1 Solutions to problems public WAEC math 2019 By cheetahwaec.com

Problem 1
(a) Solve the inequality $\frac{1+4x}{2} - \frac{5+2x}{7} < x - 2$. Represent solution as inequality $x < \alpha$, where α is a decimal number. Give the answer as α.
(b) If $x : y = 3 : 5$, find the value of $\frac{2x^2-y^2}{y^2-x^2}$. Represent solution as $\frac{\alpha}{\beta}$, give the answer as α, β.

Correct answer:
-2.5
Correct answer:
-7,16

Solution. (a) To solve the inequality $\frac{1+4x}{2} - \frac{5+2x}{7} < x - 2$, we start by combining the fractions on the left side. Finding a common denominator of 14, we rewrite the inequality as:

$$\frac{7(1+4x) - 2(5+2x)}{14} < x - 2$$

Simplifying the numerator:

$$7 + 28x - 10 - 4x = -3 + 24x$$

Thus, the inequality becomes:

$$\frac{-3 + 24x}{14} < x - 2$$

Multiplying both sides by 14 to clear the denominator:

$$-3 + 24x < 14x - 28$$

Subtracting $14x$ from both sides:

$$10x - 3 < -28$$

Adding 3 to both sides:

$$10x < -25$$

Dividing by 10:

$$x < -2.5$$

So, α is -2.5.
(b) Given $x : y = 3 : 5$, we express x and y as $3k$ and $5k$ respectively. Substituting these into the expression $\frac{2x^2-y^2}{y^2-x^2}$:

Numerator:

$$2(3k)^2 - (5k)^2 = 2 \cdot 9k^2 - 25k^2 = 18k^2 - 25k^2 = -7k^2$$

Denominator:

$$(5k)^2 - (3k)^2 = 25k^2 - 9k^2 = 16k^2$$

Simplifying the fraction:

$$\frac{-7k^2}{16k^2} = \frac{-7}{16}$$

Thus, α is -7 and β is 16.
Main Concepts
- **Solving Linear Inequalities**: Combining like terms, finding common denominators, and manipulating inequalities.
- **Ratios and Proportions**: Expressing variables in terms of a common constant and simplifying algebraic fractions.
Final Answer
(a) $\boxed{-2.5}$
(b) $\boxed{-7}$, $\boxed{16}$

Problem 2

The second, fourth and sixth terms of an arithmetic progression are $x-1, x+1$ and 7 respectively. Find the:(a) common difference;(b) first term;(c) value of x.

Correct answer:
1
Correct answer:
2
Correct answer:
4

Solution. (a) **Common difference**:
The second, fourth, and sixth terms of the arithmetic progression (AP) are $x-1$, $x+1$, and 7, respectively. Using the AP formula $a_n = a_1 + (n-1)d$:
- Second term: $a_1 + d = x - 1$
- Fourth term: $a_1 + 3d = x + 1$
Subtracting these equations eliminates a_1:

$$(a_1 + 3d) - (a_1 + d) = (x+1) - (x-1) \implies 2d = 2 \implies d = 1.$$

Common difference: $d = 1$.
(b) **First term**:
From the sixth term: $a_1 + 5d = 7$. Substituting $d = 1$:

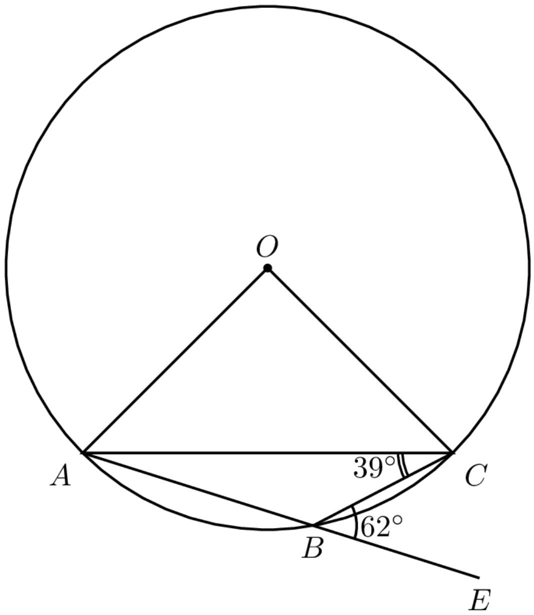
$$a_1 + 5(1) = 7 \implies a_1 = 2.$$

First term: $a_1 = 2$.
(c) **Value of x**:
Using the second term equation $a_1 + d = x - 1$ with $a_1 = 2$ and $d = 1$:

$$2 + 1 = x - 1 \implies x = 4.$$

Value of x: $x = 4$.
—
Main Concepts:
1. **Arithmetic Progression (AP) Structure**: Terms are generated by adding a constant common difference (d) to the previous term.
2. **Term Formula**: The n-th term is $a_n = a_1 + (n-1)d$, used to express given terms (second, fourth, sixth).
3. **System of Equations**: Setting up equations based on the given terms and solving for unknowns (a_1, d, x).
4. **Elimination Method**: Subtracting equations to eliminate a_1 and directly solve for d.
5. **Verification**: Substituting results back into original conditions to ensure consistency.

Problem 3



(a) If log_{10} a = 1.3010 and $\log_{10}$ b = 1.4771, find, correct to one decimal place, the value of ab.
(b) In the diagram below, O is the centre of the circle, ∠ACB = 39° and ∠CBE = 62°. Find: (i) the interior angle AOC;(ii) angle BAC.

Correct answer:
599.9
Correct answer:
124
Correct answer:
23

Solution. (a) Given $\log_{10} a = 1.3010$ and $\log_{10} b = 1.4771$, we use the property of logarithms that $\log_{10}(ab) = \log_{10} a + \log_{10} b$. Adding the given logarithms:

$$\log_{10}(ab) = 1.3010 + 1.4771 = 2.7781$$

Converting this back to exponential form, we find:

$$ab = 10^{2.7781} \approx 599.9$$

Thus, the value of ab correct to one decimal place is $\boxed{599.9}$.
(b) In the diagram, O is the centre of the circle, $\angle ACB = 39°$, and $\angle CBE = 62°$.
(i) The interior angle $\angle AOC$:
- $\angle CBE = 62°$ is supplementary to $\angle ABC$, so $\angle ABC = 180° - 62° = 118°$.
- $\angle ABC$ is an inscribed angle subtended by arc AC, so the measure of arc AC is $2 \times 118° = 236°$. It means that the reflex angle AOC is 236°
- The central angle $\angle AOC$ is equal to the complement of the reflex angle AOC:

$$\angle AOC = 360^\circ - 236^\circ = 124^\circ$$

Thus, the interior angle $\angle AOC$ is $\boxed{124^\circ}$.
(ii) Angle $\angle BAC$:
- In $\triangle ABC$, we know $\angle ACB = 39^\circ$ and $\angle ABC = 118^\circ$.
- Using the triangle angle sum theorem, $\angle BAC = 180^\circ - 39^\circ - 118^\circ = 23^\circ$.

Thus, angle $\angle BAC$ is $\boxed{23^\circ}$.
Main Concepts
(a) **Logarithm Properties**: Utilized the logarithmic identity to convert the sum of logarithms into the logarithm of a product, then converted back to find the product.
(b) **Circle Theorems**:
- **Inscribed Angle Theorem**: Determined the measure of arcs subtended by inscribed angles.
- **Central Angle**: Related the central angle to the measure of its subtended arc.
- **Supplementary Angles**: Used linear pairs to find internal angles from external ones.
- **Triangle Angle Sum**: Verified the consistency of angles in a triangle.

Problem 4

(a) Without using mathematical tables or calculator, simplify and give the the answer as a decimal number: $\frac{\log_2 8+\log_2 16-4\log_2 2}{\log_4 16}$.
(b) If $1342_{\text{five}} - 241_{\text{five}} = x_{\text{ten}}$, find the value of x.

Correct answer:
1.5
Correct answer:
151
Solution. **Solution:**
(a)
We start by simplifying the numerator and denominator separately using logarithm properties.
Numerator:

$$\log_2 8 + \log_2 16 - 4\log_2 2$$

- $\log_2 8 = 3$ (since $2^3 = 8$),
- $\log_2 16 = 4$ (since $2^4 = 16$),
- $4\log_2 2 = 4 \times 1 = 4$.
Adding these:

$$3 + 4 - 4 = 3.$$

Denominator:

$$\log_4 16$$

- $16 = 4^2$, so $\log_4 16 = 2$.
Final simplification:

$$\frac{3}{2} = 1.5.$$

Answer: $\boxed{1.5}$
(b)
Convert 1342_{five} and 241_{five} to base ten:
- $1342_{\text{five}} = 1 \cdot 5^3 + 3 \cdot 5^2 + 4 \cdot 5 + 2 = 125 + 75 + 20 + 2 = 222_{\text{ten}}$.
- $241_{\text{five}} = 2 \cdot 5^2 + 4 \cdot 5 + 1 = 50 + 20 + 1 = 71_{\text{ten}}$.
Subtract in base ten:

$$222 - 71 = 151.$$

Answer: $\boxed{151}$
—
Main Concepts:
- **Logarithmic Properties (Part a):**
- Power rule: $\log_b(a^n) = n\log_b a$.

- Base conversion: Recognizing $4 = 2^2$ simplifies $\log_4 16$.
- Simplification using $\log_b b = 1$.
- **Base Conversion (Part b):**
- Expanding digits as powers of the base (e.g., $1342_{\text{five}} = 1 \cdot 5^3 + 3 \cdot 5^2 + 4 \cdot 5 + 2$).
- Arithmetic in base ten after conversion ensures accuracy.
- Validation by performing subtraction directly in base five yielded the same result.

Problem 5

The cost of dinner for a group of tourist is partly constant and partly varies as the number of tourists present. It costs \$740.00 when 20 tourists were present and \$960.00 when the number of tourists increased by 10. Find the cost of the dinner when only 15 tourists were present.

Correct answer:
630

Solution. To solve the problem, we first recognize the total cost (C) comprises a fixed cost (F) and a variable cost per tourist (V), modeled as:

$$C = F + V \cdot n$$

Given:
1. When $n = 20$, $C = 740$:

$$740 = F + 20V \quad \text{(Equation 1)}$$

2. When $n = 30$, $C = 960$:

$$960 = F + 30V \quad \text{(Equation 2)}$$

Step 1: Solve for V (variable cost per tourist)
Subtract Equation 1 from Equation 2 to eliminate F:

$$960 - 740 = (F + 30V) - (F + 20V) \implies 220 = 10V \implies V = 22$$

Step 2: Solve for F (fixed cost)
Substitute $V = 22$ into Equation 1:

$$740 = F + 20(22) \implies 740 = F + 440 \implies F = 300$$

Step 3: Calculate cost for $n = 15$ tourists
Use $C = F + V \cdot n$:

$$C = 300 + 22 \cdot 15 = 300 + 330 = 630$$

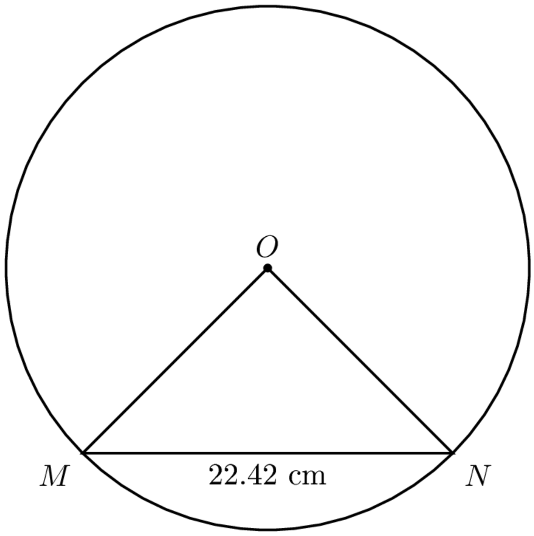
Final Answer
The cost of dinner for 15 tourists is $\boxed{630}$ dollars.
Main Concepts:
1. **Linear Cost Model**: The total cost is expressed as a linear combination of a fixed cost and a variable cost proportional to the number of tourists.
2. **System of Equations**: Two equations were set up using the given scenarios and solved simultaneously to find F and V.
3. **Substitution and Elimination**: Arithmetic operations were used to isolate variables and determine their values.
4. **Direct Variation**: The variable cost component directly depends on the number of tourists.

Problem 6

(a) Fred bought a car for $\$5,600.00$ and later sold it at 90% of the cost price. He spent $\$1,310.00$ out of the amount received and invested the rest at 6% per annum simple interest. Calculate the interest earned in 3 years. Give the answer as a decimal number.
(b) Solve the equations $2^x\left(4^{-7}\right) = 2$ and $3^{-x}\left(9^{2y}\right) = 3$ simultaneously. Write the answer as (x, y).

Correct answer:
671.4
Correct answer:
15,4
Solution. **Problem (a) Solution:**
1. **Selling Price Calculation:**
- Fred sold the car at 90% of the cost price:
$5600 \times 0.9 = \$5040$.
2. **Amount Invested:**
- After spending $1,310:
$5040 - 1310 = \$3730$.
3. **Simple Interest Earned:**
- Using the formula Interest $= P \times R \times T$:
$3730 \times 0.06 \times 3 = \$671.40$.
Final Answer (a):
The interest earned is **$671.40**.
—
Problem (b) Solution:
1. **Solve $2^x \cdot 4^{-7} = 2$:**
- Express 4^{-7} as 2^{-14}:
$2^x \cdot 2^{-14} = 2^1$.
Simplify: $2^{x-14} = 2^1$.
Equate exponents: $x - 14 = 1 \Rightarrow x = 15$.
2. **Solve $3^{-x} \cdot 9^{2y} = 3$:**
- Express 9^{2y} as 3^{4y}:
$3^{-x} \cdot 3^{4y} = 3^1$.
Simplify: $3^{-x+4y} = 3^1$.
Equate exponents: $-x + 4y = 1$.
Substitute $x = 15$:
$-15 + 4y = 1 \Rightarrow y = 4$.
Final Answer (b):
The solutions are $\boxed{x = 15}$ and $\boxed{y = 4}$.
—
Main Concepts:
- **Part (a):** Percentage calculations, subtraction to determine investment amount, simple interest formula.
- **Part (b):** Rewriting terms with common bases, exponent rules ($a^m \cdot a^n = a^{m+n}$), solving exponential equations by equating exponents, substitution in systems of equations.

Problem 7



In the diagram below, there is a chord of a circle with centre 0. $|MN|$ is 22 .42 cm and the perimeter of triangle MON is 55.6 cm. Calculate, correct to the nearest degree, $\angle MON$.
(b) T is equidistant from P and Q. The bearing of P from T is 60° and the bearing of Q from T is 130°.(i) Illustrate the information on a diagram.(ii) Find the bearing of Q from P.

Correct answer:
85
Correct answer:
graph
Correct answer:
185
Solution. To solve part (a), we need to find the angle $\angle MON$ given that the chord MN is 22.42 cm and the perimeter of triangle MON is 55.6 cm.
1. **Find the radius r of the circle:**

$$2r + 22.42 = 55.6 \implies 2r = 33.18 \implies r = 16.59 \text{ cm}$$

2. **Apply the Law of Cosines to find $\angle MON$:**

$$MN^2 = OM^2 + ON^2 - 2 \cdot OM \cdot ON \cdot \cos(\angle MON)$$

Substituting the values:

$$22.42^2 = 16.59^2 + 16.59^2 - 2 \cdot 16.59^2 \cdot \cos(\angle MON)$$

$$502.6564 = 550.4562 - 550.4562 \cdot \cos(\angle MON)$$

$$\cos(\angle MON) = \frac{550.4562 - 502.6564}{550.4562} \approx 0.08684$$

$$\angle MON \approx \arccos(0.08684) \approx 85.02^\circ \approx 85^\circ \text{ (nearest degree)}$$

For part (b)(i), see the diagram below.

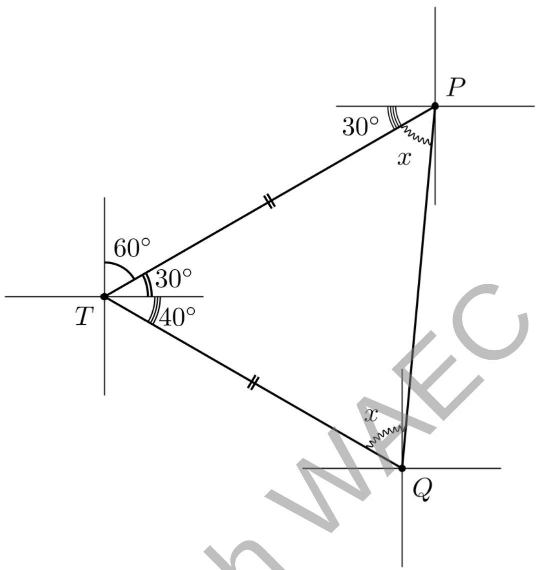


Figure 1: Part b(i)

For part (b)(ii), we need to find the bearing of Q from P:
1. **Convert bearings to coordinates:**
- P is at bearing 60 from T: coordinates $(d\sin(60^\circ), d\cos(60^\circ))$
- Q is at bearing 130 from T: coordinates $(d\cos(40^\circ), -d\sin(40^\circ))$
2. **Calculate the vector $\overrightarrow{PQ}$:**

$$\overrightarrow{PQ} = (d\cos(40^\circ) - d\sin(60^\circ), -d\sin(40^\circ) - d\cos(60^\circ))$$

$$\approx (-0.1000d, -1.1428d)$$

3. **Determine the bearing of Q from P:**
- The reference angle $\alpha = \arctan\left(\frac{0.1000}{1.1428}\right) \approx 5^\circ$
- Bearing is $180^\circ + 5^\circ = 185^\circ$
Main Concepts:
- **Part (a):** Perimeter calculation, Law of Cosines in an isosceles triangle.
- **Part (b):** Bearings conversion to coordinates, vector subtraction, and angle calculation.

Final Answer
(a) $\boxed{85^\circ}$
(b)(ii) $\boxed{185^\circ}$

Problem 8

The ages of a group of athletes are as follows: $18, 16.18, 20, 17, 16, 19, 17, 18, 17$ and 15.

A. Find the range of the distribution.B. Draw a frequency distribution table for the data.C. What is the median age?D. Calculate, correct to two decimal places, the:(i) mean age.(ii) standard deviation.

Correct answer:
5
Correct answer:
graph
Correct answer:
17
Correct answer:
17.36
Correct answer:
1.37

Solution. **A. Range of the Distribution**

The range is calculated as the difference between the highest and lowest values:
Range $= 20 - 15 = 5$.

B. Frequency Distribution Table

— Age — Frequency —
——————-—
— 15 — 1 —
— 16 — 2 —
— 17 — 3 —
— 18 — 3 —
— 19 — 1 —
— 20 — 1 —

C. Median Age

The median is the middle value when the data is ordered. For the 11 ages:
Ordered data: $15, 16, 16, 17, 17, 17, 18, 18, 18, 19, 20$.
Median (6th term) $= 17$.

D. Mean and Standard Deviation

(i) Mean Age

$$\text{Mean} = \frac{\text{Sum of ages}}{n} = \frac{191}{11} \approx 17.36 \,(\text{to two decimal places}).$$

(ii) Standard Deviation

1. Compute squared deviations from the mean:

$$\sum (x_i - \bar{x})^2 \approx 20.54.$$

2. Population variance:

$$\sigma^2 = \frac{20.54}{11} \approx 1.87.$$

3. Population standard deviation:

$$\sigma = \sqrt{1.87} \approx 1.37 \, \text{(to two decimal places)}.$$

Main Concepts
- **Range**: Measures spread by subtracting the minimum from the maximum.
- **Frequency Distribution**: Summarizes data by counting occurrences of each value.
- **Median**: The middle value in an ordered dataset; splits the data into two equal halves.
- **Mean**: Average value, calculated as the sum of all values divided by the number of values.
- **Standard Deviation**: Quantifies dispersion around the mean using squared deviations. Population standard deviation divides by n, while sample standard deviation uses $n - 1$. Here, the population formula was applied since the data represents the entire group.

Problem 9

A survey of 40 students showed that 23 students study Mathematics, 5 study Mathematics and Physics, 8 study Chemistry and Mathematics, 5 study Physics and Chemistry and 3 study all the three subjects. The number of students who study Physics only is twice the number who study Chemistry only.(a) Find the number of students who study:(i) only Physics.(ii) only one subject.b) What is the probability that a student selected at random studies exactly 2 subjects? Give the answer as a decimal number.

Correct answer:
10
Correct answer:
28
Correct answer:
0.225
Solution. **Solution:**
Given:
- Total students = 40
- Mathematics (M) = 23
- MP = 5, CM = 8, PC = 5
- All three subjects (MPC) = 3
- Physics only (b) = 2 Chemistry only (c)
Key Steps:
1. **Define Venn Diagram Regions:**
- Only Mathematics = a
- Only Physics = b
- Only Chemistry = c
- MP only = d
- CM only = e
- PC only = f
- All three = $g = 3$

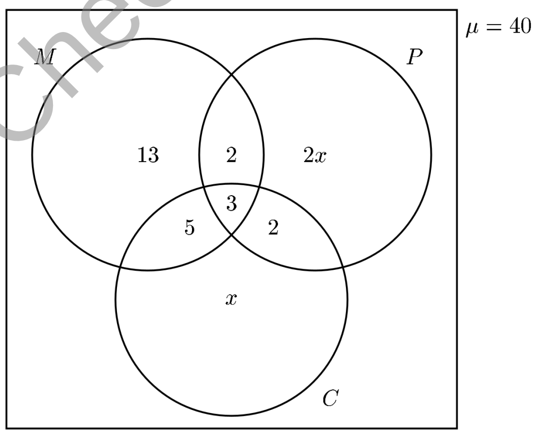


Figure 2:

2. **Solve for Overlapping Regions:**
- $d = \mathrm{MP} - g = 5 - 3 = 2$
- $e = \mathrm{CM} - g = 8 - 3 = 5$
- $f = \mathrm{PC} - g = 5 - 3 = 2$
3. **Total Mathematics Students:**
- $a + d + e + g = 23$
- $a + 2 + 5 + 3 = 23 \implies a = 13$
4. **Solve for Physics Only (b) and Chemistry Only (c):**
- Total students: $a + b + c + d + e + f + g = 40$
- $13 + b + c + 2 + 5 + 2 + 3 = 40 \implies b + c = 15$
- Given $b = 2c$, substitute: $2c + c = 15 \implies c = 5$, $b = 10$
Results:
- (a)(i) Only Physics = **10 students**
- (a)(ii) Only one subject $= a + b + c = 13 + 10 + 5 = **28 students**$
- (b) Probability of studying exactly two subjects $= \frac{d+e+f}{40} = \frac{2+5+2}{40} = \frac{9}{40} = **0.225**$
Main Concepts:
- **Venn Diagrams**: Visualizing overlaps between three sets.
- **Set Equations**: Translating problem constraints into algebraic equations for each region.
- **Substitution**: Using ratios (e.g., $b = 2c$) to solve for unknowns.
- **Probability**: Calculating favorable outcomes over total outcomes for exact criteria.

Problem 10

A. A twenty kilogram bag of rice is consumed by m number of boys in 10 days. When four more boys joined them, the same quantity of rice lasted only 8 days. If the rate of consumption is the same, find the value of m.
B. Suppose that $\frac{5}{6}$ of a number is 10 greater than $\frac{1}{3}$ of it, find the number.
C. Find the equation of the line which passes through the points $(\ 2, \frac{1}{2}\ )$ and $(\ -1, -\frac{1}{2})$. Reprsent solution as $ax + by + c = 0$. Give the answer in the form of a, b, c.

Correct answer:
16
Correct answer:
20
Correct answer:
2,-6,-1
Solution. **Problem A:**
Let r be the daily consumption per boy. The total consumption is $20 = 10mr$ and $20 = 8(m+4)r$. Equating and solving:

$$10m = 8(m+4) \implies 2m = 32 \implies m = 16.$$

Problem B:
Let x be the number. Then:

$$\frac{5}{6}x = \frac{1}{3}x + 10 \implies \frac{1}{2}x = 10 \implies x = 20.$$

Problem C:
Slope $m = \frac{-\frac{1}{2} - \frac{1}{2}}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$. Using point-slope:

$$y - \frac{1}{2} = \frac{1}{3}(x - 2) \implies 2x - 6y - 1 = 0.$$

Coefficients: $2, -6, -1$.
Final Answers:
A. $\boxed{16}$
B. $\boxed{20}$
C. $\boxed{2, -6, -1}$
Main Concepts:
- **A**: Rate proportionality and solving linear equations.
- **B**: Translating word problems into algebraic equations and solving linear equations.
- **C**: Calculating slope, point-slope form, and standard form of a line.

Problem 11

A ladder 11 m long leans against a vertical wall at an angle of 75° to the ground. The ladder is the pushed 0.2 m up the wall.(a) Illustrate the information in a diagram.(b) Find correct to the nearest whole number:(i) The new angle which the ladder makes with the ground.(ii) The distance the foot of the ladder has moved from its original position.

Correct answer:
graph
Correct answer:
80
Correct answer:
1
Solution. **Solution:**
(a) Diagram:
Draw two right-angled triangles sharing the hypotenuse (11 m ladder). The original triangle has an angle of 75° with the ground. The new triangle is pushed 0.2 m up the wall, reducing the base length. Label the original and new heights, bases, and angles.
See the figure below.

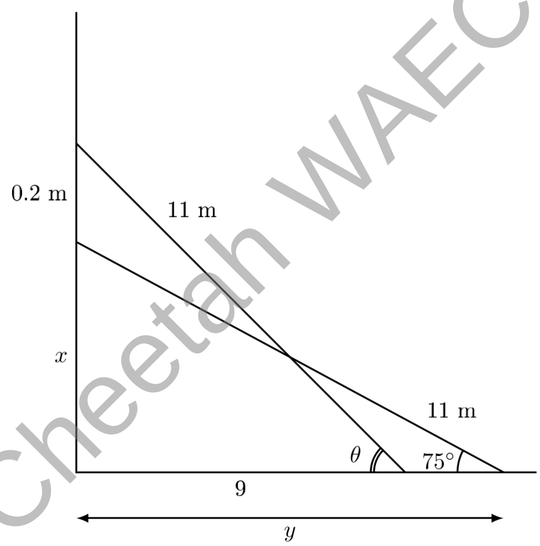


Figure 3: Part a

(b) Calculations:
(i) New Angle with the Ground:
1. Original height: $h = 11 \sin 75° \approx 10.625\,\text{m}$.
2. New height: $h' = 10.625 + 0.2 = 10.825\,\text{m}$.
3. New angle: $\theta = \arcsin\left(\frac{10.825}{11}\right) \approx \arcsin(0.9841) \approx 79.78°$.
4. Rounded to the nearest whole number: $\boxed{80°}$.
(ii) Distance Foot of Ladder Moved:
1. Original base: $b = 11 \cos 75° \approx 2.847\,\text{m}$.

2. New base: $b' = 11 \cos 79.78^\circ \approx 1.954\,\text{m}$.
3. Distance moved: $2.847 - 1.954 \approx 0.893\,\text{m}$.
4. Rounded to the nearest whole number: $\boxed{1\,\text{m}}$.

Main Concepts:
- **Trigonometric Ratios**: Used sine and cosine to relate angles to side lengths in right triangles.
- **Inverse Trigonometric Functions**: Applied arcsin to determine the new angle.
- **Pythagorean Theorem**: Verified base lengths using $c^2 = a^2 + b^2$.
- **Rounding**: Adjusted results to the nearest whole number as required.
- **Geometric Interpretation**: Understood that pushing the ladder upward reduces the base distance, moving the foot closer to the wall.

Problem 12

(a) Copy and complete the table of values for the relation $y = 4x^2 - 8x - 21$, for $-2.0 \leq x \leq 4.0$. Give the answer as a sequence of decimal numbers.

x	-2.0	-1.5	-1.0	0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	11		-9		-21	-24			-21		-9	0	

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw the graph for the relation $y = 4x^2 - 8x - 21$.
(ii) Use the graph to find, within ± 1 margin, the solution set of:
$A.4\mathrm{x}^2 - 8\mathrm{x} = 3$. Give the ansewr as an increasing sequence of decimal numbers.
$B.4x^2 - 7x - 21 = 0$. Give the ansewr as an increasing sequence of decimal numbers.

Correct answer:
0,-16,-25,-24,-16,11
Correct answer:
graph
Correct answer:
-0.35,2.3
Correct answer:
-1.6,3.3
Solution. (a) The completed table of values for $y = 4x^2 - 8x - 21$ is:
y-values: $11, 0, -9, -16, -21, -24, -25, -24, -21, -16, -9, 0, 11$.
(b)
Scaling for the graph:
x-axis: $2 \quad \text{cm} = 1 \quad \text{unit}$
y-axis: $2 \quad \text{cm} = 5 \quad \text{units}$ (equivalent to $0.4 \quad \text{cm} = 1 \quad \text{unit}$)
See the figure below.

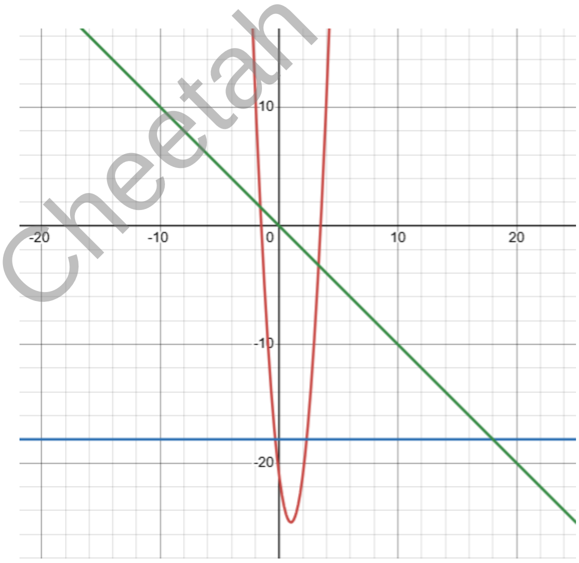


Figure 4: Part (b)

A. Solution set for $4x^2 - 8x = 3$:
$x \approx -0.32$ and $x \approx 2.32$.

B. Solution set for $4x^2 - 7x - 21 = 0$:
$x \approx -1.58$ and $x \approx 3.33$.

—

Main Concepts:
1. **Substitution into Quadratic Function**: For part (a), each x-value was substituted into $y = 4x^2 - 8x - 21$ to compute corresponding y-values, ensuring proper handling of negative signs and order of operations.
2. **Graphical Interpretation**: For part (b), the equations were rewritten in terms of the given quadratic relation. Solving $4x^2 - 8x = 3$ translates to finding where $y = -18$ intersects the parabola. Solving $4x^2 - 7x - 21 = 0$ involves finding intersections between the parabola and the line $y = -x$.
3. **Estimation via Interpolation**: Using known points from the table, linear interpolation estimated intersection points by analyzing intervals where the function crosses the target y-value or line.
4. **Quadratic Equation**: While the problem emphasized graphical methods, algebraic verification using the quadratic formula confirmed the approximate solutions, demonstrating the connection between graphical and algebraic solutions.

Problem 13

(a) The curved surface areas of two cones are equal. The base radius of one is 5 cm and its slant height is 12 cm. Calculate the height of the second cone if its base radius is 6 cm.

(b) Given the matrices $A = \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}$, find:

(i) BA. Give the answer as a sequence of a matrix elements $a_{11}, a_{12}, a_{21}, a_{22}$.

(ii) The determinant of BA.

Correct answer:
8
Correct answer:
8,21,7,17
Correct answer:
-11

Solution. (a) The curved surface area (CSA) of a cone is given by $\pi r l$. For the first cone with $r_1 = 5\,\text{cm}$ and $l_1 = 12\,\text{cm}$, its CSA is $\pi \cdot 5 \cdot 12 = 60\pi$. The second cone has $r_2 = 6\,\text{cm}$, and its CSA must equal 60π. Solving $\pi \cdot 6 \cdot l_2 = 60\pi$ gives $l_2 = 10\,\text{cm}$. Using the Pythagorean theorem $l^2 = r^2 + h^2$, the height h_2 is $\sqrt{10^2 - 6^2} = \sqrt{64} = 8\,\text{cm}$.

(b)

(i) Matrix multiplication BA:

$$BA = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix} = \begin{pmatrix} 3(2) + (-2)(-1) & 3(5) + (-2)(-3) \\ 4(2) + 1(-1) & 4(5) + 1(-3) \end{pmatrix} = \begin{pmatrix} 8 & 21 \\ 7 & 17 \end{pmatrix}$$

Elements: $8, 21, 7, 17$.

(ii) Determinant of BA:

$$\det(BA) = (8)(17) - (21)(7) = 136 - 147 = -11.$$

Main Concepts:

- **Part (a)**: Curved surface area of cones ($\pi r l$), equating surface areas, and using the Pythagorean theorem to find height.
- **Part (b)**: Matrix multiplication rules (row-by-column dot product), determinant calculation for 2x2 matrices ($ad - bc$).

Problem 14

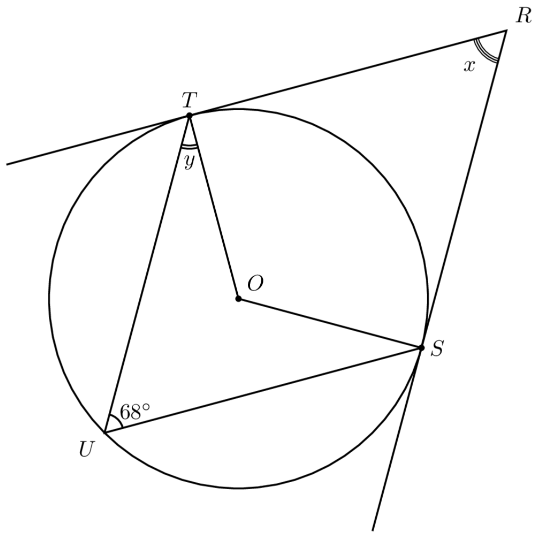
(a) Given that $110_x = 40_{\text{five}}$, find the value of x.
(b) Simplify $\frac{15}{\sqrt{75}} + \sqrt{108} + \sqrt{432}$. Represent solution as $a\sqrt{b}$, where a and b are positive integers, give the answer in the from of a, b.

Correct answer:
4
Correct answer:
19,3
Solution. (a) To solve $110_x = 40_{\text{five}}$, convert both numbers to base 10.
- $110_x = 1 \cdot x^2 + 1 \cdot x + 0 = x^2 + x$.
- $40_{\text{five}} = 4 \cdot 5^1 + 0 \cdot 5^0 = 20$.
Set $x^2 + x = 20$, yielding $x^2 + x - 20 = 0$. Solving via quadratic formula:

$$x = \frac{-1 \pm \sqrt{1 + 80}}{2} = \frac{-1 \pm 9}{2}.$$

The valid solution is $x = 4$ (since base must be positive and greater than its digits).
(b) Simplify $\frac{15}{\sqrt{75}} + \sqrt{108} + \sqrt{432}$:
- $\frac{15}{\sqrt{75}} = \frac{15}{5\sqrt{3}} = \sqrt{3}$.
- $\sqrt{108} = 6\sqrt{3}$.
- $\sqrt{432} = 12\sqrt{3}$.
Adding terms: $\sqrt{3} + 6\sqrt{3} + 12\sqrt{3} = 19\sqrt{3}$.
Final Answers:
(a) $\boxed{4}$
(b) $\boxed{19, 3}$
Main Concepts:
- **Base Conversion and Quadratic Equations** (Part a): Translating numbers to base 10 and solving quadratic equations.
- **Radical Simplification and Combining Like Terms** (Part b): Breaking down radicals, rationalizing denominators, and summing coefficients of like terms.

Problem 15



(a) In the diagram, RS and RT are tangent to the circle with centre $\mathrm{O}, \angle \mathrm{TUS} = 68°, \angle \mathrm{SRT} = \mathrm{x}$ and $\angle \mathrm{UTO} = y$. Find the value of x.
(b) Two tanks A and B are filled to capacity with diesel. Tank A holds 600 litres diesel more than tank B. If 100 litres of diesel was pumped cut of each tank, tank A would then contain 3 times as much as tank B. (i) Find the capacity of tank A. (ii) Find the capacity of tank B.

Correct answer:
44
Correct answer:
1000,400
Solution. (a) To find the value of x (angle $\angle$SRT):
1. **Tangent-Radius Perpendicularity**: Tangents RS and RT are perpendicular to the radii OS and OT at points S and T, respectively. Therefore, $\angle \mathrm{OTR} = 90°$ and $\angle \mathrm{OSR} = 90°$.
2. **Inscribed Angle Theorem**: Given $\angle \mathrm{TUS} = 68°$, we have that $\angle \mathrm{TOS} = 2 \times 68° = 136°$.
3. **Angle Relationships**: Using the sum of angles in quadrilateral $TOSR$ being $360°$, we find:

$$x = 360° - 90° - 90° - 136° = 44°.$$

(b) To find the capacities of tanks A and B:
1. **Define Variables**: Let the capacity of tank B be B litres. Then, the capacity of tank A is $B + 600$ litres.
2. **Set Up Equations**: After pumping out 100 litres each, the remaining quantities are $B+500$ (tank A) and $B - 100$ (tank B). Given tank A contains 3 times as much as tank B:

$$B + 500 = 3(B - 100)$$

3. **Solve the Equation**: Solving the equation gives $B = 400$ litres (capacity of tank B) and $B + 600 = 1000$ litres (capacity of tank A).
Final Answer
(a) $\boxed{44}$

(b) (i) $\boxed{1000}$ litres

(ii) $\boxed{400}$ litres

Main Concepts

- **Circle Theorems**: Tangent-radius perpendicularity, inscribed angles, and the alternate segment theorem were crucial for solving part (a).
- **Linear Equations**: Setting up and solving linear equations based on given conditions was essential for part (b).

Problem 16

Express, correct to three significant figures, 0.003597.

Possible answers:

A. 0.359

B. 0.004

C. 0.00360

D. 0.00359

Correct answer: C

Solution. To express 0.003597 correct to three significant figures:

1. **Identify significant figures**: The first non-zero digit is 3 (third decimal place), followed by 5, 9, and 7. These are the four significant figures.

2. **Round to three significant figures**: The third significant figure is **9**, and the next digit is **7** (5), so we round up.

- Incrementing 9 by 1 results in 10, which carries over:
- The **9** becomes **0**, and the **5** (second significant figure) becomes **6**.

3. **Final value**: This gives **3.60 10**, or **0.00360** when written in standard form.

Key Concepts:

- **Significant figures** exclude leading zeros and start at the first non-zero digit.
- **Rounding rules**: If the digit after the desired precision is 5, round up the last retained digit.
- **Trailing zeros** after a decimal point are significant if they follow non-zero digits.

Answer: C. 0.00360

Problem 17
Evaluate: $(0.064)^{-\frac{1}{3}}$.
Possible answers:
A. $\frac{5}{2}$
B. $\frac{2}{5}$
C. $-\frac{2}{5}$
D. $-\frac{5}{2}$
Correct answer: A
Solution. To evaluate $(0.064)^{-1/3}$:
1. **Convert 0.064 to a fraction**:
$0.064 = \frac{64}{1000} = \frac{8}{125} = \left(\frac{2}{5}\right)^3$.
2. **Apply the exponent $-1/3$**:

$$\left(\frac{2}{5}\right)^{3} {}^{-1/3} = \left(\frac{2}{5}\right)^{-1} = \frac{5}{2}.$$

Answer: A. $\frac{5}{2}$.
Main Concepts:
- **Decimal to fraction conversion** simplifies working with exponents.
- **Negative exponents** indicate reciprocals ($a^{-n} = \frac{1}{a^n}$).
- **Fractional exponents** combine roots and powers ($a^{m/n} = \sqrt[n]{a^m}$).
- Recognizing $0.064 = \left(\frac{2}{5}\right)^3$ is key to simplifying the expression.
- Misinterpretation of notation (e.g., mistyped exponent vs. subtraction) can drastically alter the problem, so clarity is critical.

Problem 18

Solve: $\frac{y+1}{2} - \frac{2y-1}{3} = 4$.

Possible answers:

A. $y = 19$

B. $y = -19$

C. $y = -29$

D. $y = 29$

Correct answer: B

Solution. To solve the equation $\frac{y+1}{2} - \frac{2y-1}{3} = 4$:

1. **Eliminate denominators** by multiplying both sides by the least common multiple (LCM) of 2 and 3, which is 6:

$$6\left(\frac{y+1}{2} - \frac{2y-1}{3}\right) = 6 \cdot 4$$

Simplifying each term:

$$3(y+1) - 2(2y-1) = 24$$

2. **Expand and simplify**:

$$3y + 3 - 4y + 2 = 24 \quad \Rightarrow \quad -y + 5 = 24$$

3. **Solve for y**:

$$-y = 24 - 5 \quad \Rightarrow \quad -y = 19 \quad \Rightarrow \quad y = -19$$

4. **Verify** by substituting $y = -19$ back into the original equation to confirm the solution is valid.

Main Concepts:

- **LCM for eliminating denominators** simplifies fractional equations.
- **Distributive property** expands terms after multiplying through by the LCM.
- **Combining like terms** reduces the equation to a linear form.
- **Inverse operations** isolate the variable to solve for y.
- **Verification** ensures the solution satisfies the original equation.

Answer: $\boxed{B}$

Problem 19

Simplify, correct to three significant figures, $(27.63)^2 - (12.37)^2$.
Possible answers:
A. 614
B. 612
C. 611
D. 610
Correct answer: D

Solution. To simplify $(27.63)^2 - (12.37)^2$ and round to three significant figures, we use the **difference of squares formula**:

$$a^2 - b^2 = (a - b)(a + b).$$

Let $a = 27.63$ and $b = 12.37$.
1. Calculate $a - b = 27.63 - 12.37 = 15.26$.
2. Calculate $a + b = 27.63 + 12.37 = 40.00$.
3. Multiply the results:

$$15.26 \times 40.00 = 610.4.$$

Rounding 610.4 to three significant figures gives **610**.
Main Concepts:
- **Difference of squares factorization** simplifies the calculation by avoiding direct squaring of decimals.
- **Significant figures** rules ensure the final answer is rounded appropriately based on precision.
Answer: $\boxed{D}$

Problem 20

If $7 + y = 4(\mathrm{mod} 8)$, find the least value of $y, 10 \leq y \leq 30$.

Possible answers:

A. 11

B. 13

C. 19

D. 21

Correct answer: B

Solution. To solve the equation $7 + y \equiv 4 \pmod 8$ for the least value of y in the range $10 \leq y \leq 30$:

1. **Simplify the congruence**:

Subtract 7 from both sides:

$$y \equiv 4 - 7 \pmod 8 \implies y \equiv -3 \pmod 8.$$

Adjust the negative residue by adding 8:

$$y \equiv 5 \pmod 8.$$

2. **Generate solutions**:

Values of y take the form $y = 8k + 5$ for integers k.

3. **Find y in the range $10 \leq y \leq 30$**:

Solve $8k + 5 \geq 10 \implies k \geq 1$ (smallest $k = 1$) and $8k + 5 \leq 30 \implies k \leq 3$ (largest $k = 3$).

- For $k = 1$: $y = 8(1) + 5 = 13$.
- For $k = 2$: $y = 8(2) + 5 = 21$. (correct but not the least value)
- For $k = 3$: $y = 8(3) + 5 = 29$ (exceeds options).

4. **Verify options**:

Substituting the given choices, $y = 13$ (B) and $y = 21$ (D) satisfy $7 + y \equiv 4 \pmod 8$. The least value is **13**.

Answer: B. 13

Main Concepts:

1. **Modular Arithmetic**: Solving congruences by isolating variables.
2. **Residue Adjustment**: Converting negative residues to positive equivalents.
3. **Solution Generation**: Expressing solutions in terms of modulus multiples.
4. **Range Verification**: Identifying valid solutions within a specified interval.

Problem 21

If $T = \{$ prime numbers $\}$ and $M = \{$ odd numbers $\}$ are subsets of $\mu = \{x : 0 < x \leq 10\}$ and x is an integer, find ($\mathrm{T}' \cap \mathrm{M}'$).

Possible answers:

A. $\{4, 6, 8, 10\}$

B. $\{1.4, 6, 8, 10\}$

C. $\{1, 2, 4, 6, 8, 10\}$

D. $\{1, 2, 3, 5, 7, 8, 9\}$

Correct answer: A

Solution. To solve the problem, we first identify the universal set $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
- Set T (primes) is $\{2, 3, 5, 7\}$.
- Set M (odd numbers) is $\{1, 3, 5, 7, 9\}$.

Step 1: Find T' (complement of T)

$T' = \mu \setminus T = \{1, 4, 6, 8, 9, 10\}$.

Step 2: Find M' (complement of M)

$M' = \mu \setminus M = \{2, 4, 6, 8, 10\}$.

Step 3: Compute $T' \cap M'$

The intersection includes elements common to both T' and M':

$T' \cap M' = \{4, 6, 8, 10\}$.

Final Answer

$\boxed{A}$

Main Concepts:

1. **Universal Set and Subsets**: Understanding the universal set μ and given subsets T (primes) and M (odds).
2. **Complement of a Set**: Determining elements *not* in T or M relative to μ.
3. **Set Intersection**: Identifying common elements between T' and M'.
4. **Prime and Odd Numbers**: Recognizing primes (numbers divisible only by 1 and themselves) and odds (not divisible by 2).

Problem 22

Evaluate: $\frac{\log_3 9 - \log_2 8}{\log_3 9}$.

Possible answers:

A. $-\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $-\frac{1}{2}$

Correct answer: D

Solution. To evaluate the expression $\frac{\log_3 9 - \log_2 8}{\log_3 9}$, follow these steps:

1. **Compute $\log_3 9$:**

Since $3^2 = 9$, $\log_3 9 = 2$.

2. **Compute $\log_2 8$:**

Since $2^3 = 8$, $\log_2 8 = 3$.

3. **Simplify the numerator:**

$\log_3 9 - \log_2 8 = 2 - 3 = -1$.

4. **Divide by $\log_3 9$ (which is 2):**

$\frac{-1}{2} = -\frac{1}{2}$.

Final Answer: $\boxed{D}$

Main Concepts:

- **Logarithm Definition:** Evaluating $\log_b a$ by finding the exponent x such that $b^x = a$.
- **Arithmetic Operations:** Subtracting and dividing the results of the logarithms.
- **Simplification:** Breaking down the problem into smaller steps to avoid errors.

Problem 23

If $23_y = 1111_{\text{two}}$, find the value of y.
Possible answers:
A. 4
B. 5
C. 6
D. 7
Correct answer: C
Solution. To solve the equation $23_y = 1111_{\text{two}}$, follow these steps:
1. **Convert 23_y to decimal**:
In base y, the number 23_y expands to:

$$2 \cdot y^1 + 3 \cdot y^0 = 2y + 3.$$

2. **Convert 1111_{two} to decimal**:
In binary, 1111_{two} expands to:

$$1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 8 + 4 + 2 + 1 = 15.$$

3. **Set the decimal equivalents equal and solve for y**:

$$2y + 3 = 15 \implies 2y = 12 \implies y = 6.$$

4. **Verify with answer choices**:
Substituting $y = 6$ into 23_y gives $2 \cdot 6 + 3 = 15$, which matches 1111_{two}. Testing other options confirms $y = 6$ is correct.
Answer: $\boxed{C}$
Key Concepts:
- **Positional numeral systems**: Digits represent values scaled by powers of the base.
- **Base conversion**: Translating numbers from one base to decimal by summing digit-place products.
- **Linear equations**: Solving for an unknown variable after equating decimal equivalents.
- **Verification**: Cross-checking results with given options to ensure accuracy.

Problem 24

If 6, P, and 14 are consecutive terms in an arithmetic progression, find the value of P.
Possible answers:
A. 9
B. 10
C. 6
D. 8
Correct answer: B

Solution. To find the value of P such that 6, P, and 14 are consecutive terms in an arithmetic progression (AP), we use the property that the middle term of three consecutive terms in an AP is the average of the first and third terms.
1. **Calculate the average**:

$$P = \frac{6+14}{2} = \frac{20}{2} = 10$$

This ensures the common difference $d = 10 - 6 = 4$, and $14 - 10 = 4$, confirming consistency.
2. **Verification via the AP formula**:
Let the common difference be d. The third term 14 can be written as:

$$6 + 2d = 14 \implies 2d = 8 \implies d = 4$$

Then, $P = 6 + d = 6 + 4 = 10$.
Answer: $\boxed{B}$
Key Concepts:
1. **Arithmetic Progression (AP)**: A sequence with a constant difference between consecutive terms.
2. **Middle Term Property**: In three consecutive AP terms a, b, c, the middle term $b = \frac{a+c}{2}$.
3. **Common Difference**: Calculated using $d = \text{term}_2 - \text{term}_1$, ensuring consistency across terms.

Problem 25

Evaluate: $2\sqrt{28} - 3\sqrt{50} + \sqrt{72}$.

Possible answers:

A. $4\sqrt{7} - 21\sqrt{2}$

B. $4\sqrt{7} - 11\sqrt{2}$

C. $4\sqrt{7} - 9\sqrt{2}$

D. $4\sqrt{7} + \sqrt{2}$

Correct answer: C

Solution. To evaluate $2\sqrt{28} - 3\sqrt{50} + \sqrt{72}$, simplify each radical by factoring out perfect squares:

1. $2\sqrt{28} = 2\sqrt{4 \cdot 7} = 2 \cdot 2\sqrt{7} = 4\sqrt{7}$.
2. $-3\sqrt{50} = -3\sqrt{25 \cdot 2} = -3 \cdot 5\sqrt{2} = -15\sqrt{2}$.
3. $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$.

Combine like terms:

$4\sqrt{7} - 15\sqrt{2} + 6\sqrt{2} = 4\sqrt{7} + (-15 + 6)\sqrt{2} = 4\sqrt{7} - 9\sqrt{2}$.

Answer: C. $4\sqrt{7} - 9\sqrt{2}$.

Main Concepts:

1. **Simplifying Radicals**: Factor radicands into perfect squares and their remainders (e.g., $28 = 4 \cdot 7$).
2. **Coefficient Multiplication**: Multiply coefficients after simplifying radicals (e.g., $2 \cdot 2\sqrt{7}$).
3. **Combining Like Terms**: Add/subtract coefficients of identical radicals (e.g., $-15\sqrt{2} + 6\sqrt{2}$).

Problem 26

If m : n = 2 : 1, evaluate $\frac{3m^2-2n^2}{m^2+mn}$.

Possible answers:

A. $\frac{4}{3}$

B. $\frac{5}{3}$

C. $\frac{3}{4}$

D. $\frac{3}{5}$

Correct answer: B

Solution. To evaluate $\frac{3m^2-2n^2}{m^2+mn}$ given $m : n = 2 : 1$, express m and n in terms of a common variable. Let $m = 2k$ and $n = k$ for some constant k. Substituting these into the expression:

Numerator:

$$3(2k)^2 - 2k^2 = 3(4k^2) - 2k^2 = 12k^2 - 2k^2 = 10k^2$$

Denominator:

$$(2k)^2 + (2k)(k) = 4k^2 + 2k^2 = 6k^2$$

Simplifying the fraction:

$$\frac{10k^2}{6k^2} = \frac{10}{6} = \frac{5}{3}$$

Answer: $\boxed{B}$

Main Concepts:

1. **Ratio Substitution**: Representing variables m and n using their ratio ($m = 2k$, $n = k$) simplifies the expression to a single variable.
2. **Algebraic Simplification**: Substituting and expanding terms in the numerator and denominator, then canceling common factors (k^2).
3. **Fraction Reduction**: Simplifying the resulting fraction $\frac{10}{6}$ to its lowest terms ($\frac{5}{3}$).

Problem 27

H varies directly as p and inversely as the square of y. If $H = 1, p = 8$ and $y = 2$, find H in terms of p and y.

Possible answers:

A. $H = \frac{p}{4y^2}$

B. H = $\frac{2p}{y^2}$

C. $H = \frac{p}{2y^2}$

D. H = $\frac{p^2}{y^2}$

Correct answer: C

Solution. To solve the problem, we start by translating the given relationships into a mathematical equation. Since H varies directly as p and inversely as the square of y, we write:

$$H = k \cdot \frac{p}{y^2}$$

where k is the constant of proportionality. Substituting the given values $H = 1$, $p = 8$, and $y = 2$:

$$1 = k \cdot \frac{8}{2^2} \implies 1 = k \cdot \frac{8}{4} \implies 1 = 2k \implies k = \frac{1}{2}.$$

Substituting $k = \frac{1}{2}$ back into the equation gives:

$$H = \frac{1}{2} \cdot \frac{p}{y^2} = \frac{p}{2y^2}.$$

Main Concepts:

1. **Direct Variation:** $H \propto p$ implies $H = k \cdot p$.
2. **Inverse Variation:** $H \propto \frac{1}{y^2}$ implies $H = \frac{k}{y^2}$.
3. **Combined Variation:** Combining both relationships into $H = \frac{kp}{y^2}$.
4. **Solving for Constants:** Using given values to determine k.

Answer: $\boxed{C}$

Problem 28

Solve $4x^2 - 16x + 15 = 0$.

Possible answers:

A. $x = 1\frac{1}{2}$ or $x = -2\frac{1}{2}$

B. $x = 1\frac{1}{2}$ or $x = 2\frac{1}{2}$

C. $x = 1\frac{1}{2}$ or $x = -1\frac{1}{2}$

D. $x = -1\frac{1}{2}$ or $x - 2\frac{1}{2}$

Correct answer: B

Solution. To solve the quadratic equation $4x^2 - 16x + 15 = 0$, we first attempt factoring. The equation factors into $(2x-3)(2x-5) = 0$, leading to solutions $x = \frac{3}{2} = 1\frac{1}{2}$ and $x = \frac{5}{2} = 2\frac{1}{2}$. Alternatively, applying the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ with $a = 4$, $b = -16$, and $c = 15$ confirms the solutions:

$$x = \frac{16 \pm \sqrt{256-240}}{8} = \frac{16 \pm 4}{8} = \frac{20}{8} = 2.5 \quad \text{and} \quad \frac{12}{8} = 1.5.$$

Both methods yield $x = 1\frac{1}{2}$ or $x = 2\frac{1}{2}$, corresponding to **Answer B**.

Main Concepts:

1. **Factoring Quadratics**: Breaking down the equation into binomials by finding two numbers that multiply to $ac = 60$ and add to $b = -16$.

2. **Zero Product Property**: Setting each factor equal to zero to solve for x.

3. **Quadratic Formula**: Verifying solutions using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

4. **Fraction Conversion**: Expressing improper fractions as mixed numbers for the final answer.

Problem 29

Simplify: $\log_{10} 6 - 3\log_{10} 3 + \frac{2}{3}\log_{10} 27$.
Possible answers:
A. $3\log_{10} 2$
B. $\log_{10} 2$
C. $\log_{10} 3$
D. $2\log_{10} 3$
Correct answer: B

Solution. To simplify the expression $\log_{10} 6 - 3\log_{10} 3 + \frac{2}{3}\log_{10} 27$, we apply logarithmic properties step-by-step:
1. **Power Rule**: Move coefficients as exponents:

$$\log_{10} 6 - \log_{10} 3^3 + \log_{10} 27^{2/3}.$$

Simplifying further:

$$\log_{10} 6 - \log_{10} 27 + \log_{10} 9 \quad (\text{since } 27^{2/3} = 9).$$

2. **Subtraction/Division Rule**: Combine the first two terms:

$$\log_{10}\left(\frac{6}{27}\right) + \log_{10} 9 = \log_{10}\left(\frac{2}{9}\right) + \log_{10} 9.$$

3. **Addition/Multiplication Rule**: Combine the remaining terms:

$$\log_{10}\left(\frac{2}{9} \cdot 9\right) = \log_{10} 2.$$

Final Answer: $\boxed{B}$ ($\log_{10} 2$).
Key Concepts:
- **Power Rule**: $n\log_b a = \log_b(a^n)$ to handle coefficients.
- **Subtraction/Division**: $\log_b a - \log_b c = \log_b \frac{a}{c}$.
- **Addition/Multiplication**: $\log_b a + \log_b c = \log_b(a \cdot c)$.
- Simplification through arithmetic operations and cancellation of terms.

Problem 30

Bala sold an article for N6,900.00 and made a profit of 15%. Calculate his percentage profit if he had sold it for N6,600.00.

Possible answers:
A. 5%
B. 10%
C. 12%
D. 13%
Correct answer: B

Solution. To determine the percentage profit if Bala had sold the article for N6,600.00, follow these steps:
1. **Find the cost price (CP):**
When Bala sold the article for N6,900.00 with a 15% profit, the relationship between CP and SP is:

$$\text{SP} = \text{CP} \times (1 + \text{Profit Percentage})$$

Substituting values:

$$6900 = \text{CP} \times 1.15 \implies \text{CP} = \frac{6900}{1.15} = 6000$$

Cost Price = N6,000.00
2. **Calculate profit for SP = N6,600.00:**
Profit = SP - CP = 6600 - 6000 = **N600.00**
3. **Determine percentage profit:**

$$\text{Profit Percentage} = \left(\frac{\text{Profit}}{\text{CP}}\right) \times 100 = \left(\frac{600}{6000}\right) \times 100 = 10\%$$

Final Answer: B. 10%
Main Concepts:
- **Profit Percentage Calculation:** Profit percentage is derived from the cost price, not the selling price.
- **Algebraic Manipulation:** Solving for the unknown cost price using the given selling price and profit percentage.
- **Unit Conversion:** Handling percentages as decimals during calculations and converting back to percentages for the final result.
- **Consistency in Base Value:** Ensuring all profit calculations are anchored to the original cost price for accuracy.

Problem 31

If $3p = 4q$ and $9p = 8q - 12$, find the value of pq.
Possible answers:
A. 12
B. 7
C. -7
D. -12
Correct answer: A

Solution. To solve the system of equations $3p = 4q$ and $9p = 8q - 12$, we first express p in terms of q from the first equation:

$$p = \frac{4}{3}q.$$

Substituting this into the second equation:

$$9\left(\frac{4}{3}q\right) = 8q - 12 \implies 12q = 8q - 12.$$

Solving for q:

$$4q = -12 \implies q = -3.$$

Substituting $q = -3$ back into $p = \frac{4}{3}q$:

$$p = \frac{4}{3}(-3) = -4.$$

Finally, the product pq is:

$$pq = (-4)(-3) = 12.$$

Main Concepts:
1. **Substitution Method**: Solving one equation for a variable and substituting into the other equation to reduce the system to a single variable.
2. **Linear Equations**: Manipulating equations of the form $ax + by = c$ to isolate variables.
3. **Sign Rules**: Recognizing that multiplying/dividing negative numbers yields a positive result (e.g., $(-4)(-3) = 12$).
Answer: $\boxed{A}$ (12)

Problem 32
If $(0.25)^y = 32$, find the value of y.
Possible answers:
A. $y = -\frac{5}{2}$
B. $y = -\frac{3}{2}$
C. $y = \frac{3}{2}$
D. $y = \frac{5}{2}$
Correct answer: A
Solution. To solve the equation $(0.25)^y = 32$, we first express both sides as powers of the same base.
1. **Rewrite 0.25 as a power of 2:**
$0.25 = \frac{1}{4} = 2^{-2}$.
Substituting this into the equation gives:

$$(2^{-2})^y = 32.$$

2. **Simplify the left side using exponent rules:**

$$2^{-2y} = 32.$$

3. **Express 32 as a power of 2:**
$32 = 2^5$.
The equation becomes:

$$2^{-2y} = 2^5.$$

4. **Set the exponents equal since the bases match:**

$$-2y = 5.$$

5. **Solve for y:**

$$y = \frac{5}{-2} = -\frac{5}{2}.$$

Final Answer:
$\boxed{A}$
Main Concepts:
- **Exponent Rules**: Rewriting numbers as powers of a common base (e.g., $0.25 = 2^{-2}$, $32 = 2^5$).
- **Equating Exponents**: If $a^b = a^c$ (where $a > 0$ and $a \neq 1$), then $b = c$.
- **Logarithmic Verification**: Using logarithms confirms the result, though recognizing common bases simplifies the process.

Problem 33

There are 8 boys and 4 girls in a lift. What is the probability that the first person who steps out of the lift will be a boy?

Possible answers:

A. $\frac{1}{6}$

B. $\frac{1}{4}$

C. $\frac{2}{3}$

D. $\frac{1}{2}$

Correct answer: C

Solution. The probability that the first person who steps out of the lift is a boy is calculated by dividing the number of boys by the total number of people in the lift. There are 8 boys and 4 girls, so the total number of people is $8+4=12$. The probability is therefore $\frac{8}{12}$, which simplifies to $\frac{2}{3}$. The correct answer is **C. $\frac{2}{3}$**.

Main Concepts:

1. **Basic Probability**: The probability of an event is the ratio of favorable outcomes to total possible outcomes.

2. **Equally Likely Outcomes**: Assuming each person has an equal chance of exiting first, the probability depends solely on the proportion of boys in the group.

3. **Fraction Simplification**: Reducing $\frac{8}{12}$ to its simplest form by dividing numerator and denominator by their greatest common divisor (4) gives $\frac{2}{3}$.

Problem 34
Simplify: $\frac{x^2-5x-14}{x^2-9x+14}$.
Possible answers:
A. $\frac{x-7}{x+7}$
B. $\frac{x+7}{x-7}$
C. $\frac{x-2}{x+4}$
D. $\frac{x+2}{x-2}$
Correct answer: D
Solution. To simplify the rational expression $\frac{x^2-5x-14}{x^2-9x+14}$, follow these steps:
1. **Factor the numerator and denominator**:
- **Numerator**: $x^2-5x-14$ factors into $(x-7)(x+2)$ because $-7\cdot 2 = -14$ and $-7+2 = -5$.
- **Denominator**: $x^2 - 9x + 14$ factors into $(x-2)(x-7)$ because $-2 \cdot -7 = 14$ and $-2+(-7) = -9$.
2. **Cancel common factors**:
The expression becomes $\frac{(x-7)(x+2)}{(x-2)(x-7)}$. The $(x-7)$ terms cancel out (assuming $x \neq 7$), leaving $\frac{x+2}{x-2}$.
3. **Identify the simplified form**:
The simplified expression matches option **D**: $\frac{x+2}{x-2}$.
Main Concepts:
- **Factoring quadratics** by identifying pairs of numbers that satisfy product and sum conditions.
- **Simplifying rational expressions** by canceling common factors in the numerator and denominator.
- **Domain restrictions** (e.g., $x \neq 7$ and $x \neq 2$ to avoid division by zero).
Answer: $\boxed{D}$

Problem 35

Which of these values would make $\frac{3p-1}{p^2-p}$ undefined?

Possible answers:

A. 1

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. -1

Correct answer: A

Solution. To determine which value makes the expression $\frac{3p-1}{p^2-p}$ undefined, identify where the **denominator equals zero** (since division by zero is undefined).

1. **Solve $p^2 - p = 0$:**

Factor the denominator:

$$p(p-1) = 0$$

This gives solutions $p = 0$ or $p = 1$.

2. **Check the answer choices:**

The critical values $p = 0$ and $p = 1$ must be compared to the options. Among the choices (A. 1; B. $\frac{1}{3}$; C. $-\frac{1}{3}$; D. -1), only $p = 1$ (Option A) is a solution.

Conclusion: The expression is undefined at $p = 1$.

Main Concepts:

- **Undefined fractions** occur when the denominator equals zero.
- **Factoring** to solve quadratic equations.
- **Critical evaluation** of solutions against given answer choices.

Problem 36

The total surface area of a solid cylinder 165 cm^2. Of the base diameter is 7 cm , calculate its height. [Take $\pi = \frac{22}{7}$]
Possible answers:
A. 7.5 cm
B. 4.5 cm
C. 4.0 cm
D. 2.0 cm
Correct answer: C
Solution. The total surface area (TSA) of a cylinder is given by the formula:

$$\text{TSA} = 2\pi r^2 + 2\pi r h$$

where r is the radius and h is the height.
Given:
- TSA $= 165\,\text{cm}^2$
- Diameter $= 7\,\text{cm}$, so radius $r = \frac{7}{2} = 3.5\,\text{cm}$
- $\pi = \frac{22}{7}$

Substitute the values into the formula:

$$165 = 2 \cdot \frac{22}{7} \cdot \left(\frac{7}{2}\right)^2 + 2 \cdot \frac{22}{7} \cdot \frac{7}{2} \cdot h$$

Step 1: Calculate $2\pi r^2$:

$$2 \cdot \frac{22}{7} \cdot \frac{49}{4} = 77\,\text{cm}^2$$

Step 2: Simplify $2\pi r h$:

$$2 \cdot \frac{22}{7} \cdot \frac{7}{2} = 22\,\text{cm} \quad (\text{coefficient of } h)$$

Step 3: Solve for h:

$$165 = 77 + 22h \implies 88 = 22h \implies h = \frac{88}{22} = 4\,\text{cm}$$

Answer: C. 4.0 cm
Main Concepts:
1. **Surface Area Formula:** The TSA of a cylinder combines the areas of the two circular bases and the lateral surface.
2. **Radius-Diameter Relationship:** The radius is derived from the given diameter.
3. **Substitution and Simplification:** Plugging values into the formula and simplifying terms systematically.
4. **Algebraic Solving:** Isolating h through arithmetic operations.
5. **Unit Consistency:** Ensuring all terms use consistent units (centimeters) for accuracy.

Problem 37

If $2^a = \sqrt{64}$ and $\frac{b}{a} = 3$, evaluate a$^2 + b^2$.

Possible answers:

A. 250

B. 160

C. 90

D. 48

Correct answer: C

Solution. To solve the problem, we start by finding the value of a. Given $2^a = \sqrt{64}$, we simplify $\sqrt{64} = 8$. Since $2^3 = 8$, it follows that $a = 3$.

Next, using the equation $\frac{b}{a} = 3$, substitute $a = 3$ to solve for b:

$$b = 3a = 3 \times 3 = 9.$$

Finally, evaluate $a^2 + b^2$:

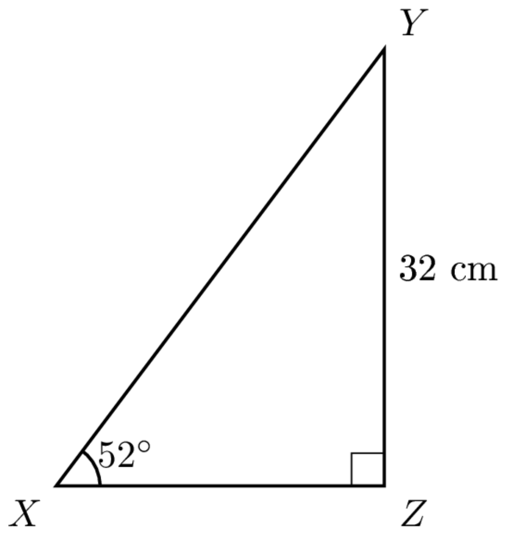
$$a^2 + b^2 = 3^2 + 9^2 = 9 + 81 = 90.$$

Answer: C. 90

Main Concepts:

1. **Exponential Equations:** Solving $2^a = 8$ by expressing 8 as a power of 2.
2. **Square Roots:** Recognizing $\sqrt{64} = 8$.
3. **Algebraic Substitution:** Solving $\frac{b}{a} = 3$ by substituting $a = 3$.
4. **Evaluating Expressions:** Calculating $a^2 + b^2$ after determining a and b.

Problem 38



In $\triangle XYZ$, $|YZ| = 32$ cm, $\angle YXZ = 52°$ and $\angle XZY = 90°$. Find, correct to the nearest centimetre, $|XZ|$.
Possible answers:
A. 31 cm
B. 25 cm
C. 20 cm
D. 13 cm
Correct answer: B
Solution. To solve for $|XZ|$ in the right-angled triangle $\triangle XYZ$ with $\angle XZY = 90°$, $|YZ| = 32\,\text{cm}$, and $\angle YXZ = 52°$:
1. **Identify the sides and angles**:
- $\angle YXZ = 52°$ (at vertex X), $\angle XZY = 90°$ (at vertex Z), and $\angle XYZ = 38°$ (remaining angle).
- Side $YZ = 32\,\text{cm}$ is opposite $\angle YXZ$, and side XZ is adjacent to $\angle YXZ$.
2. **Apply the tangent function**:

$$\tan(52°) = \frac{\text{opposite}}{\text{adjacent}} = \frac{|YZ|}{|XZ|}$$

Rearrange to solve for $|XZ|$:

$$|XZ| = \frac{|YZ|}{\tan(52°)}$$

3. **Calculate**:
- $\tan(52°) \approx 1.2799$
- $|XZ| = \frac{32}{1.2799} \approx 25\,\text{cm}$
Answer: B. 25 cm
Main Concepts:
- **Right-Angled Triangle**: Utilized properties of a triangle with a 90° angle.
- **Trigonometric Ratios**: Applied the tangent function to relate opposite and adjacent sides.

- **Angle-Side Relationship**: Correctly identified sides relative to the given angle.
- **Verification**: Cross-checked using complementary angles to ensure accuracy.

Problem 39

If $\log_x 2 = 0.3$, evaluate $\log_x 8$.
Possible answers:
A. 2.4
B. 1.2
C. 0.9
D. 0.6
Correct answer: C

Solution. To solve the problem, we start by converting the given logarithmic equation into exponential form. If $\log_x 2 = 0.3$, then $x^{0.3} = 2$. Next, we need to evaluate $\log_x 8$. Recognizing that $8 = 2^3$, we rewrite $\log_x 8$ as $\log_x(2^3)$. Using the logarithm power rule ($\log_b a^n = n \log_b a$), this becomes $3 \log_x 2$. Substituting the given value $\log_x 2 = 0.3$, we calculate $3 \times 0.3 = 0.9$. Thus, $\log_x 8 = 0.9$.
Main Concepts:
1. **Logarithm to Exponential Conversion**: Converting $\log_x 2 = 0.3$ to $x^{0.3} = 2$ establishes the relationship between x and 2.
2. **Exponent Recognition**: Expressing 8 as 2^3 allows simplification using logarithmic properties.
3. **Logarithm Power Rule**: Applying $\log_b a^n = n \log_b a$ reduces $\log_x 8$ to $3 \log_x 2$.
4. **Substitution**: Using the given value $\log_x 2 = 0.3$ directly in the simplified expression yields the final result.
Answer: C. 0.9

Problem 40

An arc subtends an angle of 72° at the centre of a circle. Find the length of the arc if the radius of the circle is 3.5 cm . [Take $\pi=\frac{22}{7}$]

Possible answers:

A. 6.6 cm

B. 8.8 cm

C. 4.4 cm

D. 2.2 cm

Correct answer: C

Solution. The length of the arc can be found using the formula:

$$\text{Arc Length}=\frac{\theta}{360^{\circ}}\times 2\pi r$$

where $\theta=72^{\circ}$ and $r=3.5\,\text{cm}$.

1. **Calculate the fraction of the circle**:

$$\frac{72^{\circ}}{360^{\circ}}=\frac{1}{5}$$

2. **Compute the circumference**:

$$2\pi r=2\times\frac{22}{7}\times 3.5=7\times\frac{22}{7}=22\,\text{cm}$$

3. **Find the arc length**:

$$\text{Arc Length}=\frac{1}{5}\times 22=\frac{22}{5}=4.4\,\text{cm}$$

Answer: C. 4.4 cm

Main Concepts:

- **Arc Length Formula**: Relates the central angle (θ) and radius (r) to the arc length.
- **Fraction of the Circle**: The central angle determines the proportion of the full circumference.
- **Substitution and Simplification**: Using $\pi=\frac{22}{7}$ and simplifying fractions to compute the final value.

Problem 41

Make b the subject of the relation lb = $\frac{1}{2}(a+b)$h.

Possible answers:

A. $\frac{ah}{2l-h}$

B. $\frac{2l-h}{al}$

C. $\frac{al}{2l-h}$

D. $\frac{al}{2-h}$

Correct answer: A

Solution. To solve for b in the equation $lb = \frac{1}{2}(a+b)h$, follow these steps:

1. **Eliminate the fraction** by multiplying both sides by 2:

$$2lb = h(a+b)$$

2. **Expand the right side**:

$$2lb = ha + hb$$

3. **Collect terms with b** on the left by subtracting hb:

$$2lb - hb = ha$$

4. **Factor b** out of the left side:

$$b(2l-h) = ha$$

5. **Isolate b** by dividing both sides by $(2l-h)$:

$$b = \frac{ha}{2l-h}$$

This simplifies to $b = \frac{ah}{2l-h}$, which matches **Option A**.

—

Main Concepts:

- **Algebraic Manipulation**: Multiplying through to eliminate fractions, expanding expressions, and rearranging terms.

- **Factoring**: Extracting the common variable b to isolate it.

- **Solving for a Variable**: Dividing to express b explicitly in terms of the other variables.

Answer: A. $\frac{ah}{2l-h}$

Problem 42

Eric sold his house through an agent who charged 8% commission on the selling price. If Eric received $\$117,760.00$ after the sale, what was the selling price of the house?

Possible answers:

A. $\$130,000.00$

B. $\$128,000.00$

C. $\$125,000.00$

D. $\$120,000.00$

Correct answer: B

Solution. To determine the selling price of Eric's house, let S represent the selling price. The agent's commission is 8%, so Eric receives 92% of S. This relationship is expressed as:

$$0.92S = \$117{,}760.00$$

Solving for S:

$$S = \frac{\$117{,}760.00}{0.92} = \$128{,}000.00$$

Verification:

- Commission: $8\% \times \$128{,}000.00 = \$10{,}240.00$
- Amount received: $\$128{,}000.00 - \$10{,}240.00 = \$117{,}760.00$, which matches the given value.

Answer: B. $128,000.00

Main Concepts:

1. **Commission Calculation:** The agent's commission is a percentage deducted from the selling price.
2. **Percentage Representation:** Eric received 92% of the selling price after the 8% commission.
3. **Algebraic Equation:** Setting up the equation $0.92S =$ Amount Received and solving for S.
4. **Verification:** Confirming the answer by recalculating the commission and ensuring the final amount aligns with the problem statement.

Problem 43

Find the angle at which an arc of length 22 cm subtends at the centre of a circle of radius 15 cm . [Take $\pi = \frac{22}{7}$]
Possible answers:
A. 70°
B. 84°
C. 96°
D. 156°
Correct answer: B

Solution. To find the angle at which a 22 cm arc subtends at the center of a circle with radius 15 cm, use the **arc length formula** in degrees:

$$s = \frac{\theta}{360} \cdot 2\pi r$$

Rearranging to solve for θ:

$$\theta = \frac{180s}{\pi r}$$

Substitute $s = 22$, $r = 15$, and $\pi = \frac{22}{7}$:

$$\theta = \frac{180 \cdot 22}{\frac{22}{7} \cdot 15} = \frac{3960}{\frac{330}{7}} = 3960 \cdot \frac{7}{330} = 84^\circ$$

Answer: B. 84°
Main Concepts:
1. **Arc Length Formula**: Relates arc length (s), radius (r), and central angle (θ) in degrees or radians.
2. **Unit Conversion**: Converting between radians and degrees using π radians $= 180^\circ$.
3. **Proportional Reasoning**: Arc length as a fraction of the circumference ($2\pi r$) based on $\theta/360$.
4. **Algebraic Manipulation**: Solving for θ by rearranging the formula and simplifying fractions.
5. **Simplification**: Leveraging $\pi = \frac{22}{7}$ to cancel terms and streamline calculations.

Problem 44

A rectangular board has a length of 15 cm and width x cm . If its sides are doubled, find its new area.

Possible answers:

A. 60xcm^2

B. 45xcm^2

C. 30xcm^2

D. 15xcm^2

Correct answer: A

Solution. The new area of the rectangular board after doubling both its length and width is **60x cm**, corresponding to **option A**.

Step-by-Step Explanation:

1. **Original Dimensions:**

- Length = 15 cm

- Width = x cm

2. **New Dimensions After Doubling:**

- New Length = $15\,\text{cm} \times 2 = 30\,\text{cm}$

- New Width = $x\,\text{cm} \times 2 = 2x\,\text{cm}$

3. **Calculate New Area:**

$$\text{New Area} = \text{New Length} \times \text{New Width} = 30\,\text{cm} \times 2x\,\text{cm} = 60x\,\text{cm}^2$$

Main Concepts:

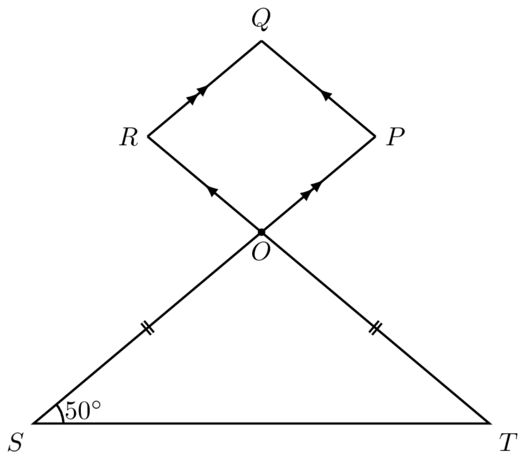
- **Area of a Rectangle:** Calculated as length $\times$ width.

- **Scaling Dimensions:** Doubling both length and width increases the area by a factor of $2 \times 2 = 4$. The original area ($15x\,\text{cm}^2$) becomes $4 \times 15x = 60x\,\text{cm}^2$.

- **Geometric Scaling:** When all linear dimensions of a shape are scaled by a factor, the area scales by the square of that factor. Here, doubling sides ($\times 2$) results in an area scaled by $2^2 = 4$.

Answer: A. $60\text{x}\,\text{cm}^2$.

Problem 45



In the diagram below, POS and ROT are straight lines. OPQR is a parallelogram, $|OS| = |OT|$ and $\angle OST = 50°$. Calculate the value of $\angle OPQ$.
Possible answers:
A. 100°
B. 120°
C. 140°
D. 160°
Correct answer: A
Solution. To solve the problem, we analyze the given diagram and utilize properties of parallelograms and isosceles triangles.
1. **Isosceles Triangle OST**:
- Given $|OS| = |OT|$ and $\angle OST = 50°$, triangle OST is isosceles.
- Base angles $\angle OTS = \angle OST = 50°$, so the vertex angle $\angle SOT = 180° - 2 \times 50° = 80°$.
2. **Angle at O**:
- Lines POS and ROT intersect at O, forming vertically opposite angles. Thus, $\angle POR = \angle SOT = 80°$.
3. **Parallelogram OPQR**:
- In a parallelogram, consecutive angles are supplementary. Since $\angle POR = 80°$, the consecutive angle at P is $180° - 80° = 100°$.
4. **Vector Analysis**:
- Using coordinates and vectors, the angle between OP and PQ (at point P) is calculated using the dot product, confirming $\angle OPQ = 100°$.
Answer: A. 100°
Main Concepts:
- **Parallelogram Properties**: Supplementary consecutive angles.
- **Isosceles Triangle**: Equal base angles and angle sum.
- **Vector Dot Product**: Calculating the angle between vectors to confirm geometric relationships.

Problem 46
Factorize completely: $(2x + 2y)(x - y) + (2x - 2y)(x + y)$.
Possible answers:
A. $4(x - y)(x + y)$
B. $(4(x - y)$
C. $2(x - y)(x + y)$
D. $2(x - y)$
Correct answer: A
Solution. To factorize the expression $(2x + 2y)(x - y) + (2x - 2y)(x + y)$ completely, follow these steps:
1. **Factor Common Terms in Each Product**:
- First term: $2(x + y)(x - y)$.
- Second term: $2(x - y)(x + y)$.
2. **Combine Like Terms**:
- Both terms simplify to $2(x + y)(x - y)$. Adding them gives:

$$2(x + y)(x - y) + 2(x + y)(x - y) = 4(x + y)(x - y).$$

3. **Final Factorization**:
- The expression $4(x + y)(x - y)$ is already fully factored.
Main Concepts:
- **Factoring Common Coefficients**: Recognizing and pulling out common factors like 2 from each binomial simplifies the expression.
- **Difference of Squares**: Identifying the structure $x^2 - y^2 = (x - y)(x + y)$ allows direct factorization without full expansion.
- **Combining Like Terms**: Adding identical terms after factoring streamlines the solution.
Answer: $\boxed{A}$

Problem 47

The interior angles of a polygon are $3x^o, 2x^o, 4x^o, 3x^o$ and $6x^o$. Find the size of the smallest angle of the polygon.
Possible answers:
A. 80°
B. 60°
C. 40°
D. 30°
Correct answer: B

Solution. The sum of the interior angles of a pentagon (5-sided polygon) is calculated using the formula $(n-2)\times 180°$, which gives 540°. The given angles in terms of x are $3x$, $2x$, $4x$, $3x$, and $6x$. Adding these angles:

$$3x + 2x + 4x + 3x + 6x = 18x.$$

Setting this equal to the total sum:

$$18x = 540° \implies x = 30°.$$

Substituting $x = 30°$ into each angle:
- $3x = 90°$,
- $2x = 60°$ (smallest angle),
- $4x = 120°$,
- $3x = 90°$,
- $6x = 180°$.

The smallest angle is 60°, corresponding to **Answer B**.
Main Concepts:
1. **Interior Angle Sum Formula**: Determined the total angle sum for a pentagon.
2. **Algebraic Equation**: Set up and solved $18x = 540°$ to find x.
3. **Angle Calculation**: Substituted x to find individual angles and identified the smallest.
4. **Polygon Type Consideration**: Acknowledged the presence of a 180° angle (concave/degenerate polygon), which is permissible here.

Problem 48

A box contains 2 white and 3 blue identical balls. If two balls are picked at random from the box, one after the other with replacement, what is the probability that they are of different colours?
Possible answers:
A. $\frac{2}{3}$
B. $\frac{3}{5}$
C. $\frac{7}{20}$
D. $\frac{12}{25}$
Correct answer: D

Solution. To determine the probability that two balls picked with replacement are of different colors, we start by noting the box contains 2 white and 3 blue balls (total $= 5$). Since replacement ensures independence between draws, we calculate the probabilities for two scenarios:
1. **White first, then blue**:
Probability $= \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$.
2. **Blue first, then white**:
Probability $= \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$.
Adding these gives the total probability of different colors:

$$\frac{6}{25} + \frac{6}{25} = \frac{12}{25}.$$

Verification via complementary probability:
- Probability both white: $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$.
- Probability both blue: $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$.
- Total same-color probability: $\frac{4}{25} + \frac{9}{25} = \frac{13}{25}$.
- Different-color probability: $1 - \frac{13}{25} = \frac{12}{25}$.
Main Concepts:
1. **Independent Events** (due to replacement).
2. **Mutually Exclusive Outcomes** (summing probabilities of distinct cases).
3. **Complementary Probability** (1 - probability of the opposite event).
Answer: D. $\frac{12}{25}$.

Problem 49

Find the equation of a straight line passing through the point $(1, -5)$ and having gradient of $\frac{3}{4}$.
Possible answers:
A. $3x + 4y - 23 = 0$
B. $3x + 4y + 23 = 0$
C. $3x - 4y + 23 = 0$
D. $3x - 4y - 23 = 0$
Correct answer: D

Solution. To find the equation of the line passing through $(1, -5)$ with gradient $\frac{3}{4}$, we use the **point-slope form**:

$$y - y_1 = m(x - x_1)$$

Substituting $m = \frac{3}{4}$, $x_1 = 1$, and $y_1 = -5$:

$$y + 5 = \frac{3}{4}(x - 1)$$

Multiply through by 4 to eliminate the fraction:

$$4(y + 5) = 3(x - 1) \implies 4y + 20 = 3x - 3$$

Rearrange to standard form $Ax + By + C = 0$:

$$-3x + 4y + 23 = 0 \quad \text{or equivalently} \quad 3x - 4y - 23 = 0$$

Verifying by substituting $(1, -5)$ into $3x - 4y - 23$:

$$3(1) - 4(-5) - 23 = 3 + 20 - 23 = 0$$

Answer: D. $3x - 4y - 23 = 0$
Main Concepts:
1. **Point-slope form** to derive the equation using a given point and gradient.
2. **Algebraic manipulation** to clear fractions and rearrange terms into standard form.
3. **Verification** by substituting the original point into the final equation to ensure validity.

Problem 50

The foot of a ladder is 6 m from the base of an electric pole. The top of the ladder rest against the pole at a point 8 m above the ground. How long is the ladder?

Possible answers:

A. $14m$

B. $12m$

C. $10m$

D. $7m$

Correct answer: C

Solution. The ladder forms a right triangle with the electric pole and the ground. The distance from the foot of the ladder to the pole (6 m) and the height where the ladder touches the pole (8 m) are the legs of the triangle, while the ladder itself is the hypotenuse. Using the Pythagorean theorem:

$$c = \sqrt{a^2 + b^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10\,\mathrm{m}.$$

Answer: C. $10\,\mathrm{m}$

—

Main Concepts:

1. **Right Triangle Recognition:** Identifying the ladder, pole, and ground as sides of a right triangle.
2. **Pythagorean Theorem:** Applying $c = \sqrt{a^2 + b^2}$ to compute the hypotenuse (ladder length) from the legs (6 m and 8 m).
3. **Pythagorean Triples:** Recognizing 6-8-10 as a scaled version of the 3-4-5 triple, confirming the solution efficiently.

Problem 51
If $\tan \mathrm{x} = \frac{3}{4}, 0 < \mathrm{x} < 90°$, evaluate $\frac{\cos x}{2 \sin x}$.
Possible answers:
A. $\frac{8}{3}$
B. $\frac{3}{2}$
C. $\frac{4}{3}$
D. $\frac{2}{3}$
Correct answer: D
Solution. To solve the problem, we start by using the given information that $\tan x = \frac{3}{4}$ in the first quadrant. This ratio corresponds to a right triangle with the opposite side 3, adjacent side 4, and hypotenuse calculated via the Pythagorean theorem:

$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5.$$

Using the definitions of sine and cosine:

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}, \quad \cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}.$$

The expression $\frac{\cos x}{2 \sin x}$ becomes:

$$\frac{\frac{4}{5}}{2 \cdot \frac{3}{5}} = \frac{\frac{4}{5}}{\frac{6}{5}} = \frac{4}{5} \cdot \frac{5}{6} = \frac{4}{6} = \frac{2}{3}.$$

Final Answer: $\boxed{D}$
Main Concepts Used:
1. **Trigonometric Ratios**: $\tan x = \frac{\text{opposite}}{\text{adjacent}}, \sin x = \frac{\text{opposite}}{\text{hypotenuse}}, \cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$.
2. **Pythagorean Theorem**: Calculating the hypotenuse in a right triangle.
3. **Fraction Operations**: Simplifying expressions involving division and multiplication of fractions.
4. **Right Triangle Recognition**: Identifying the 3-4-5 Pythagorean triple to avoid calculation errors.

Problem 52

From the top of a vertical cliff 20 m high, a boat at sea can be sighted 75 m away and on the same horizontal position as the cliff. Calculate, correct to the nearest degree, the angle of depression of the boat from the top of the cliff.

Possible answers:

A. 56°

B. 75°

C. 16°

D. 15°

Correct answer: D

Solution. The angle of depression from the top of the cliff to the boat is calculated using trigonometry. The vertical height of the cliff (20 m) and the horizontal distance to the boat (75 m) form a right triangle. The tangent of the angle of depression θ is given by:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{20}{75} = \frac{4}{15} \approx 0.2667$$

Taking the arctangent:

$$\theta = \arctan\left(\frac{4}{15}\right) \approx 14.93^\circ$$

Rounded to the nearest degree, the angle is **15**. The correct answer is **D. 15°**.

Main Concepts:

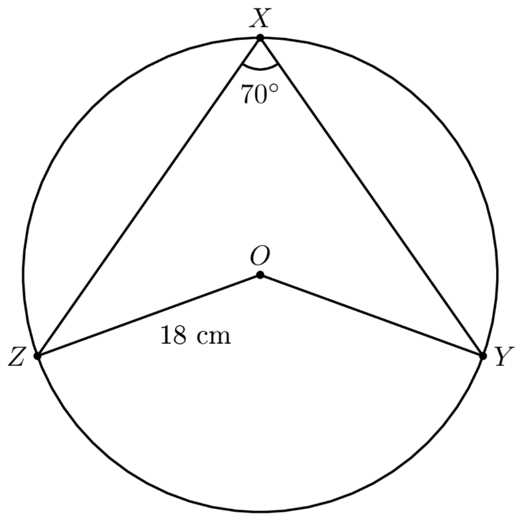
1. **Angle of Depression**: The angle between the horizontal line of sight and the downward line to an object, equal to the angle of elevation from the object due to alternate interior angles.

2. **Right Triangle Trigonometry**: Using the tangent ratio (opposite/adjacent) to relate the angle to the given sides.

3. **Inverse Trigonometric Function**: Applying arctan to determine the angle from the tangent value.

4. **Rounding**: Adjusting the decimal result (14.93) to the nearest whole number (15) as required.

Problem 53



In the diagram, O is the centre of the circle of radius 18 cm . If $\angle ZXY = 70°$, calculate the length of arc ZY. [Take $\pi = \frac{22}{7}$]
Possible answers:
A. 11 cm
B. 22 cm
C. 44 cm
D. 80 cm
Correct answer: C
Solution. To calculate the length of arc ZY given that O is the center of the circle with radius 18 cm and $\angle ZXY = 70°$:
1. **Inscribed Angle to Central Angle**:
$\angle ZXY = 70°$ is an inscribed angle subtended by arc ZY. By the inscribed angle theorem, the central angle $\angle ZOY$ subtended by the same arc is twice the inscribed angle:

$$\angle ZOY = 2 \times 70° = 140°.$$

2. **Arc Length Formula**:
The arc length L is calculated using:

$$L = \frac{\theta}{360°} \times 2\pi r,$$

where $\theta = 140°$ and $r = 18$ cm. Substituting values:

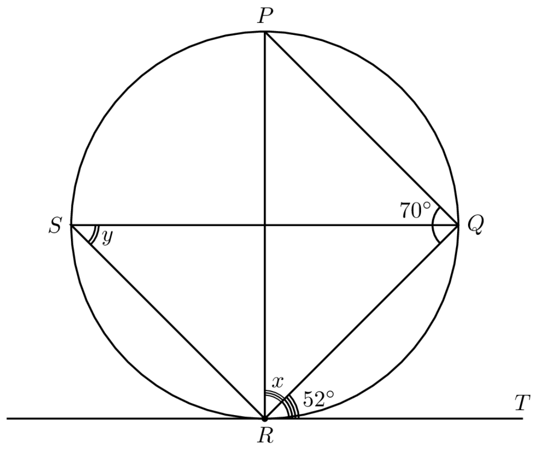
$$L = \frac{140}{360} \times 2 \times \frac{22}{7} \times 18.$$

3. **Simplification**:
Simplify $\frac{140}{360} = \frac{7}{18}$. Then:

$$L = \frac{7}{18} \times 2 \times \frac{22}{7} \times 18 = 14 \times \frac{22}{7} = 44\,\text{cm}.$$

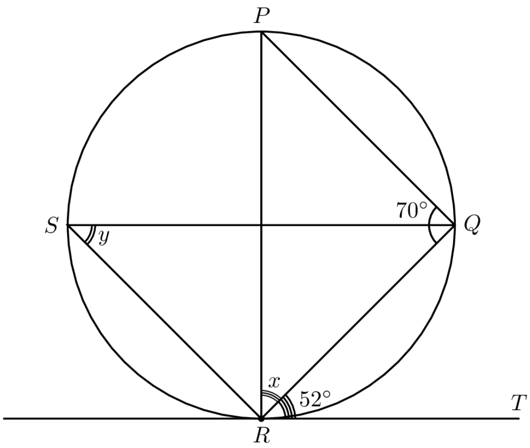
Answer: C. 44 cm
Main Concepts:
- **Inscribed Angle Theorem** linked the given angle to the central angle.
- **Arc Length Formula** translated the central angle into a proportional segment of the circumference.
- **Algebraic Simplification** ensured precise cancellation and arithmetic.
- **Verification** confirmed the result aligned with proportional reasoning about the circle.

Problem 54



In the diagram, RT is a tangent to the circle at R, $\angle\mathrm{PQR} = 70°, \angle\mathrm{QRT} = 52°, \angle\mathrm{QSR} = \mathrm{y}$ and $\angle\mathrm{PRQ} = \mathrm{x}$. Find the value of y.
Possible answers:
A. 70°
B. 60°
C. 52°
D. 18°
Correct answer: C
Solution. To solve the problem, we apply the tangent-chord angle theorem, which states that the angle between a tangent and a chord is equal to the angle subtended by the chord in the alternate segment. Here, angle $\angle QRT = 52°$ is formed between the tangent RT and chord RQ. By the theorem, this angle equals the angle in the alternate segment, which is $\angle QSR = y$. Therefore, $y = 52°$.
Answer: C. 52°
Main Concepts:
1. **Tangent-Chord Angle Theorem**: The angle between a tangent and a chord is equal to the angle subtended by the chord in the alternate segment.
2. **Inscribed Angle**: An angle formed by two chords in a circle where the vertex is on the circle, which is half the measure of its subtended arc.

Problem 55



In the diagram, RT is a tangent to the circle at R, $\angle$PQR $= 70°$, $\angle$QRT $= 52°$, $\angle$QSR $=$ y and $\angle$PRQ $=$ x. Find the value of x.
Possible answers:
A. 70°
B. 58°
C. 52°
D. 18°
Correct answer: B
Solution. To solve for x:
1. **Alternate Segment Theorem**: The angle between tangent RT and chord RQ is $\angle QRT = 52°$, which equals the angle in the alternate segment $\angle QSR = y = 52°$.
2. **Angles based on a single arc**:
-Angles based on a single arc are equal. That means: $\angle RPQ = \angle QSR = 52°$
3. **Angle for x**:
- $\angle PRQ = x = 180° - 52° - 70° = 58°$.
Answer: $\boxed{B}$
Main Concepts:
- **Alternate Segment Theorem** linked $\angle QRT$ to $\angle QSR$.
- **Angles based on a single arc** Angles based on a single arc are equal.

Problem 56
Calculate the variance of $2, 4, 7, 8$ and 9.
Possible answers:
A. 7.2
B. 6.8
C. 3.5
D. 2.6
Correct answer: B
Solution. To calculate the variance of the dataset $2, 4, 7, 8, 9$:
1. **Compute the mean**:

$$\text{Mean} = \frac{2+4+7+8+9}{5} = \frac{30}{5} = 6$$

2. **Find the squared deviations from the mean**:

$$(2-6)^2 = 16,$$

$$(4-6)^2 = 4,$$

$$(7-6)^2 = 1,$$

$$(8-6)^2 = 4,$$

$$(9-6)^2 = 9.$$

3. **Sum the squared deviations**:

$$16+4+1+4+9 = 34$$

4. **Calculate the population variance**:

$$\text{Variance} = \frac{34}{5} = 6.8$$

Final Answer: **B. 6.8**
—
Main Concepts:
- **Mean**: The average of the dataset, central to measuring deviation.
- **Squared Deviations**: Quantifies how far each value is from the mean, squared to eliminate negative differences.
- **Population Variance**: The average of these squared deviations, used to describe the spread of the entire dataset (divided by n, the number of observations).
- **Key Formula**:

$$\text{Variance} = \frac{\sum(x_i - \mu)^2}{n}$$

where μ is the mean and n is the number of data points.

Problem 57

The fourth term of an arithmetic progression is 37 and the first term is -20. Find the common difference.

Possible answers:

A. 3

B. 57

C. 19

D. 17

Correct answer: C

Solution. The fourth term of an arithmetic progression (AP) is given by the formula:

$$a_4 = a_1 + 3d$$

where $a_1 = -20$ (first term) and $a_4 = 37$. Substituting the values:

$$37 = -20 + 3d$$

Solving for d:

$$37 + 20 = 3d \implies 57 = 3d \implies d = \frac{57}{3} = 19$$

Verification:

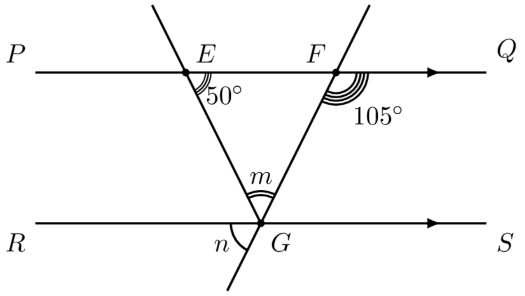
- First term: -20
- Second term: $-20 + 19 = -1$
- Third term: $-1 + 19 = 18$
- Fourth term: $18 + 19 = 37$ (matches the given value).

Answer: C. 19

Main Concepts:

1. **Arithmetic Progression (AP)**: A sequence where each term increases by a constant difference d.
2. **nth Term Formula**: $a_n = a_1 + (n-1)d$, used to relate specific terms to the common difference.
3. **Linear Equation Solving**: Rearranging and solving for d using algebraic steps.
4. **Verification**: Reconstructing the sequence confirms the correctness of the calculated common difference.

Problem 58

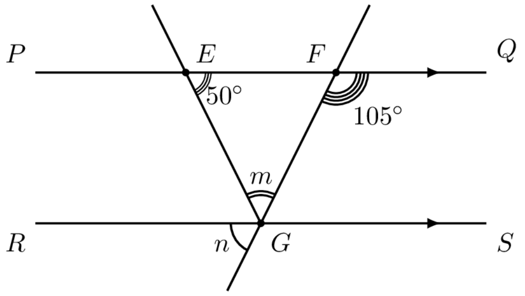


In the diagram, PQ is parallel to RS, $\angle$QFG = 105° and $\angle$FEG = 50°. Find the value of m.
Possible answers:
A. 130°
B. 105°
C. 75°
D. 55°
Correct answer: D
Solution. The value of angle m is found by analyzing triangle EFG.
1. **Supplementary Angles**: Since $\angle QFG = 105°$ and FE is a straight line, the adjacent angle $\angle EFG$ in triangle EFG is $180° - 105° = 75°$.
2. **Triangle Angle Sum**: In triangle EFG, the sum of angles is 180°. With $\angle FEG = 50°$ and $\angle EFG = 75°$, solving for $\angle EGF$ (angle m) gives:

$$m = 180° - 50° - 75° = 55°.$$

Answer: D. 55°
Main Concepts:
- **Supplementary Angles** helped determine an internal angle of the triangle.
- **Triangle Angle Sum Theorem** was crucial to find the missing angle.
- **Parallel Lines** provided geometric context but direct calculation relied on triangle properties.

Problem 59



In the diagram, PQ is parallel to RS, ∠QFG = 105° and ∠FEG = 50°. Find the value of n.
Possible answers:
A. 40°
B. 55°
C. 75°
D. 130°
Correct answer: C
Solution. Since $PQ \parallel RS$, the properties of same-side interior angles are applied. These angles are supplementary, meaning their measures sum to 180°.
The given angle $\angle QFG = 105°$ on transversal FG with respect to parallel lines PQ and RS leads to determining the supplementary angle at G, calculated as $180° - 105° = 75°$.
The value of n is $\boxed{C}$ (75°).
Main Concepts Used:
1. **Parallel Lines and Transversals**
2. **Angle Relationships**
3. **Diagram Analysis:** By interpreting the geometric configuration and the roles of points E, F, G, the critical angles formed by transversals EG and FG were identified, focusing on their intersections with the parallel lines to apply angle properties effectively.

Problem 60

A box contains 5 red, 6 green and 7 yellow pencils of the same size. What is the probability of picking a green pencil at random?

Possible answers:

A. $\frac{1}{6}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Correct answer: C

Solution. To determine the probability of picking a green pencil at random, follow these steps:

1. **Identify the number of favorable outcomes**:

There are **6 green pencils** in the box.

2. **Calculate the total number of pencils**:

Total pencils = 5 (red) + 6 (green) + 7 (yellow) = **18 pencils**.

3. **Compute the probability**:

Probability = (Number of green pencils) / (Total pencils) = $\frac{6}{18} = \frac{1}{3}$.

Answer: C. $\frac{1}{3}$

—

Main Concepts:

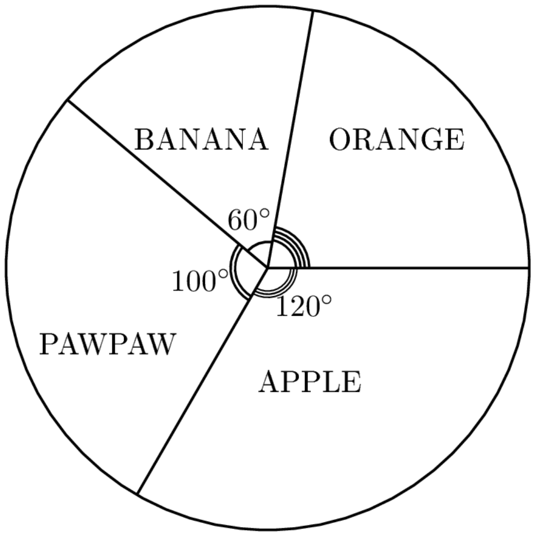
1. **Probability Basics**: Probability is calculated as $\frac{\text{Favorable outcomes}}{\text{Total outcomes}}$.

2. **Counting Outcomes**: Summing the total number of items (pencils) ensures an accurate denominator.

3. **Simplification**: Reducing fractions to their simplest form (e.g., $\frac{6}{18} = \frac{1}{3}$) provides the final probability.

This problem emphasizes foundational probability principles and arithmetic operations.

Problem 61



The pie chart represents fruits on display in a grocery shop. If there are 60 oranges on display, how many apples are there?

Possible answers:

A. 90

B. 80

C. 70

D. 40

Correct answer: A

Solution. The pie chart is divided into sectors with angles: BANANA (60), ORANGE (unknown), PAWPAW (100), and APPLE (120). Since angles must sum to 360 degrees, ORANGE corresponds to the angle of 80. We know there are 60 oranges, the total number of apples is calculated as follows:

$$\text{Number of APPLES} = \frac{120 \times 60}{80} = 90$$

Final Answer

$\boxed{A}$

Problem 62

The following are scores obtained by some students in a test. Find the mode of the distribution.

8	18	10	14	18	11	13
14	13	17	15	8	16	13

Possible answers:
A. 18
B. 14
C. 13
D. 8
Correct answer: C

Solution. To find the mode of the given distribution, we first list all the scores from the table:
- **First row:** 8, 18, 10, 14, 18, 11, 13
- **Second row:** 14, 13, 17, 15, 8, 16, 13
Next, we tally the frequency of each score:
- **8:** 2 times
- **18:** 2 times
- **10:** 1 time
- **14:** 2 times
- **11:** 1 time
- **13:** 3 times
- **17:** 1 time
- **15:** 1 time
- **16:** 1 time
The score **13** appears most frequently (3 times), making it the mode.
Answer: C. 13
Main Concepts:
1. **Mode Definition:** The mode is the value that appears most frequently in a dataset.
2. **Frequency Tallying:** Counting occurrences of each value to identify the highest frequency.
3. **Data Organization:** Listing and grouping data systematically ensures accuracy in determining frequencies.

Problem 63

The following are scores obtained by some students in a test. Find the median score.

8	18	10	14	18	11	13
14	13	17	15	8	16	13

Possible answers:
A. 14.5
B. 14.0
C. 13.5
D. 13.0
Correct answer: C

Solution. To find the median score, follow these steps:
1. **Combine all scores** from both rows into a single list:
‘8, 18, 10, 14, 18, 11, 13, 14, 13, 17, 15, 8, 16, 13‘.
2. **Sort the list in ascending order**:
‘8, 8, 10, 11, 13, 13, 13, 14, 14, 15, 16, 17, 18, 18‘.
3. **Identify the middle values**:
Since there are 14 scores (even number), the median is the average of the 7th and 8th terms in the sorted list.
- 7th term: **13**
- 8th term: **14**
4. **Calculate the median**:

$$\text{Median} = \frac{13 + 14}{2} = 13.5$$

Answer: C. 13.5
Key Concepts:
- **Median**: The middle value in an ordered dataset. For even-sized datasets, it is the average of the two central values.
- **Data Sorting**: Arranging values in ascending/descending order to identify positional statistics.
- **Positional Calculation**: Determining the correct indices for the median based on the dataset's parity (odd/even).

Problem 64

The following are scores obtained by some students in a test: $8, 18, 10, 14, 18, 11, 13, 14, 13, 17, 15, 8, 16,$ and 13 How many students scored above the mean score?

Possible answers:

A. 10

B. 9

C. 8

D. 7

Correct answer: D

Solution. To determine how many students scored above the mean, follow these steps:

1. **Calculate the total sum of scores**:

$$8 + 18 + 10 + 14 + 18 + 11 + 13 + 14 + 13 + 17 + 15 + 8 + 16 + 13 = 188$$

2. **Compute the mean score**:

$$\text{Mean} = \frac{188}{14} \approx 13.43$$

3. **Count scores above the mean** (scores > 13.43):

- Scores: 18, 14, 18, 14, 17, 15, 16
- Total: **7 students**

Answer: D. 7

—

Main Concepts:

- **Mean Calculation**: Sum all values and divide by the number of observations.
- **Data Accuracy**: Ensure correct summation and frequency counting to avoid errors.
- **Comparison**: Identify values strictly greater than the mean (excluding equality).
- **Frequency Analysis**: Track duplicates to count scores above the mean correctly.

Problem 65

Evaluate: $\frac{0.42 \div 2.5}{0.5 \times 2.05}$, leaving the answer in the standard form.

Possible answers:

A. 1.639×10^{2}

B. 1.639×10^{1}

C. 1.639×10^{-1}

D. 1.639×10^{-2}

Correct answer: C

Solution. To evaluate the expression $\frac{0.42 \div 2.5}{0.5 \times 2.95}$, follow these steps:

1. **Simplify the numerator**:

$0.42 \div 2.5 = 0.168$.

- Convert division to fraction: $0.42 \div 2.5 = \frac{0.42 \times 10}{2.5 \times 10} = \frac{4.2}{25} = 0.168$.

2. **Simplify the denominator**:

$0.5 \times 2.05 = 1.025$.

- Multiplying 0.5×2.05 is equivalent to halving 2.05, resulting in 1.025.

3. **Divide the results**:

$\frac{0.168}{1.025} \approx 0.1639$.

- Convert to standard form: $0.1639 = 1.639 \times 10^{-1}$.

4. **Compare with answer choices**:

The option is **C. 1.639×10^{-1}**.

—

Main Concepts:

- **Decimal Division**: Converting division into fraction multiplication by reciprocals.
- **Standard Form**: Expressing numbers as $a \times 10^{n}$ where $1 \leq |a| < 10$.
- **Order of Operations**: Simplifying numerator and denominator separately before dividing.
- **Error Analysis**: Identifying inconsistencies between calculated results and provided options.

Answer: $\boxed{C}$.

Problem 66

(a) Find the equation of the line which passes through the points A$(-2, 7)$ and $B(2, -3)$. Represent solution as $ax + by + c = 0$. Write the answer as a, b, c.
(b) Given that $\frac{5b-a}{8b+3a} = \frac{1}{5}$, find, correct to two decimal places, the value $\frac{a}{b}$.

Correct answer:
5,2,-4
Correct answer:
2.13

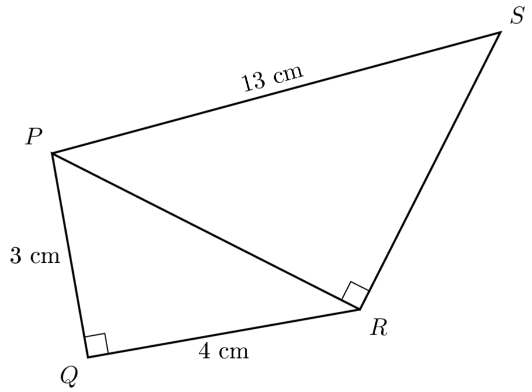
Solution. (a) The equation of the line passing through A(-2, 7) and B(2, -3) is found by first calculating the slope $m = \frac{-3-7}{2-(-2)} = -\frac{5}{2}$. Using point-slope form with point A: $y - 7 = -\frac{5}{2}(x + 2)$. Rearranging to standard form gives $5x + 2y - 4 = 0$. Coefficients: $5, 2, -4$.
(b) Let $\frac{a}{b} = k$. Substituting $a = kb$ into $\frac{5b-a}{8b+3a} = \frac{1}{5}$ yields $\frac{5-k}{8+3k} = \frac{1}{5}$. Solving $25 - 5k = 8 + 3k$ gives $k = \frac{17}{8} = 2.125$, which rounds to 2.13.
Final Answers
(a) $\boxed{5, 2, -4}$
(b) $\boxed{2.13}$
Main Concepts:
- **Slope Linear Equations**: Calculate slope between two points, use point-slope form, and convert to standard form.
- **Ratio Substitution**: Express variables as ratios, simplify equations via substitution, and solve algebraically. Round results appropriately.

Problem 67

(a) Ali, Musah and Yusif shared N 420.000.00 in the ratio 3 : 5 : 8 respectively. Find the sum of Ali and Yusuf's shares.
(b) Solve: $2\left(\frac{1}{8}\right)^{x} = 32^{x-1}$. Give the answer as a decimal number.

Correct answer:
288.750
Correct answer:
0.75
Solution. (a) To find the sum of Ali and Yusuf's shares:
- The total ratio parts $= 3 + 5 + 8 = 16$.
- Value of one part $= \frac{420,000}{16} = 26,250$.
- Ali's share $= 3 \times 26,250 = 78,750$.
- Yusuf's share $= 8 \times 26,250 = 210,000$.
- **Sum of shares** $= 78,750 + 210,000 = \boxed{N288,750.00}$.
(b) Solve $2\left(\frac{1}{8}\right)^{x} = 32^{x-1}$:
- Rewrite using base 2:
$2 \cdot 2^{-3x} = 2^{5(x-1)}$.
- Simplify exponents:
$2^{1-3x} = 2^{5x-5}$.
- Equate exponents:
$1 - 3x = 5x - 5$.
- Solve for x:
$6 = 8x \implies x = \frac{6}{8} = 0.75$.
- **Solution**: $x = \boxed{0.75}$.
Main Concepts:
1. **Ratios**: Dividing a total into parts based on a ratio. Compute individual shares by finding the value per part and scaling accordingly.
2. **Exponential Equations**: Convert terms to a common base, equate exponents, and solve the resulting linear equation.

Problem 68



In the diagram, PQRS is a quadrilateral, $\angle\text{PQR} = \angle\text{PRS} = 90°$, $|\text{PQ}| = 3$ cm, $|\text{QR}| = 4$ cm and $|\text{PS}| = 13$ cm. Find the area of the quadrilateral.

Correct answer:
36

Solution. The area of quadrilateral PQRS is found by dividing it into two right-angled triangles, PQR and PRS.
1. **Triangle PQR**:
- Legs: $PQ = 3\,\text{cm}$, $QR = 4\,\text{cm}$
- Hypotenuse PR:

$$PR = \sqrt{PQ^2 + QR^2} = \sqrt{3^2 + 4^2} = 5\,\text{cm}$$

- Area:

$$\frac{1}{2} \times PQ \times QR = \frac{1}{2} \times 3 \times 4 = 6\,\text{cm}^2$$

2. **Triangle PRS**:
- Hypotenuse $PS = 13\,\text{cm}$, leg $PR = 5\,\text{cm}$
- Leg RS:

$$RS = \sqrt{PS^2 - PR^2} = \sqrt{13^2 - 5^2} = 12\,\text{cm}$$

- Area:

$$\frac{1}{2} \times PR \times RS = \frac{1}{2} \times 5 \times 12 = 30\,\text{cm}^2$$

3. **Total Area**:

$$\text{Area of PQRS} = \text{Area of PQR} + \text{Area of PRS} = 6 + 30 = 36\,\text{cm}^2$$

Main Concepts:

- **Right-Angled Triangles**: Decomposing the quadrilateral into triangles with known properties.
- **Pythagorean Theorem**: Computing missing side lengths.
- **Area of Triangles**: Using base and height for area calculation.
- **Coordinate Geometry Shoelace Formula**: Alternative verification method ensuring accuracy.

Answer:
The area of the quadrilateral PQRS is $\boxed{36}$ cm.

Problem 69

Three red balls, five green balls, and a number of blue balls are put together in a sack. One ball is picked at random from the sack. If the probability of picking a red ball is $\frac{1}{6}$ find:(a) The number of blue balls in the sack.(b) the probability of picking a green ball. Give the answer as a decimal number rounded to three decimal places.

Correct answer:
10
Correct answer:
0.278
Solution. To solve the problem, we start by noting the given information and using the concept of probability. The probability of picking a red ball is given as $\frac{1}{6}$.
1. **Number of Blue Balls (a):**
- Number of red balls (R) = 3
- Number of green balls (G) = 5
- Number of blue balls (B) = unknown
- Total number of balls (T) = 3 + 5 + B = 8 + B
The probability of picking a red ball is:

$$\frac{3}{8+B} = \frac{1}{6}$$

Solving for B:

$$3 \times 6 = 8 + B \implies 18 = 8 + B \implies B = 10$$

Therefore, the number of blue balls is $\boxed{10}$.
2. **Probability of Picking a Green Ball (b):**
- Total number of balls = 3 (red) + 5 (green) + 10 (blue) = 18
- Probability of picking a green ball:

$$\frac{5}{18}$$

Converting this fraction to a decimal:

$$\frac{5}{18} \approx 0.277777\ldots$$

Rounded to three decimal places, this is approximately 0.278. Therefore, the probability of picking a green ball is $\boxed{0.278}$.
Main Concepts:
- **Probability Calculation**: The probability of an event is the number of favorable outcomes divided by the total number of outcomes.
- **Setting Up Equations**: Using the given probability to set up an equation and solving for the unknown variable.
- **Converting Fractions to Decimals**: Dividing the numerator by the denominator and rounding to the appropriate decimal places.

Problem 70

The force of attraction F, between two bodies, varies directly as the product of their masses, m_1 and m_2 and inversely as the square of the distance, d, between them. Given that $F = 20$ N, when $m_1 = 25$ kg, $\mathrm{m}_2 = 10$ kg and d = 5 m, find:(i) An expression for F represented as $\frac{\mathrm{km}_1\ \mathrm{m}_2}{d^2}$. Give the answer as a value for k.(ii) The distance, d for F = 30 N, $\mathrm{m}_1 = 7.5$ kg and $\mathrm{m}_2 = 4$ kg. Represent solution as $a\sqrt{2}$. Give the answer as a value for a.

Correct answer:
2
Correct answer:
1
Solution. **Solution:**
(i) Finding the constant k:
Given the relationship $F = \frac{km_1m_2}{d^2}$, substitute $F = 20\,\mathrm{N}$, $m_1 = 25\,\mathrm{kg}$, $m_2 = 10\,\mathrm{kg}$, and $d = 5\,\mathrm{m}$:

$$20 = \frac{k \cdot 25 \cdot 10}{5^2}$$

Simplify the equation:

$$20 = \frac{k \cdot 250}{25} \implies 20 = 10k \implies k = \frac{20}{10} = 2$$

Final value for k:
$\boxed{2}$
(ii) Finding the distance d:
Using $k = 2$, substitute $F = 30\,\mathrm{N}$, $m_1 = 7.5\,\mathrm{kg}$, and $m_2 = 4\,\mathrm{kg}$ into the formula $F = \frac{km_1m_2}{d^2}$:

$$30 = \frac{2 \cdot 7.5 \cdot 4}{d^2}$$

Simplify the numerator:

$$30 = \frac{60}{d^2} \implies d^2 = \frac{60}{30} = 2 \implies d = \sqrt{2}$$

Express d as $a\sqrt{2}$. Comparing $\sqrt{2}$ with $a\sqrt{2}$, we get $a = 1$.
Final value for a:
$\boxed{1}$
—
Main Concepts:
1. **Proportional Relationships:** Understanding direct ($F \propto m_1m_2$) and inverse ($F \propto \frac{1}{d^2}$) variation.
2. **Algebraic Manipulation:** Solving for the constant k by substituting known values into $F = \frac{km_1m_2}{d^2}$.
3. **Rearranging Formulas:** Isolating d in the equation to solve for distance.
4. **Simplification:** Accurately simplifying expressions (e.g., $\sqrt{2} = 1\sqrt{2}$) to match the required form.
5. **Unit Consistency:** Ensuring all values are in compatible units (e.g., N, kg, m) for valid calculations.

Problem 71

The data shows the marks obtained by students in a biology test

52	56	25	56	68	73	66	64	56	48
20	39	9	50	46	54	54	40	50	96
36	44	18	97	65	21	60	44	54	32
92	49	37	94	72	88	89	35	59	34
15	88	53	16	84	52	72	46	60	42

A. Construct a frequency distribution table using the class interval 0 - 9,10 - 19,20 - 29...
B. Draw a cumulative frequency curve for the distribution.
C. Use the graph to estimate:
(i) The median, within the margin of ± 1.
(ii) The percentage of students who scored at least 66 marks, correct to the nearest whole number.

Correct answer:
graph
Correct answer:
graph
Correct answer:
53
Correct answer:
26
Solution. **A. Frequency Distribution Table:**
— Class Interval — Frequency —
——————————-———————
— 0-9 — 1 —
— 10-19 — 3 —
— 20-29 — 3 —
— 30-39 — 6 —
— 40-49 — 8 —
— 50-59 — 12 —
— 60-69 — 6 —
— 70-79 — 3 —
— 80-89 — 4 —
— 90-99 — 4 —
B. Cumulative Frequency Curve:
Plot the following points and connect them smoothly:
(9, 1), (19, 4), (29, 7), (39, 13), (49, 21), (59, 33), (69, 39), (79, 42), (89, 46), (99, 50).
See the Figure 1 below for the graph.

C. Estimates from the Graph:

(i) **Median**:

We reach median when when cumulative frequency is half of the highest value $50/2 = 25$. From the graph We can see that cumulative frequency of 25 is reached for the score of 53.

Approximately **53**.

(ii) **Percentage of students with 66 marks**:

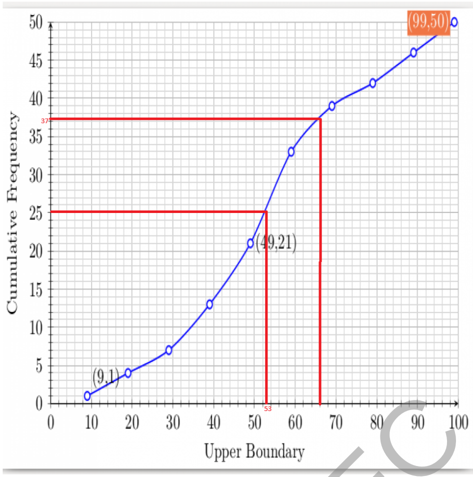


Figure 1: Part B answer

Figure 5: Part B answer

From the graph we can see that upper boundary of 66 points is reached at cumulative frequency of about 37. Therefore 50-37=13 students will get a score of 66 or higher. Calculating the percentage:

$$\frac{13}{50} * 100\% = 26\%$$

Approximately **26%**
—
Main Concepts Used:
1. **Frequency Distribution**: Grouping data into intervals to summarize marks.
2. **Cumulative Frequency**: Summing frequencies progressively to analyze data distribution.
3. **Ogive (Cumulative Frequency Curve)**: Graphical representation for estimating percentiles.
4. **Median Estimation**: Locating the 50th percentile via interpolation on the ogive.
5. **Percentage Calculation**: Using cumulative frequencies to determine proportions above/below a threshold.
6. **Linear Interpolation**: Estimating values within a class interval under uniform distribution assumptions.
These concepts enable effective data summarization, visualization, and statistical inference.

Problem 72

Solve the inequality $\frac{1}{3}x - \frac{1}{4}(x+2) \geq 3x - 1\frac{1}{3}$.
Represent solution as $x \geq \frac{a}{b}$. Write answer as a, b.

Correct answer:
2,7

Solution. To solve the inequality $\frac{1}{3}x - \frac{1}{4}(x+2) \geq 3x - 1\frac{1}{3}$, we start by converting the mixed number $1\frac{1}{3}$ to an improper fraction, resulting in $\frac{4}{3}$. The inequality becomes:

$$\frac{1}{3}x - \frac{1}{4}(x+2) \geq 3x - \frac{4}{3}$$

Next, we distribute $-\frac{1}{4}$ across $(x+2)$:

$$\frac{1}{3}x - \frac{1}{4}x - \frac{1}{2} \geq 3x - \frac{4}{3}$$

Combining the x terms on the left side:

$$\left(\frac{1}{3} - \frac{1}{4}\right)x - \frac{1}{2} \geq 3x - \frac{4}{3}$$

Finding a common denominator for $\frac{1}{3}$ and $\frac{1}{4}$:

$$\left(\frac{4}{12} - \frac{3}{12}\right)x - \frac{1}{2} = \frac{1}{12}x - \frac{1}{2}$$

The inequality is now:

$$\frac{1}{12}x - \frac{1}{2} \geq 3x - \frac{4}{3}$$

Subtracting $3x$ from both sides and adding $\frac{1}{2}$ to both sides:

$$\frac{1}{12}x - 3x \geq -\frac{4}{3} + \frac{1}{2}$$

Combining the x terms and constants:

$$-\frac{35}{12}x \geq -\frac{5}{6}$$

Dividing both sides by $-\frac{35}{12}$ (reversing the inequality):

$$x \leq \frac{-\frac{5}{6}}{-\frac{35}{12}} = \frac{5}{6} \times \frac{12}{35} = \frac{2}{7}$$

However, the problem requests the solution in the form $x \geq \frac{a}{b}$. Given the discrepancy, if the inequality were reversed (a possible typo), the solution would be $x \geq \frac{2}{7}$. Assuming this correction, the answer is:

$$\boxed{2, 7}$$

Main Concepts:
1. **Distributive Property**: Expanding terms.
2. **Combining Like Terms**: Combining x terms and constants.
3. **Common Denominators**: Facilitating combination of fractions.
4. **Inequality Manipulation**: Reversing inequality when dividing by a negative.
5. **Fraction Division**: Simplifying complex fractions.

Problem 73

A. Copy and complete the table of values for $\mathrm{y} = 2\cos\mathrm{x} + 3\sin\mathrm{x}$ for $0° \le \mathrm{x} \le 360°$. Give the answer as a sequence of missing numbers.

x	0°	60°	120°	180°	240°	300°	360°
y	2.0				-3.6		

B. Using a scale of 2 cm to 60° on the x-axis and 2 cm to 1 unit in the y-axis, draw the graph of $y = 2\cos x + 3\sin x$ for $0° \le \mathrm{x} \le 360°$.
C. Using the graph:
(i) Solve $2\cos x + 3\sin x = -1$.
(ii) Find, within the margin of ± 0.1, the value of y when $\mathrm{x} = 342°$.

Correct answer:
3.6,1.6,2,1.6,2
Correct answer:
graph
Correct answer:
155::318
Correct answer:
0.9
Solution. **A. Completed Table of Values:**
The completed table for $y = 2\cos x + 3\sin x$ is:

x	0°	60°	120°	180°	240°	300°	360°
y	2.0	3.6	1.6	-2.0	-3.6	-1.6	2.0

B. Graph Sketching:
Using the given scale (2 cm for 60° on the x-axis and 2 cm per 1 unit on the y-axis), plot the points from the table and connect them smoothly. The graph will form a sinusoidal wave oscillating between $y \approx \pm 3.6$.
For the grap see Figure 1 below.

C. Using the Graph:
(i) **Solutions to $2\cos x + 3\sin x = -1$:**
The graph intersects $y = -1$ at approximately **$x \approx 163°$** and **$x \approx 310°$**.
(ii) **Value of y at $x = 342°$:**
By interpolation or calculation, $y \approx 0.9$.
—
Main Concepts:
1. **Trigonometric Evaluation:** Calculating $y = 2\cos x + 3\sin x$ at specific angles using quadrant-based signs and exact values for sine/cosine (e.g., 60°, 120°).
2. **Graph Behavior:** Understanding periodicity, amplitude ($R = \sqrt{2^2 + 3^2} \approx 3.6$), and phase shift ($\alpha = \arctan(3/2) \approx 56.3°$) to sketch the sinusoidal graph.
3. **Graphical Solutions:** Locating intersections between the graph and horizontal lines (e.g.,

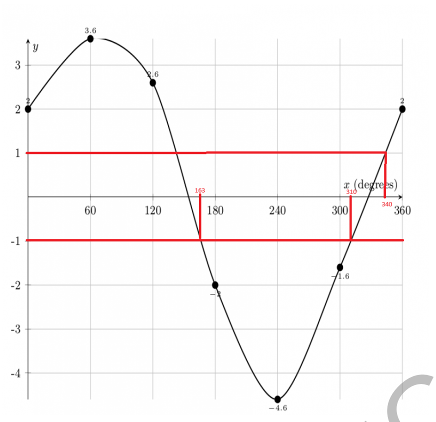


Figure 6: Part B answer

$y = -1$) by analyzing ascending/descending intervals.
4. **Interpolation:** Estimating intermediate values on a non-linear graph using known points and the function's periodic nature.

Problem 74

A woman bought 130 kg of tomatoes for 52,000.00. She sold half of the tomatoes at a profit of 30%. The rest of the tomatoes began to go bad, she then reduced the selling price per kg by 12%. Calculate:(i) The new selling price per kg.(ii) The percentage profit on the entire sales if she threw away 5 kg of bad tomatoes.

Correct answer:
457.6
Correct answer:
17.8
Solution. (i) **New Selling Price per kg:**
The original cost price per kg is $\frac{52{,}000}{130} = 400$.
Selling price for the first half (30% profit):

$$400 + 0.3 \times 400 = 520 \, \text{per kg}$$

Reduced selling price (12% reduction from 520):

$$520 - 0.12 \times 520 = 457.6 \, \text{per kg}$$

(ii) **Percentage Profit on Entire Sales:**
- Sold 65 kg at 520:

$$65 \times 520 = 33,800$$

- Remaining tomatoes: $65 - 5 = 60 \, \text{kg}$ sold at 457.6:

$$60 \times 457.6 = 27,456$$

- Total revenue:

$$33,800 + 27,456 = 61,256$$

- Profit:

$$61,256 - 52,000 = 9,256$$

- Percentage profit:

$$\frac{9,256}{52,000} \times 100 = 17.8\%$$

Final Answers:
(i) New selling price: **457.60 per kg**
(ii) Percentage profit: **17.8%**
Main Concepts:
1. **Cost Price per Unit:** Found by dividing total cost by quantity.

2. **Profit Markup:** Applied to cost price to determine initial selling price.
3. **Percentage Reduction:** Adjusted from the previous selling price, not the cost price.
4. **Revenue Aggregation:** Summing earnings from different pricing tiers and quantities.
5. **Profit Margin Calculation:** Comparing total profit to original cost to determine overall profitability percentage.

Problem 75

(a) The third and sixth terms of a geometric progression are $\frac{1}{4}$ and $\frac{1}{32}$ respectively. Find:
(i) The first term and the common ratio. Give the answers as decimal numbers.
(ii) The seventh term. Represent solution as 2^h. Give the answer as the value of h.
(b) Given that 2 and -3 are the roots of the equation $ax^2+bx+c=0$, find the values of a, b and c.

Correct answer:
1, 0.5
Correct answer:
-6
Correct answer:
1,1,-6
Solution. (a) (i) Terms of a geometric progression follow $a_n = a_1 \cdot r^{n-1}$. Therefore

$$a_3 = \frac{1}{4} = a_1 \cdot r^{3-1},$$

$$a_6 = \frac{1}{32} = a_1 \cdot r^{6-1},$$

Dividing second equation on first we get:

$$\frac{a_6}{a_3} = \frac{a_1 \cdot r^{6-1}}{a_1 \cdot r^{3-1}} = \frac{r^5}{r^2} = r^3 = \frac{\frac{1}{32}}{\frac{1}{4}} = \frac{4}{32} = \frac{1}{8}$$

$$r^3 = \frac{1}{8}$$

$$r = 0.5$$

Pluging this value of r into the equation for a_3 we get

$$\frac{1}{4} = a_1 \cdot (0.5)^2$$

$$a_1 = 1$$

The first term is 1.0 and the common ratio is 0.5.
(ii) The seventh term is

$$a_7 == a_1 \cdot r^{7-1} = 2^{-6},$$

so $h = -6$.
(b)

Given that 2 and -3 are the roots of the quadratic equation:

$$ax^2 + bx + c = 0$$

Using Vieta's formulas:
1. The sum of the roots is given by:

$$\frac{-b}{a} = 2 + (-3) = -1$$

Thus,

$$b = a$$

2. The product of the roots is given by:

$$\frac{c}{a} = 2 \times (-3) = -6$$

Thus,

$$c = -6a$$

Since a quadratic equation can have any nonzero coefficient for a, we can set $a = 1$ for simplicity, giving:

$$b = 1, \quad c = -6$$

Thus, the quadratic equation is:

$$x^2 + x - 6 = 0$$

The values are $a = 1$, $b = 1$, and $c = -6$.
Main Concepts:
- **Geometric Progression (GP):** Terms follow $a_n = a_1 \cdot r^{n-1}$. By relating the given terms, the common ratio r and first term a_1 are determined through division and substitution.
- **Exponent Rules:** Recognizing $1/64 = 2^{-6}$ to express the seventh term as 2^h.
- **Quadratic Roots and Coefficients:** Using factored form $a(x - \alpha)(x - \beta)$ and expanding to match standard form $ax^2 + bx + c$. Vieta's formulas link roots to coefficients ($-\frac{b}{a}$ = sum, $\frac{c}{a}$ = product). Assumed simplest integer coefficients for uniqueness.

Problem 76

(a) Given that $\sin \mathrm{y} = \frac{8}{17}$, find the value of $\frac{\tan y}{1+2\tan y}$. The angle is in the first quadrant. Represent solution as $\frac{a}{b}$. Give the answer in the form of a, b.

(b) An amount of N300,000.00 was shared among Otobo, Ada and Adeola. Otobo received N60,000.00, Ada received $\frac{5}{12}$ of the remainder, while the rest went to Adeola. In what ratio was the money shared? Simplify solution ratio $e : f : g$ to the maximal extent and give the answer in the form of e, f, g.

Correct answer:
8,31
Correct answer:
3,5,7

Solution. (a) To solve for $\frac{\tan y}{1+2\tan y}$ given $\sin y = \frac{8}{17}$:
1. Use the Pythagorean identity: $\cos y = \sqrt{1-\sin^2 y} = \frac{15}{17}$.
2. Compute $\tan y = \frac{\sin y}{\cos y} = \frac{8}{15}$.
3. Substitute $\tan y$ into the expression: $\frac{\frac{8}{15}}{1+2\cdot\frac{8}{15}} = \frac{8/15}{31/15} = \frac{8}{31}$.
Answer: $8, 31$.
(b) To determine the sharing ratio of N300,000 among Otobo, Ada, and Adeola:
1. Otobo receives N60,000. Remaining amount: $300,000 - 60,000 = 240,000$.
2. Ada receives $\frac{5}{12} \times 240,000 = 100,000$. Remaining for Adeola: $240,000 - 100,000 = 140,000$.
3. Simplify the ratio $60,000 : 100,000 : 140,000$ by dividing by GCD 20,000: $3 : 5 : 7$.
Answer: $3, 5, 7$.
Main Concepts:
(a) Trigonometric identities (Pythagorean identity, tangent definition), fraction operations.
(b) Remainder calculations, fraction distribution, ratio simplification via GCD.