000$
B. $600,000$
C. $355,000$
D. $250,000$
Correct answer: C
Solution. The pie chart divides the population into three sectors with central angles of 120, 169, and 71. To find the number of men:
1. **Determine the men's sector angle**: The smallest angle (71) corresponds to men based on the labels and conventions (smallest slice for men, largest for children).
2. **Calculate the proportion**:

$$\text{Men} = \left(\frac{71}{360}\right) \times 1,800,000 \approx 355,000$$

Answer: C. $355,000$
Main Concepts:
- **Pie Chart Interpretation**: Central angles represent proportions of categories.
- **Angle-to-Population Conversion**: Use sector angles to calculate population shares.
- **Proportional Reasoning**: Relate angles to total degrees (360) for accurate division of the total quantity.
- **Label Analysis**: Context clues (e.g., CHILDREN as the largest sector) help assign angles correctly.

Problem 34

The mean of the numbers $15, 21, 17, 26, 18$ and 29 is 21. Calculate the standard deviation.
Possible answers:
A. 9
B. 6
C. 5
D. 0
Correct answer: C

Solution. The standard deviation of the numbers $15, 21, 17, 26, 18$, and 29 is calculated as follows:
1. **Verify the Mean**:
Given the mean is 21, we confirm by summing the numbers and dividing by the count:

$$\frac{15+21+17+26+18+29}{6} = \frac{126}{6} = 21.$$

2. **Calculate Squared Deviations**:
Subtract the mean from each value and square the result:

$$(15-21)^2 = (-6)^2 = 36,$$

$$(21-21)^2 = 0^2 = 0,$$

$$(17-21)^2 = (-4)^2 = 16,$$

$$(26-21)^2 = 5^2 = 25,$$

$$(18-21)^2 = (-3)^2 = 9,$$

$$(29-21)^2 = 8^2 = 64.$$

3. **Sum Squared Deviations**:

$$36+0+16+25+9+64 = 150.$$

4. **Compute Variance**:
Divide the total squared deviations by the number of data points ($n = 6$):

$$\text{Variance} = \frac{150}{6} = 25.$$

5. **Determine Standard Deviation**:
Take the square root of the variance:

$$\text{Standard Deviation} = \sqrt{25} = 5.$$

Answer: C. 5
Main Concepts:
1. **Mean Calculation**: The average of the data set.
2. **Squared Deviations**: Measuring how each value differs from the mean, squared to eliminate negative values.
3. **Variance**: The average of these squared deviations.
4. **Standard Deviation**: The square root of variance, representing spread in the original units.

Problem 35

Find the sum of the interior angle of a pentagon.
Possible answers:
A. $340°$
B. $350°$
C. $540°$
D. $550°$
Correct answer: C

Solution. To find the sum of the interior angles of a pentagon, use the formula for the sum of interior angles of a polygon:

$$\text{Sum} = (n - 2) \times 180°$$

where n is the number of sides. For a pentagon ($n = 5$):

$$\text{Sum} = (5 - 2) \times 180° = 3 \times 180° = 540°$$

Verification via triangulation: A pentagon can be divided into 3 triangles by drawing non-intersecting diagonals from one vertex. Each triangle contributes $180°$, so:

$$3 \times 180° = 540°$$

Answer: C. $540°$
—
Main Concepts:
1. **Formula Application**: The formula $(n - 2) \times 180°$ generalizes the sum of interior angles for any n-sided polygon.
2. **Triangulation**: Dividing a polygon into triangles confirms the formula, as each triangle contributes $180°$.
3. **Polygon Universality**: The formula applies to both regular and irregular convex polygons, ensuring consistency regardless of side lengths or angles.

Problem 36

The diameter of a sphere is $12cm$. Calculate, correct of the nearest cm^3, the volume of the sphere. [Take $\pi = \frac{22}{7}$]

Possible answers:

A. 903 cm^3

B. 904 cm^3

C. 905 cm^3

D. 906 cm^3

Correct answer: C

Solution. To calculate the volume of a sphere with a diameter of 12 cm, follow these steps:

1. **Find the radius**:

The radius r is half the diameter:

$$r = \frac{12}{2} = 6\,\text{cm}.$$

2. **Apply the volume formula**:

The formula for the volume V of a sphere is:

$$V = \frac{4}{3}\pi r^3.$$

Substitute $r = 6\,\text{cm}$ and $\pi = \frac{22}{7}$:

$$V = \frac{4}{3} \times \frac{22}{7} \times 6^3.$$

3. **Compute 6^3**:

$$6^3 = 6 \times 6 \times 6 = 216.$$

4. **Simplify the expression**:

$$V = \frac{4}{3} \times \frac{22}{7} \times 216 = \frac{4 \times 22 \times 216}{3 \times 7}.$$

Simplify $\frac{4 \times 216}{3}$:

$$\frac{4 \times 216}{3} = 4 \times 72 = 288.$$

Now multiply by $\frac{22}{7}$:

$$288 \times \frac{22}{7} = \frac{6336}{7} \approx 905.14\,\text{cm}^3.$$

5. **Round to the nearest whole number**:

$905.14\,\text{cm}^3$ rounds to **$905\,\text{cm}^3$**.

Final Answer:

$\boxed{C}$

Main Concepts:

1. **Volume of a Sphere**: The formula $V = \frac{4}{3}\pi r^3$ requires the radius.
2. **Diameter to Radius Conversion**: The radius is half the diameter ($r = \frac{\text{diameter}}{2}$).
3. **Fraction Arithmetic**: Handling operations with fractions and simplifying step-by-step.
4. **Approximation and Rounding**: Using $\pi = \frac{22}{7}$ for calculation and rounding to the nearest whole number.

Problem 37

A box contains 12 identical balls of which 5 are red, 4 blue, and the rest are green. If a ball is selected at random from the box, what is the probability that it is green?

Possible answers:

A. $\frac{3}{4}$
B. $\frac{1}{2}$
C. $\frac{1}{3}$
D. $\frac{1}{4}$

Correct answer: D

Solution. To determine the probability of selecting a green ball from the box, follow these steps:
1. **Total Balls**: The box contains 12 balls in total.
2. **Non-Green Balls**: There are 5 red and 4 blue balls, totaling $5 + 4 = 9$ non-green balls.
3. **Green Balls**: Subtract the non-green balls from the total:

$$12 - 9 = 3 \text{ green balls.}$$

4. **Probability Calculation**: The probability of selecting a green ball is:

$$\frac{\text{Number of green balls}}{\text{Total balls}} = \frac{3}{12} = \frac{1}{4}.$$

Final Answer: $\boxed{D}$

Main Concepts:

- **Total vs. Favorable Outcomes**: Probability is calculated as the ratio of favorable outcomes (green balls) to total possible outcomes (all balls).
- **Subtraction for Remaining Quantity**: The number of green balls is found by subtracting the sum of red and blue balls from the total.
- **Simplification**: Reducing fractions to their simplest form ensures clarity in the final answer.

Problem 38

A box contains 12 identical balls of which 5 are red, 4 blue, and the rest are green. If two balls are selected at random one after the other with replacement, what is the probability that both are red?

Possible answers:

A. $\frac{25}{144}$

B. $\frac{5}{33}$

C. $\frac{5}{6}$

D. $\frac{103}{132}$

Correct answer: A

Solution. The probability that both selected balls are red when drawing with replacement is calculated as follows:

1. **Determine the probability of drawing a red ball each time:**

- There are 5 red balls out of 12 total.
- Probability of red on the first draw: $\frac{5}{12}$.
- Since the ball is replaced, the probability of red on the second draw remains $\frac{5}{12}$.

2. **Multiply the probabilities for independent events:**

$$P(\text{both red}) = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}.$$

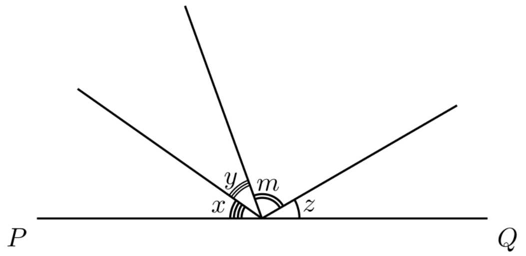
Answer: A. $\frac{25}{144}$.

—

Main Concepts:

- **Independent Events:** Replacement ensures the outcome of the first draw does not affect the second.
- **Probability Multiplication Rule:** For independent events, $P(A \text{ and } B) = P(A) \times P(B)$.
- **Fraction Multiplication:** Numerators and denominators are multiplied directly for sequential probabilities.
- **Replacement Significance:** Maintains constant total outcomes (12 balls) and favorable outcomes (5 red) for both draws.

Problem 39



In the diagram, PQ is a straight line. If $m = \frac{1}{2}(x + y + z)$, find value of m.
Possible answers:
A. 45°
B. 60°
C. 90°
D. 100°
Correct answer: B
Solution. 1. The sum of angles around point O on the straight line PQ is 180°.
2. Using the given relationship $m = \frac{1}{2}(x + y + z)$ to substitute $x + y + z = 2m$ into the total angle equation.
3. Simplifying $2m + m = 180°$ to $3m = 180°$, yielding $m = 60°$.
The value of m is $\boxed{B}$ (60).
Main Concepts:
1. **Straight Line Angle:** The sum of angles around point O on the straight line PQ is 180°.
2. **Substitution:** Using the given relationship $m = \frac{1}{2}(x + y + z)$ to substitute $x + y + z = 2m$ into the total angle equation.
3. **Algebraic Manipulation:** Simplifying $2m + m = 180°$ to $3m = 180°$, yielding $m = 60°$.

Problem 40

x	6.20	6.85	7.50
y	3.90	5.20	6.50

The points on a linear graph are as shown in the table. Find the gradient (slope) of the line.

Possible answers:
A. $2\frac{1}{2}$
B. 2
C. 1
D. $\frac{1}{2}$
Correct answer: B

Solution. To find the gradient (slope) of the line through the given points, we use the formula:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

First two points (6.20, 3.90) and (6.85, 5.20):

$$\Delta y = 5.20 - 3.90 = 1.30, \quad \Delta x = 6.85 - 6.20 = 0.65$$

$$\text{slope} = \frac{1.30}{0.65} = 2$$

Second and third points (6.85, 5.20) and (7.50, 6.50):

$$\Delta y = 6.50 - 5.20 = 1.30, \quad \Delta x = 7.50 - 6.85 = 0.65$$

$$\text{slope} = \frac{1.30}{0.65} = 2$$

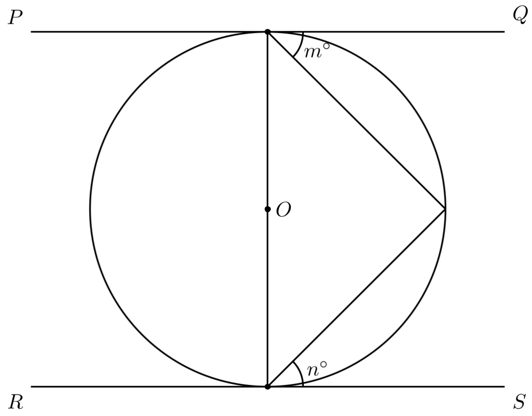
First and third points (6.20, 3.90) and (7.50, 6.50):

$$\Delta y = 6.50 - 3.90 = 2.60, \quad \Delta x = 7.50 - 6.20 = 1.30$$

$$\text{slope} = \frac{2.60}{1.30} = 2$$

Main Concepts:
1. **Slope Formula**: The gradient of a line is calculated using the ratio of the vertical change (Δy) to the horizontal change (Δx) between two points.
2. **Consistency in Linear Relationships**: For a linear graph, the slope remains constant across all pairs of points, confirming the line's linearity.
3. **Unit Conversion/Simplification**: Dividing decimals by aligning place values or converting to fractions ensures accurate arithmetic.
Answer: B. 2

Problem 41



In the diagram below, O is the center of the circle. PQ and RS are tangents to the circle. Find the value of $(m + n)^\circ$.
Possible answers:
A. 120°
B. 90°
C. 75°
D. 60°
Correct answer: B
Solution. 1. According to Thales' theorem, every inscribed angle based on the diameter of a circle is a right angle.
2. Tangents PQ, RS are perpendicular to radii OA, OB at points of contact A, B, establishing vertical radii and horizontal tangents.
3. From 1. and 2. follows that there is a trinagle with angles $90^\circ - m, 90^\circ - n, 90^\circ$.
4. Therefore:

$$90^\circ - m + 90^\circ - n + 90^\circ = 180^\circ,$$

$$m + n = 90^\circ.$$

The value of $(m + n)^\circ$ is $\boxed{B}$.
Main Concepts:
1. **Tangents and Radii:** Tangents (PQ, RS) are perpendicular to radii (OA, OB) at points of contact (A, B), establishing vertical radii and horizontal tangents.
2. **Thales' theorem:** According to Thales' theorem, every inscribed angle based on the diameter of a circle is a right angle.
3. **Triangle Properties:** Angles in a triangle sum to 180°.

Problem 42

Which of the following is not a sufficient condition for two triangles to be congruent?
Possible answers:
A. AAS
B. SSS
C. SAS
D. SSA
Correct answer: D

Solution. The answer is **D. SSA**.

To determine which condition is not sufficient for congruence, we analyze each option:
- **A. AAS (Angle-Angle-Side):** If two angles and a non-included side are equal, the triangles are congruent. The third angle is determined by the sum of angles (180), effectively reducing it to ASA.
- **B. SSS (Side-Side-Side):** All three sides being equal guarantees congruence, as the triangles are identical in shape and size.
- **C. SAS (Side-Angle-Side):** Two sides and the included angle uniquely determine the triangle's structure.
- **D. SSA (Side-Side-Angle):** Two sides and a non-included angle may produce ambiguity (the ambiguous case), allowing two distinct triangles to satisfy the conditions. Thus, it is not a valid congruence criterion.
Main Concepts:
1. **Triangle Congruence Criteria:** SSS, SAS, ASA, AAS, and HL (Hypotenuse-Leg for right triangles).
2. **Ambiguity in SSA:** Unlike SAS, SSA does not guarantee congruence because the non-included angle can correspond to two different triangle configurations.
3. **Angle-Sum Property:** In AAS, the third angle is determined, converting it to ASA for congruence.
4. **Unique Determination:** SSS, SAS, and ASA/AAS uniquely fix a triangle's dimensions, while SSA does not.

Problem 43

A woman received a discount of 20% on a piece of cloth she purchased from a shop. If she paid \$525.00, what was the original price?

Possible answers:

A. \$675.25

B. \$660.25

C. \$656.25

D. \$616.25

Correct answer: C

Solution. The woman paid 80% of the original price after a 20% discount. Let P be the original price.

$$0.80P = 525 \implies P = \frac{525}{0.80} = \frac{525 \times 5}{4} = 656.25$$

Verifying:

20% of $656.25 = 131.25$, so $656.25 - 131.25 = 525$, confirming the calculation.

Answer: C. \$656.25

Main Concepts:

1. **Percentage Discounts:** Recognizing that a 20% discount means paying 80% of the original price.
2. **Algebraic Equation:** Setting up the equation $0.80P = 525$ to represent the relationship between the discounted price and the original price.
3. **Solving for Variables:** Isolating P by dividing both sides of the equation by 0.80.
4. **Verification:** Checking the solution by recalculating the discount and confirming it matches the amount paid.

Problem 44

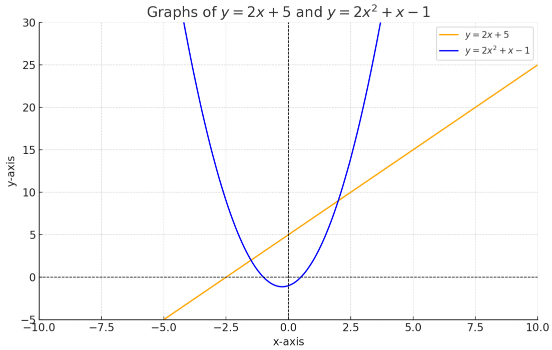
The interquartile range of distribution is 7. If the 25th percentile is 16, find the upper quartile.
Possible answers:
A. 35
B. 30
C. 23
D. 9
Correct answer: C

Solution. The interquartile range (IQR) is calculated as $Q_3 - Q_1 = 7$, where Q_1 (the 25th percentile) is given as 16. Solving for Q_3:

$$Q_3 = Q_1 + \text{IQR} = 16 + 7 = 23.$$

Thus, the upper quartile Q_3 is **23**, corresponding to **Option C**.
Main Concepts:
1. **Interquartile Range (IQR)**: Defined as $Q_3 - Q_1$, it measures the spread of the middle 50% of data.
2. **Percentiles and Quartiles**: The 25th percentile is equivalent to Q_1, and the 75th percentile is Q_3.
3. **Algebraic Manipulation**: Rearranging the IQR formula to solve for Q_3 given Q_1 and the IQR.

Problem 45



The graph of the equations $y = 2x + 5$ and $y = 2x2 + x - 1$ are shown. Use the information above to find the points of intersection of the two graphs.
Possible answers:
A. $(2.0, 9.0)$ and $(-1.5, 2.0)$
B. $(2.0, 8.5)$ and $(-1.5, 2.0)$
C. $(2.0, 8.0)$ and $(-1.5, 2.5)$
D. $(2.0, 7.5)$ and $(-1.5, 2.5)$
Correct answer: A

Solution. To find the points of intersection between the graphs of $y = 2x+5$ and $y = 2x^2+x-1$, set the equations equal:

$$2x + 5 = 2x^2 + x - 1.$$

Rearranging terms gives the quadratic equation:

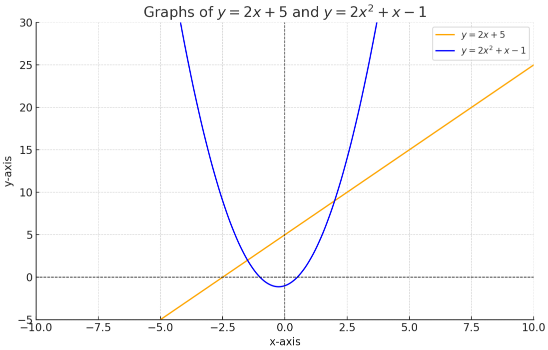
$$2x^2 - x - 6 = 0.$$

Factoring the quadratic:

$$(2x + 3)(x - 2) = 0,$$

yields solutions $x = -\frac{3}{2}$ (-1.5) and $x = 2$.
Substituting these x-values into $y = 2x + 5$:
- For $x = 2$: $y = 2(2) + 5 = 9$, giving the point $(2.0, 9.0)$.
- For $x = -1.5$: $y = 2(-1.5) + 5 = 2$, giving the point $(-1.5, 2.0)$.
Verification in $y = 2x^2 + x - 1$ confirms these points.
Answer: A. $(2.0, 9.0)$ and $(-1.5, 2.0)$.
Main Concepts:
1. **Equating Functions:** Intersections occur where y-values match, so set the equations equal.
2. **Quadratic Solving:** Rearrange to form a quadratic equation and solve by factoring or quadratic formula.
3. **Factoring:** Split the middle term and factor by grouping.
4. **Substitution:** Use solved x-values in one of the original equations to find y.
5. **Verification:** Ensure solutions satisfy both equations to confirm accuracy.

Problem 46



The graph of the equations $y = 2x + 5$ and $y = 2x^2 + x - 1$ are shown. Use the information above to answer this question. If $x = -2.5$, what is the value of y on the curve?
Possible answers:
A. $y = 8.0$
B. $y = 8.5$
C. $y = 9.0$
D. $y = 9.5$
Correct answer: C
Solution. To find the value of y on the curve when $x = -2.5$, we substitute $x = -2.5$ into the quadratic equation $y = 2x^2 + x - 1$:
1. Calculate x^2:

$$(-2.5)^2 = 6.25$$

2. Multiply by 2:

$$2 \times 6.25 = 12.5$$

3. Add the x-term:

$$12.5 + (-2.5) = 10$$

4. Subtract 1:

$$10 - 1 = 9$$

Thus, when $x = -2.5$, the value of y on the curve is $\boxed{C}$ $(y = 9.0)$.
Main Concepts:
1. **Identifying the Correct Equation:** The term curve refers to the quadratic equation $y = 2x^2 + x - 1$, distinguishing it from the linear equation.
2. **Substitution:** Plugging $x = -2.5$ into the quadratic equation requires precise arithmetic with decimals and negative numbers.
3. **Order of Operations:** Correctly applying exponentiation before multiplication, addition, and subtraction.
4. **Verification:** Rechecking calculations ensures accuracy, especially critical with multiple

operations and negative values.

Problem 47

If $(x+2)$ is a factor of $x^2+px-10$, find the value of p.
Possible answers:
A. 3
B. -3
C. 7
D. -7
Correct answer: B

Solution. To solve the problem, we apply the Factor Theorem, which states that if $(x+a)$ is a factor of a polynomial $P(x)$, then $P(-a)=0$. Here, $(x+2)$ is a factor, so substituting $x=-2$ into the polynomial $x^2+px-10$ must yield zero:

$$(-2)^2+p(-2)-10=0.$$

Simplifying:

$$4-2p-10=0 \implies -6-2p=0.$$

Solving for p:

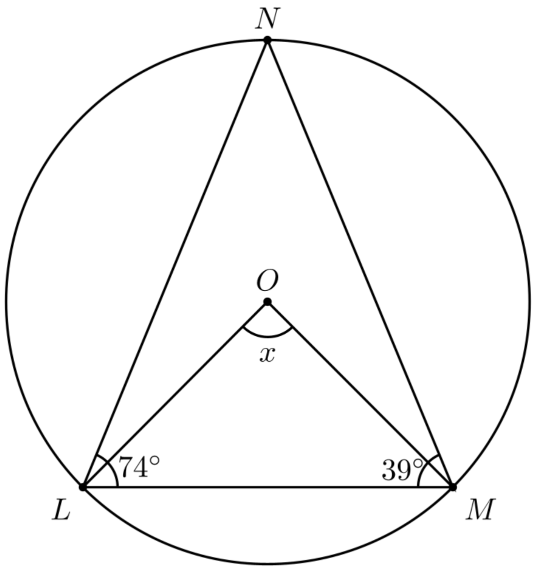
$$-2p=6 \implies p=-3.$$

Verification by factoring $x^2-3x-10$ confirms $(x+2)$ is a factor:

$$(x+2)(x-5)=x^2-3x-10.$$

Main Concepts:
1. **Factor Theorem**: Determines factors of polynomials by evaluating roots.
2. **Substitution**: Plugging $x=-2$ into the polynomial to form an equation.
3. **Solving Linear Equations**: Isolating p to find its value.
4. **Polynomial Factoring**: Verifying the result by factoring the quadratic expression.
Answer: B. -3

Problem 48



In the diagram, O is the centre of the circle. If $\angle NLM = 74°$, $\angle LMN = 39°$ and $\angle LOM = x$, find the value of x.
Possible answers:
A. 134°
B. 126°
C. 113°
D. 106°
Correct answer: A
Solution. The value of x is found by applying the Inscribed Angle Theorem and the Triangle Sum Theorem.
1. **Calculate the missing angle in triangle LMN:**

$$\angle LNM = 180° - 74° - 39° = 67°$$

2. **Apply the Inscribed Angle Theorem:**
The inscribed angle $\angle LNM$ subtends arc LM, so the measure of arc LM is:

$$2 \times 67° = 134°$$

3. **Central angle $\angle LOM$ is equal to its subtended arc LM:**

$$x = 134°$$

Answer: $\boxed{A}$
Main Concepts:
- **Inscribed Angle Theorem**: An inscribed angle is half the measure of its subtended arc.
- **Triangle Sum Theorem**: Sum of angles in a triangle equals 180°.

- Relationship between central angles and their subtended arcs.

Problem 49

Find the least value of x which satisfies the equation $4x = 7(\text{mod}9)$.

Possible answers:

A. 7

B. 6

C. 5

D. 4

Correct answer: D

Solution. To solve the equation $4x \equiv 7 \pmod 9$, we first find the multiplicative inverse of 4 modulo 9. The inverse is a number y such that $4y \equiv 1 \pmod 9$. Testing values, we find $4 \times 7 = 28 \equiv 1 \pmod 9$, so the inverse is 7. Multiplying both sides of the original equation by 7 gives:

$$x \equiv 7 \times 7 \pmod 9 \implies x \equiv 49 \pmod 9.$$

Calculating $49 \mod 9$, we get $49 - 9 \times 5 = 4$, so $x \equiv 4 \pmod 9$. The smallest positive solution is $x = 4$. Substituting $x = 4$ confirms $4 \times 4 = 16 \equiv 7 \pmod 9$. Testing other options (A, B, C) shows they do not satisfy the equation. Thus, the least value is $\boxed{D}$.

Main Concepts:

1. **Modular Arithmetic**: Solving congruences by reducing equations modulo 9.
2. **Multiplicative Inverse**: Finding the inverse of $4 \mod 9$ to isolate x.
3. **Verification by Substitution**: Checking solutions ensures correctness, especially useful for small moduli.

Problem 50

(a) If A = { multiples of 2}, B = { multiples of 3} and C = { factors of 6} are subsets of $\mu = \{x : 1 \leq x \leq 10\}$, find $A' \cap B' \cap C'$.

(b) Tickets for a movie premiere cost \$18.50 each while the bulk purchase price for 5 tickets is \$80.00. If 4 gentlemen decide to get a fifth person to join them so that they can share the bulk purchase price equally, how much would each person save?

Correct answer:
5,7
Correct answer:
2.5

Solution. To solve part (a), we first identify the universal set $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. The subsets are defined as follows:
- $A = \{2, 4, 6, 8, 10\}$ (multiples of 2)
- $B = \{3, 6, 9\}$ (multiples of 3)
- $C = \{1, 2, 3, 6\}$ (factors of 6)

Next, we find the complements of each set:
- $A' = \mu - A = \{1, 3, 5, 7, 9\}$
- $B' = \mu - B = \{1, 2, 4, 5, 7, 8, 10\}$
- $C' = \mu - C = \{4, 5, 7, 8, 9, 10\}$

The intersection of these complements $A' \cap B' \cap C'$ is the set of elements common to all three complements, which results in $\{5, 7\}$.

For part (b), we compare the cost of buying tickets individually versus in bulk. The original cost for 4 tickets is $4 \times 18.50 = 74.00$ dollars. By purchasing 5 tickets in bulk for \$80.00 and splitting the cost equally among 5 people, each person pays $80.00/5 = 16.00$ dollars. The savings per person is the difference between the individual ticket price and the shared bulk price: $18.50 - 16.00 = 2.50$ dollars.

Final Answer
(a) $\boxed{\{5, 7\}}$
(b) $\boxed{2.50}$

Main Concepts
(a) The problem involves set theory, specifically working with complements and intersections of subsets within a universal set. Key concepts include understanding set notation, determining elements of sets based on given conditions (multiples and factors), and applying De Morgan's laws to find the intersection of complements.
(b) This problem applies arithmetic and financial mathematics to calculate savings through bulk purchasing. Key concepts include calculating total costs for individual and bulk purchases, determining the cost per person when splitting the bulk price, and finding the difference (savings) between individual and shared costs.

Problem 51

(a) Given that P $= \left(\frac{rk}{Q} - ms\right)^{\frac{2}{3}}$,

(i) make Q the subject of the relation, represent the answer as $Q = \frac{rk}{p^{\frac{a}{b}}+cms}$, write the answer as a, b, c, where a, b, c are integers.

(ii) find, correct to two decimal places, the value of Q when $\mathrm{P} = 3,\ \mathrm{m} = 15,\ \mathrm{s} = 0.2, \mathrm{k} = 4$ and $\mathrm{r} = 10$.

(b) Given that $\frac{x+2y}{5} = \mathrm{x} - 2\mathrm{y}$, find x : y.

Correct answer:
3,2,1
Correct answer:
4.88
Correct answer:
3:1

Solution. **Solution:**
(a)(i)
Starting with the equation:

$$P = \left(\frac{rk}{Q} - ms\right)^{\frac{2}{3}}$$

Raise both sides to the power $\frac{3}{2}$:

$$P^{\frac{3}{2}} = \frac{rk}{Q} - ms$$

Add ms to both sides:

$$P^{\frac{3}{2}} + ms = \frac{rk}{Q}$$

Solve for Q:

$$Q = \frac{rk}{P^{\frac{3}{2}} + ms}$$

Thus, $a = 3$, $b = 2$, $c = 1$.
Answer (a)(i): $a = 3$, $b = 2$, $c = 1$.
(a)(ii)
Substitute $P = 3$, $m = 15$, $s = 0.2$, $k = 4$, $r = 10$:

$$Q = \frac{10 \cdot 4}{3^{\frac{3}{2}} + 15 \cdot 0.2}$$

Calculate $3^{\frac{3}{2}} \approx 5.196$ and $15 \cdot 0.2 = 3$:

$$Q \approx \frac{40}{5.196 + 3} \approx \frac{40}{8.196} \approx 4.88$$

Answer (a)(ii): $Q \approx 4.88$.

(b)
Given $\frac{x+2y}{5} = x - 2y$:
Multiply both sides by 5:

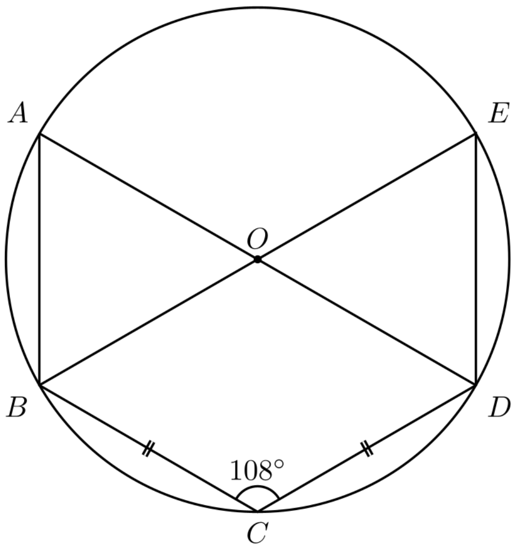
$$x + 2y = 5x - 10y$$

Rearrange terms:

$$12y = 4x \implies \frac{x}{y} = \frac{12}{4} = 3$$

Thus, $x : y = 3 : 1$.
Answer (b): $x : y = 3 : 1$.
Main Concepts:
- **Exponent Manipulation:** Raising both sides to reciprocal powers to isolate terms.
- **Algebraic Isolation:** Rearranging equations to solve for a specific variable.
- **Substitution:** Plugging numerical values into derived formulas.
- **Linear Equation Solving:** Balancing equations to find variable ratios.
- **Decimal Precision:** Applying rounding rules for final numerical answers.

Problem 52



A. In the diagram, O is the centre of the circle, $|BC| = |CD|$ and $\angle BCD = 108°$. Find $\angle CDE$.
B. Given that $\tan x = \sqrt{3}, 0° \leq x \leq 90°$, evaluate $\frac{(\cos x)^2 - \sin x}{(\sin x)^2 + \cos x}$. Represent the answer as $\frac{a+b\sqrt{3}}{c}$, where a, b, c are integers, write down the answer as a, b, c.

Correct answer:
126
Correct answer:
1,-2,5
Solution. To solve part A, we start by noting that $|BC| = |CD|$ implies triangle BCD is isolesces. Given $\angle BCD = 108°$, we have that $\angle CBD = \angle CDB$ and :

$$\angle CBD + \angle CDB + 108° = 180°.$$

Therefore, $\angle CDB = 36°$.
According to Thales' theorem, every inscribed angle based on the diameter of a circle is a right angle. That means $\angle BDE = 90°$. Hence:

$$\angle CDE = \angle CDB + \angle BDE = 36° + 90° = 126°.$$

For part B, given tan x = $\sqrt{3}$, we find x = 60. Evaluating the expression (cosx - sin x)/(sinx + cos x) with x = 60, we get:
- Numerator: (1/2) - ($\sqrt{3}$/2) = (1 - 2$\sqrt{3}$)/4
- Denominator: (3/4) + (1/2) = 5/4
Simplifying, we get (1 - 2$\sqrt{3}$)/5, leading to the integers 1, -2, 5.
Final Answer
A. $\boxed{126°}$
B. $\boxed{1, -2, 5}$
Main Concepts
- **Circle Geometry**: Understanding inscribed angles and arcs to determine angle measures.

- **Trigonometric Evaluation**: Using known angles and trigonometric identities to simplify expressions.

Problem 53

The total surface area of a cone of slant height L cm and base radius R cm is 224π cm^2. If $R : L = 2 : 5$, find: A. Correct to one decimal place, the value of R. B. Correct to the nearest whole number, the volume of the cone. [Take $\pi = \frac{22}{7}$]

Correct answer:
8
Correct answer:
1229
Solution. **Solution:**
Given the total surface area of a cone with slant height L cm and base radius R cm is $224\pi\,\text{cm}^2$, and the ratio $R : L = 2 : 5$:
Part A: Finding R
1. **Surface Area Formula**:
Total surface area $= \pi R(L + R) = 224\pi$.
Dividing both sides by π:
$R(L + R) = 224$.
2. **Substitute Ratio**:
Let $R = 2x$ and $L = 5x$. Substituting into the equation:
$2x(5x + 2x) = 224$
$2x \cdot 7x = 224$
$14x^2 = 224$
$x^2 = 16$
$x = 4$.
Thus, $R = 2x = 8.0\,\text{cm}$.
Part B: Finding the Volume
1. **Calculate Height h**:
Using the Pythagorean theorem for cones:
$L^2 = R^2 + h^2$.
Substituting $L = 20\,\text{cm}$ and $R = 8\,\text{cm}$:
$20^2 = 8^2 + h^2$
$400 = 64 + h^2$
$h = \sqrt{336} \approx 18.33\,\text{cm}$.
2. **Volume Formula**:
Volume $= \frac{1}{3}\pi R^2 h$.
Substituting $R = 8$, $h \approx 18.33$, and $\pi = \frac{22}{7}$:

$$\text{Volume} = \frac{1}{3} \cdot \frac{22}{7} \cdot 8^2 \cdot 18.33 \approx 1229\,\text{cm}^3 \text{ (nearest whole number)}.$$

Final Answers:
- **A.** $R = 8.0\,\text{cm}$
- **B.** Volume $= 1229\,\text{cm}^3$
—
Main Concepts:
1. **Surface Area of a Cone**: Combined lateral and base area formula $\pi R(L + R)$.
2. **Ratio Substitution**: Expressing variables in terms of a common ratio to simplify equations.
3. **Quadratic Equation Solving**: Derived from substituting ratios into the surface area equation.

4. **Pythagorean Theorem**: Relating slant height, radius, and vertical height.
5. **Volume Calculation**: Using $\frac{1}{3}\pi R^2 h$ with approximated π for precise results.

Problem 54

A die was rolled a number of times. The outcomes are as shown in the table.

Number	1	2	3	4	5	6
Outcomes	32	m	25	40	28	45

If the probability of obtaining 2 is 0.15, find: A. The value of m. B. The number of times the die was rolled. C. The probability of obtaining an even number, give the answer as a decimal number.

Correct answer:
30
Correct answer:
200
Correct answer:
0.575

Solution. **A. The value of m**
Given the probability of rolling a 2 is 0.15, we set up the equation:

$$\frac{m}{32+m+25+40+28+45} = 0.15$$

Simplifying the total outcomes:

$$\frac{m}{170+m} = 0.15 \implies m = 0.15(170+m)$$

Solving for m:

$$m = 25.5 + 0.15m \implies 0.85m = 25.5 \implies m = \frac{25.5}{0.85} = 30$$

Answer: $m = 30$.
B. The number of times the die was rolled
Total rolls $= 32 + 30 + 25 + 40 + 28 + 45 = 200$.
Answer: 200 times.
C. Probability of obtaining an even number
Even numbers (2, 4, 6) have outcomes $30 + 40 + 45 = 115$.
Probability:

$$\frac{115}{200} = 0.575$$

Answer: 0.575.
Main Concepts
1. **Probability Calculation**: Using Probability $= \frac{\text{Favorable Outcomes}}{\text{Total Trials}}$ to relate m to the total rolls.
2. **Linear Equation Solving**: Isolating m by manipulating the probability equation.
3. **Summing Frequencies**: Calculating total trials by summing all outcomes.
4. **Even Number Identification**: Recognizing even numbers on a die (2, 4, 6) and summing their frequencies for combined probability.

Problem 55

A. Complete the table of values for the relation y = $3\sin 2x$. Write the answer as a sequence of missing values for y coresponding to the values of x from 15° to 135°.

x	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°
y	0.0					1.5					-2.6

B. Using a scale of 2 *cm* to 15° on the x-axis and 2 *cm* to 1 unit on the y-axis, draw the graph of $y = 3\sin 2x$ for $0° \leq x \leq 150°$.

C. Use the graph to find the truth set of: (i) $3\sin 2x + 2 = 0$; (ii) $\frac{3}{2}\sin 2x = 0.25$.

Correct answer:
1.5,2.6,3,2.6,0,-1.5,-2.6,-3
Correct answer:
graph
Correct answer:
111
Correct answer:
5::85

Solution. **Answer**

A. The missing y-values for x from 15° to 135° are:
1.5, 2.6, 3.0, 2.6, 0.0, -1.5, -2.6, -3.0.

B. To draw the graph:
- **X-axis**: Use 2 cm per 15°. For example, 15° = 2 cm, 30° = 4 cm, etc.
- **Y-axis**: Use 2 cm per 1-unit. Plot points from the table and connect them smoothly to form the sine curve $y = 3\sin 2x$. The graph will oscillate between 3 and -3, completing one full cycle every 180°. See the figure below.

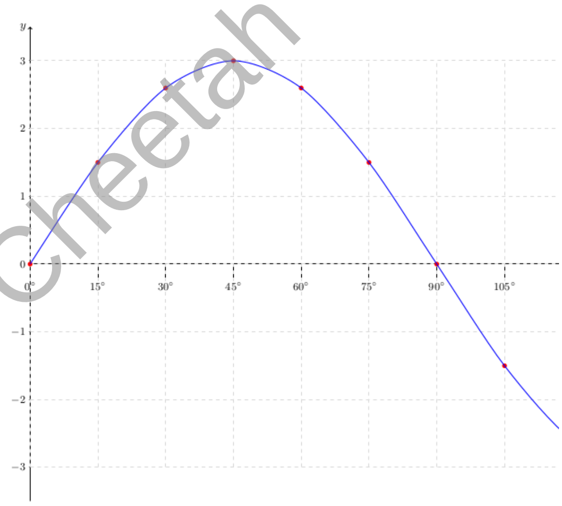


Figure 1: Part B answer

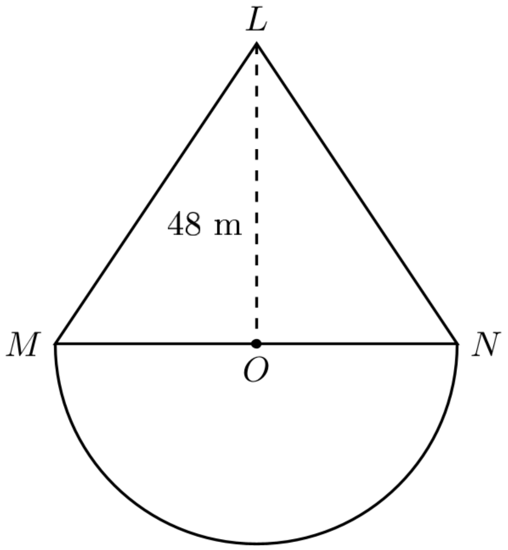
C. Truth sets using the graph:
(i) **$3\sin 2x + 2 = 0$**
Solutions occur where $y = -2$. Approximate solutions: $x \approx 111°$.

(ii) **$\frac{3}{2}\sin 2x = 0.25$**
Solutions occur where $y = 0.5$. Approximate solutions: $x \approx 5°$ and $x \approx 85°$.

—

Main Concepts
1. **Amplitude and Period**: The equation $y = 3\sin 2x$ has an amplitude of 3 (vertical stretch) and a period of 180° (horizontal compression due to $2x$).
2. **Key Angles**: Exact sine values (e.g., $\sin 30° = 0.5$) were used to compute y-values at multiples of 15°.
3. **Graph Scaling**: Converting degrees to centimeters for plotting and interpreting the sine wave visually.
4. **Equation Solving**: For $3\sin 2x + 2 = 0$, finding intersections with $y = -2$. For $\frac{3}{2}\sin 2x = 0.25$, finding intersections with $y = 0.5$, leveraging the sine function's periodicity and symmetry.

Problem 56



(a) The diagram below shows a wooden structure in the form of a cone, mounted on a hemispherical base. The vertical height of the cone is 48 m and the base radius is 14 m. Calculate, correct to three significant figures, the surface area of the structure. [Take $\pi = \frac{22}{7}$]
(b) Five years ago, Musah was twice as old as Sesay. If the sum of their ages is 100, find Sesay's present age.

Correct answer:
3430
Correct answer:
35
Solution. (a) The surface area of the structure is calculated by summing the lateral surface area of the cone and the curved surface area of the hemisphere.
- **Cone's slant height**: $l = \sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2} = 50\,\text{m}$.
- **Lateral surface area of cone**: $\pi r l = \frac{22}{7} \times 14 \times 50 = 2200\,\text{m}^2$.
- **Curved surface area of hemisphere**: $2\pi r^2 = 2 \times \frac{22}{7} \times 14^2 = 1232\,\text{m}^2$.
- **Total surface area**: $2200 + 1232 = 3432 \approx 3430\,\text{m}^2$ (to 3 significant figures).
Answer (a): $\boxed{3430\,\text{m}^2}$.
(b) Let Musah's and Sesay's current ages be M and S, respectively.
- **Equations**:
$M - 5 = 2(S - 5)$ (Five years ago)
$M + S = 100$ (Sum of ages)
- Solving: Substitute $M = 100 - S$ into the first equation:
$95 - S = 2S - 10 \Rightarrow 3S = 105 \Rightarrow S = 35$.
Answer (b): $\boxed{35\,\text{years}}$.
Key Concepts:
1. **Composite Surface Area**: Combining curved surfaces of a cone and hemisphere, excluding overlapping bases.
2. **Pythagorean Theorem**: Critical for determining the cone's slant height.
3. **Linear Systems**: Translating age conditions into algebraic equations and solving via sub-

stitution.

Problem 57

(a) Ms. Maureen spent $\frac{1}{4}$ of her monthly income at a shopping mall, $\frac{1}{3}$ at an open market and $\frac{2}{5}$ of the remaining amount at a Mechanic workshop. If she had N 225, 000.00 left, find: (i) Her monthly income. (ii) The amount spent at the open market.

(b) The third term of an arithmetic progression is $4m - 2n$. If the ninth term of the progression is $2m - 8n$, find the common difference in terms of m and n. Represent the answer as $\frac{am+bn}{c}$, where a, b, c are integers, wrire down the answer as a, b, c.

Correct answer:
900000
Correct answer:
300000
Correct answer:
-1,-3,3

Solution. (a) (i) Let x be Ms. Maureen's monthly income in Naira.
Finding Monthly Income:

1. Amount spent at the shopping mall:
$$\frac{1}{4}x$$

2. Remaining amount after shopping mall:
$$x - \frac{1}{4}x = \frac{3}{4}x$$

3. Amount spent at the open market:
$$\frac{1}{3}x$$

4. Remaining amount after open market:
$$\frac{3}{4}x - \frac{1}{3}x = \frac{9}{12}x - \frac{4}{12}x = \frac{5}{12}x$$

5. Amount spent at the mechanic workshop ($\frac{2}{5}$ of the remaining $\frac{5}{12}x$):
$$\frac{2}{5} \times \frac{5}{12}x = \frac{1}{6}x$$

6. Final remaining amount:

$$\frac{5}{12}x - \frac{1}{6}x = \frac{5}{12}x - \frac{2}{12}x = \frac{3}{12}x = \frac{1}{4}x$$

7. Given that the final remaining amount is N225,000.00:

$$\frac{1}{4}x = 225{,}000 \implies x = 225{,}000 \times 4 = \boxed{900{,}000}$$

a (ii) Amount Spent at the Open Market:

$$\text{Amount} = \frac{1}{3}x = \frac{1}{3} \times 900{,}000 = \boxed{300{,}000}$$

(b) To find the common difference of the arithmetic progression:
1. The third term $a_3 = 4m - 2n$ and the ninth term $a_9 = 2m - 8n$.
2. Using the formula for the nth term of an AP, $a_n = a_1 + (n-1)d$:
- $a_3 = a_1 + 2d = 4m - 2n$
- $a_9 = a_1 + 8d = 2m - 8n$
3. Subtracting these equations: $6d = (2m - 8n) - (4m - 2n) = -2m - 6n$
4. Solving for d: $d = \frac{-2m-6n}{6} = \frac{-m-3n}{3}$
5. Representing as $\frac{am+bn}{c}$ with integers a, b and c: The coefficients are -1, -3 and 3.
The common difference is $\boxed{-1, -3, 3}$.
Main Concepts:
1. **Fractional Expenditures and Remaining Amounts:** Understanding how successive fractional expenditures reduce the total income step-by-step.
2. **Arithmetic Progression Terms:** Using the nth term formula to set up and solve equations for the common difference.
3. **Solving Linear Equations:** Working backward from remaining amounts and solving systems of equations derived from terms of an AP.

Problem 58

Two cyclists X and Y leave town Q at the same time. Cyclist X travels at the rate of 5 km/h on a bearing of 049° and cyclist Y travels at the rate of 9 km/h on a bearing of 319°.

A. Illustrate the information on a diagram.

B. After travelling for two hours, calculate, correct to the nearest whole number:

(i) the distance between cyclist X and Y.

(ii) the bearing of cyclist X from Y.

C. If after 2 hours Y will stop and X will travel directly towards it, find the average speed at which cyclist X will get to Y in 4 hours.

Correct answer:
graph
Correct answer:
21
Correct answer:
110
Correct answer:
5

Solution. To solve the problem, we start by analyzing the movements of cyclists X and Y using bearings and trigonometry.

Part A: Diagram

Cyclist X travels at 5 km/h on a bearing of 049, which is 49 east of north. Cyclist Y travels at 9 km/h on a bearing of 319, which is 41 west of north. After 2 hours, cyclist X is 10 km from Q, and cyclist Y is 18 km from Q. The angle between their paths is 90, forming a right-angled triangle.

See the figure below.

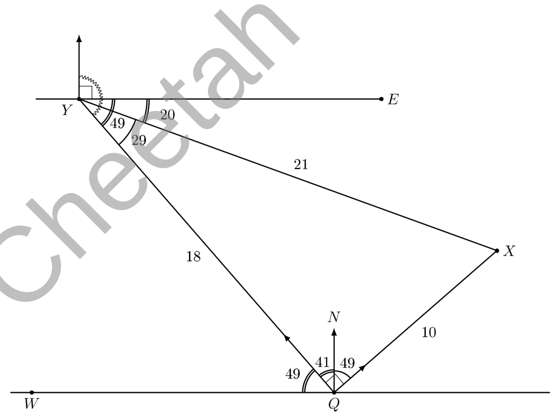


Figure 2:

Part B: Calculations

(i) Distance between cyclists after 2 hours:

Using the Pythagorean theorem:

$$\text{Distance} = \sqrt{(10\text{ km})^2 + (18\text{ km})^2} = \sqrt{100 + 324} = \sqrt{424} \approx 20.59\text{ km} \approx 21\text{ km}$$

The distance between them is $\boxed{21}$ km.
(ii) Bearing of cyclist X from Y:
Since $\angle NQX = 49^\circ$ and $\angle YQX = 90^\circ$, we have $\angle WQN = 49^\circ$ (see the diagram).
Additionally, $\angle EYQ = 49^\circ$ as angle vertical to $\angle WQNFrom\triangle QYX$:

$$\tan(\angle QYX) = \frac{10}{18}$$

$$\angle QYX = \tan^{-1}\left(\frac{10}{18}\right) = 29^\circ$$

Now, we find $\angle EYX$:

$$\angle EYX = \angle EYQ - \angle QYX$$

$$= 49^\circ - 29^\circ = 20^\circ$$

Thus, the bearing of X from Y is:

$$\angle EYX + 90^\circ = 20^\circ + 90^\circ = 110^\circ$$

$$\boxed{110^\circ}$$

Part C: Average Speed
To find the average speed for cyclist X to reach Y in 4 hours, assuming Y will stay in its place and X will travel directly towards Y. From Bi we know the distance X will have to cover in 4 hours is 21 km so

$$\text{Average speed} = \frac{21 \text{ km}}{4 \text{ h}} = 5 \text{ km/h}$$

The required average speed is $\boxed{5}$ km/h.
Main Concepts
1. **Bearings and Trigonometry**: Convert bearings to coordinate angles and use trigonometric functions.
2. **Distance Calculation**: Apply the Pythagorean theorem in right-angled triangles.
3. **Bearing Determination**: Use inverse trigonometric functions and coordinate geometry.
4. **Relative Motion**: Calculate future positions and determine required speed for interception.

Problem 59

The table below shows the distribution of marks obtained by students in an examination.

2* Marks (%)	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	7	11	17	20	29	34	30	25	21	6

A. Construct a cumulative frequency table for the distribution.
B. Draw the cumulative frequency curve for the distribution.
C. Using the curve, find correct to one decimal place:
(i) the median mark, round to an integer.
(ii) the lowest mark, round to an integer, for the distinction if 5% of the students passed with distinction.

Correct answer:

Marks	Upper Class Boundary	Frequency (F)	Cumulative Frequency (F) (CF)
0-9	9.5	7	7
10-19	19.5	11	18
20-29	29.5	17	35
30-39	39.5	20	55
40-49	49.5	29	84
50-59	59.5	34	118
60-69	69.5	30	148
70-79	79.5	25	173
80-89	89.5	21	194
90-99	99.5	6	200

Correct answer:
graph
Correct answer:
55
Correct answer:
87

Solution. **A. Cumulative Frequency Table:**
— Upper Class Boundary — Cumulative Frequency —
——————————-——————————-—
— 9 — 7 —
— 19 — 18 —
— 29 — 35 —
— 39 — 55 —
— 49 — 84 —
— 59 — 118 —
— 69 — 148 —
— 79 — 173 —
— 89 — 194 —
— 99 — 200 —
B. Cumulative Frequency Curve:
Plot the points (9, 7), (19, 18), (29, 35), (39, 55), (49, 84), (59, 118), (69, 148), (79, 173), (89, 194), (99, 200) on a graph with marks (%) on the x-axis and cumulative frequency on the y-axis. Connect the points with a smooth curve to form the ogive.

See the figure below.

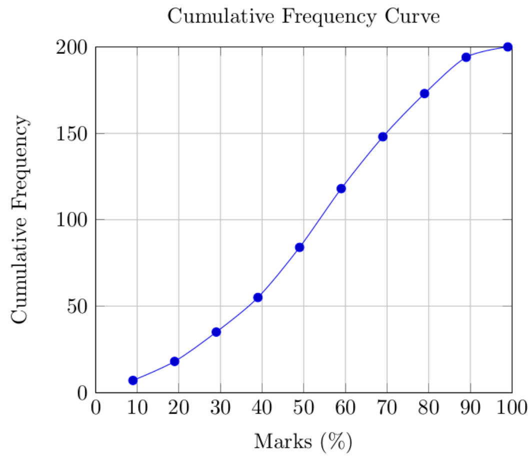


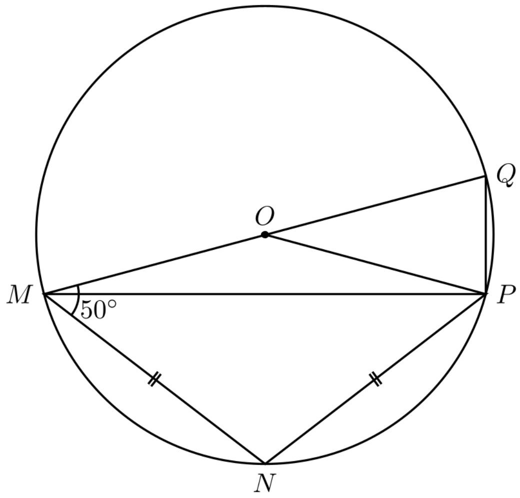
Figure 3:

C. Using the Curve:
(i) **Median Mark:** Locate $\frac{200}{2} = 100$ on the y-axis. Draw a horizontal line to the curve, then drop a vertical line to the x-axis. The median is approximately 54.7%. Round to integere, **55%**.
(ii) **Lowest Distinction Mark (Top 5%):** Locate $200 - 5\% \times 200 = 190$ on the y-axis. Draw a horizontal line to the curve, then drop a vertical line to the x-axis. The lowest distinction mark is approximately 87.1%. Round to integer, **87%**

—

Main Concepts:
1. **Cumulative Frequency:** Summing frequencies progressively to analyze data distribution.
2. **Ogive:** A curve visualizing cumulative frequencies for estimating percentiles.
3. **Median via Interpolation:** Using the median class and linear interpolation to find the middle value.
4. **Percentiles:** Determining thresholds (e.g., top 5%) by interpolating the cumulative frequency curve.
5. **Linear Interpolation:** Estimating values within intervals assuming uniform distribution.

Problem 60



(a) In the diagram, there is a circle with centre O, $|MN| = |NP|$ and $\angle OMN = 50°$. A. Find $\angle MNP$. B. Find $\angle POQ$.
(b) Find the equation of the line which has the same gradient as $8y + 4xy = 24$ and passes through the point $(-8, 12)$. Represent the answer as $ay = bx + c$, where a, b, c are integers, write down the answer as a, b, c.

Correct answer:
100
Correct answer:
20
Correct answer:
2,-1,16
Solution. To solve part (a) of the problem:
(a) A. **Finding $\angle MNP$**:
1. Given $|MN| = |NP|$, triangle MNP is isosceles with $\angle NMP = \angle NPM$.
2. Angle $OMN = 50°$. Since the sum of the opposite angles of an inscribed quadrilateral is 180 degrees, $\angle NPQ$ is 130°.
3. According to Thales' theorem, every inscribed angle based on the diameter of a circle is a right angle. Therefore, $\angle MPQ = 90°$.
4. From 3. and 4. follows that:

$$\angle NPM = 130° - 90° = 40°.$$

5. From 1. we conclude that:

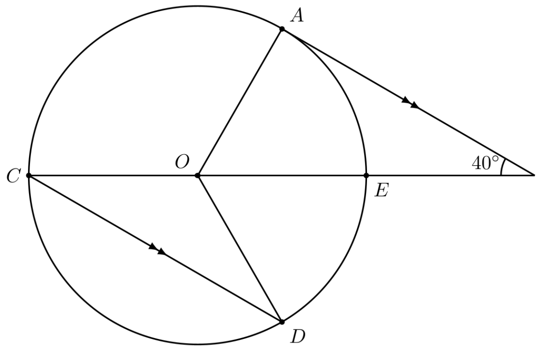
$$\angle MNP = 180° - 2 * 40° = 100°.$$

B. **Finding $\angle POQ$**:
1. Angle $MNP = 100°$. Since the sum of the opposite angles of an inscribed quadrilateral is 180 degrees, $\angle PQO$ is $80°$.
2. Given $|OQ| = |OP|$ as radii, triangle OPQ is isosceles with $\angle OPQ = \angle PQO = 80°$.
3. In triangle OPQ we have that:

$$\angle POQ + \angle PQO + \angle OPQ = 180°.$$

Therefore, $\angle POQ = 180° - \angle PQO - \angle OPQ = 180° - 2\angle PQO = 180° - 2 * 80° = 20°$.
For part (b):
(b) **Finding the equation of the line**:
1. Assuming a typo in the equation, we use 8y + 4x = 24 instead of 8y + 4xy = 24.
2. Rearranging to y = -0.5x + 3, the gradient is -0.5.
3. Using the point-slope form with point (-8, 12): y - 12 = -0.5(x + 8).
4. Converting to the form ay = bx + c: 2y = -x + 16, resulting in integers a = 2, b = -1, c = 16.
Final Answers:
(a) A. $\boxed{100°}$; B. $\boxed{20°}$
(b) The integers are $\boxed{2}$, $\boxed{-1}$, $\boxed{16}$.
Main Concepts:
- **Circle Theorems**: The sum of the opposite angles of an inscribed quadrilateral is 180 degrees, properties of isosceles triangles, Thales' theorem.
- **Linear Equations**: Gradient calculation and point-slope form for line equations.

Problem 61



A. In the diagram, AB is a tangent to the circle with centre O, and COB is a straight line. If $CD||AB$ and $\angle ABE = 40°$, find $\angle ODE$.
B. $ABCD$ is a parallelogram in which $|CD| = 7cm$, $|AD| = 5cm$ and $\angle ADC = 125°$.
(i) Illustrate the information in a diagram.
(ii) Find, correct to one decimal place, the area of the parallelogram.
C. If $x = \frac{1}{2}(1 - \sqrt{2})$. Evaluate $(2x^2 - 2x)$. Represent the answer as $a + b\sqrt{2}$, where a, b are decimal numbers, write down the answer as a, b.

Correct answer:
50
Correct answer:
graph
Correct answer:
28.7
Correct answer:
0.5,0
Solution. **A. Solution:**
Given AB is tangent at B, $CD||AB$, and $\angle ABE = 40°$:
1. **Parallel Lines:** $\angle BED = \angle DCE = 40°$ (corresponding angles).
2. **Central Angle:** Arc DE $= 2 \times \angle DCE = 80°$, so central angle $\angle DOE = 80°$.
3. **Isosceles Triangle:** In triangle ODE, $\angle ODE = (180° - 80°)/2 = 50°$.
Answer: $\boxed{50°}$
B. Solution:
(i) Diagram: Parallelogram ABCD with AD = 5 cm, DC = 7 cm, $\angle ADC = 125°$.
See the figure below.

(ii) Area Calculation:

$$\text{Area} = |AD \cdot DC \cdot \sin(125°)| = 5 \cdot 7 \cdot \sin(125°) \approx 28.7\,\text{cm}^2$$

Answer: $\boxed{28.7}$
C. Solution:
Given $x = \frac{1-\sqrt{2}}{2}$:

$$2x^2 - 2x = 2\left(\frac{3-2\sqrt{2}}{4}\right) - 2\left(\frac{1-\sqrt{2}}{2}\right) = \frac{3-2\sqrt{2}}{2} - (1-\sqrt{2}) = 0.5$$

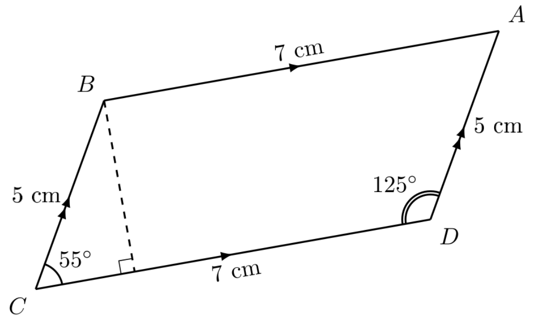


Figure 4: Part B answer

Expressed as $0.5 + 0\sqrt{2}$, so $a = 0.5, b = 0$.
Answer: $\boxed{0.5, 0}$
—
Main Concepts:
- **Problem A:** Tangent-chord angle theorem, parallel lines, central/inscribed angles, isosceles triangle properties.
- **Problem B:** Parallelogram properties, area via sides and included angle.
- **Problem C:** Algebraic expansion, simplifying radical expressions.

Problem 62

Tickets for a movie premiere cost \$18.50 each while the bulk purchase price for 5 tickets is \$80.00. If 4 gentlemen decide to get a fifth person to join them so that they can share the bulk purchase price equally, how much would each person save?

Correct answer:
2.5

Solution. To solve the problem, we first calculate the cost per person if the four gentlemen bought individual tickets and compare it to the cost when they split the bulk purchase with a fifth person.
Step 1: Individual Cost for Four Tickets
Each ticket costs \$18.50. For 4 people:

$$4 \times 18.50 = 74.00 \quad \text{(Total)}$$

Each person would pay \$18.50.
Step 2: Bulk Purchase for Five Tickets
The bulk price for 5 tickets is \$80.00. When split equally among 5 people:

$$80.00 \div 5 = 16.00 \quad \text{(Per person)}$$

Step 3: Savings Per Person
The difference between the individual price and the bulk share:

$$18.50 - 16.00 = 2.50$$

Conclusion
By purchasing the bulk package and splitting the cost equally, **each person saves \$2.50**.
Main Concepts
1. **Unit Price Comparison**: Comparing the cost per ticket when buying individually vs. in bulk.
2. **Equal Distribution**: Dividing the total bulk cost among all participants to determine the shared price per person.
3. **Savings Calculation**: Subtracting the bulk share price from the individual price to quantify savings.
The problem emphasizes understanding bulk discounts, collaborative cost-sharing, and basic arithmetic operations to evaluate financial benefits.

Problem 63
Five years ago, Musah was twice as old as Sesay. If the sum of their ages is 100, find Sesay's present age.

Correct answer:
35
Solution. Let M be Musah's current age and S be Sesay's current age.
1. **Five years ago**, Musah was twice as old as Sesay:

$$M - 5 = 2(S - 5)$$

Expanding and simplifying:

$$M - 5 = 2S - 10 \implies M = 2S - 5$$

2. **Sum of their current ages** is 100:

$$M + S = 100$$

Substitute $M = 2S - 5$ into this equation:

$$(2S - 5) + S = 100 \implies 3S - 5 = 100 \implies 3S = 105 \implies S = 35$$

Verification:
- Sesay's current age: 35
- Musah's current age: $100 - 35 = 65$
- Five years ago: Sesay was 30, Musah was 60. $60 = 2 \times 30$, which satisfies the condition.
Sesay's present age is $\boxed{35}$.
—
Main Concepts:
1. **Variable Assignment**: Representing unknown quantities (ages) with variables.
2. **Equation Formation**: Translating worded age relationships into algebraic equations.
3. **System of Equations**: Solving using substitution to express one variable in terms of another.
4. **Verification**: Checking solutions against original problem constraints to ensure validity.

Problem 64

The third term of an arithmetic progression is $4m-2n$. If the ninth term of the progression is $2m-8n$, find the common difference in terms of m and n. Represent the answer as $\frac{am+bn}{c}$, where a,b,c are integers, wrire down the answer as a,b,c.

Correct answer:
-1,-3,3

Solution. The third term of an arithmetic progression (AP) is given as $4m-2n$, and the ninth term is $2m-8n$. To find the common difference d in terms of m and n, we use the formula for the n-th term of an AP:

$$a_k = a_1 + (k-1)d$$

For the third term ($k=3$):

$$a_1 + 2d = 4m - 2n \quad (1)$$

For the ninth term ($k=9$):

$$a_1 + 8d = 2m - 8n \quad (2)$$

Subtracting equation (1) from (2) eliminates a_1:

$$6d = (2m-8n) - (4m-2n)$$

$$6d = -2m - 6n$$

$$d = \frac{-2m-6n}{6} = \frac{-m-3n}{3} **Main Concepts** : 1. **Arithmetic Progression** : Terms are separated by a constant di$$

Problem 65

Find the equation of the line which has the same gradient as $8y + 4x = 24$ and passes through the point $(-8, 12)$. Represent the answer as $ay = bx + c$, where a, b, c are integers, write down the answer as a, b, c.

Correct answer:
2,-1,16

Solution. The equation of the line with the same gradient as $8y + 4x = 24$ and passing through $(-8, 12)$ is found as follows:
1. **Analyze the Equation:** The original equation is linear.
2. **Find the Gradient:** Rearrange to $y = -\frac{1}{2}x + 3$, giving a gradient of $-\frac{1}{2}$.
3. **Use Point-Slope Form:** With point $(-8, 12)$ and slope $-\frac{1}{2}$, the equation is $y - 12 = -\frac{1}{2}(x + 8)$.
4. **Simplify to Standard Form:** Multiply by 2 to clear the fraction: $2y - 24 = -x - 8$. Rearrange to $2y = -x + 16$.
Answer: $2, -1, 16$
Main Concepts:
1. **Linear Equation Identification:** The equation is linear.
2. **Slope Determination:** Converting to slope-intercept form to find the gradient.
3. **Point-Slope Application:** Using the given point and slope to form the line equation.
4. **Algebraic Rearrangement:** Adjusting the equation to the required format and verifying correctness.

Problem 66

If $x = \frac{1}{2}(1 - \sqrt{2})$. Evaluate $(2x^2 - 2x)$. Represent the answer as $a + b\sqrt{2}$, where a, b are decimal numbers, write down the answer as a, b.

Correct answer:
0.5,0

Solution. To evaluate $2x^2 - 2x$ where $x = \frac{1}{2}(1 - \sqrt{2})$:
1. **Substitute x into the expression**:

$$x = \frac{1 - \sqrt{2}}{2}$$

2. **Compute x^2**:

$$x^2 = \left(\frac{1 - \sqrt{2}}{2}\right)^2 = \frac{(1 - \sqrt{2})^2}{4} = \frac{1 - 2\sqrt{2} + 2}{4} = \frac{3 - 2\sqrt{2}}{4}$$

3. **Calculate $2x^2$**:

$$2x^2 = 2 \cdot \frac{3 - 2\sqrt{2}}{4} = \frac{3 - 2\sqrt{2}}{2}$$

4. **Compute $-2x$**:

$$-2x = -2 \cdot \frac{1 - \sqrt{2}}{2} = -(1 - \sqrt{2}) = -1 + \sqrt{2}$$

5. **Combine the results**:

$$2x^2 - 2x = \frac{3 - 2\sqrt{2}}{2} + (-1 + \sqrt{2})$$

Convert $-1 + \sqrt{2}$ to a fraction with denominator 2:

$$-1 + \sqrt{2} = -\frac{2}{2} + \frac{2\sqrt{2}}{2} = \frac{-2 + 2\sqrt{2}}{2}$$

Add the fractions:

$$\frac{3 - 2\sqrt{2}}{2} + \frac{-2 + 2\sqrt{2}}{2} = \frac{1}{2}$$

The result simplifies to $\frac{1}{2}$, which is $0.5 + 0\sqrt{2}$. Thus, $a = 0.5$ and $b = 0$.
Main Concepts:
1. **Substitution**: Inserting the given x into the expression.
2. **Binomial Expansion**: Expanding $(1 - \sqrt{2})^2$ to compute x^2.
3. **Simplification**: Combining like terms and reducing fractions.
4. **Common Denominators**: Aligning terms for addition by expressing them with the same denominator.
5. **Verification**: Ensuring correctness through algebraic and numerical checks.

Answer:
0.5,0